BagFlip: A Certified Defense against Data Poisoning

Abstract

Machine learning models are vulnerable to data-poisoning attacks, in which an attacker maliciously modifies the training set to change the prediction of a learned model. In a *trigger-less* attack, the attacker can modify the training set but not the test inputs, while in a *backdoor* attack the attacker can also modify test inputs. Existing model-agnostic defense approaches either cannot handle backdoor attacks or do not provide effective certificates (i.e., a proof of a defense). We present BagFlip, a model-agnostic certified approach that can effectively defend against both trigger-less and backdoor attacks. We evaluate BagFlip on image classification and malware detection datasets. BagFlip is equal to or more effective than the state-of-the-art approaches for trigger-less attacks and more effective than the state-of-the-art approaches for backdoor attacks.

1 Introduction

Recent works have shown that machine learning models are vulnerable to data-poisoning attacks, where the attackers maliciously modify the training set to influence the prediction of the victim model as they desire. In a *trigger-less* attack [42, 31, 24, 3, 1, 11], the attacker can modify the training set but not the test inputs, while in a *backdoor* attack [29, 34, 41, 30, 5, 21, 25, 27] the attacker can also modify test inputs. Effective attack approaches have been proposed for various domains such as image recognition [12], sentiment analysis [25], and malware detection [30].

Consider the malware detection setting. An attacker can modify the training data by adding a special signature—e.g., a line of code—to a set of benign programs. The idea is to make the learned model correlate the presence of the signature with benign programs. Then, the attacker can sneak a piece of malware past the model by including the signature, fooling the model into thinking it is a benign program. Indeed, it has been shown that if the model is trained on a dataset with only a few poisoned examples, a backdoor signature can be installed in the learned model [30, 27]. Thus, data-poisoning attacks are of great concern to the safety and security of machine learning models and systems, particularly as training data is gathered from different sources, e.g., via web scraping.

Ideally, a defense against data poisoning should fulfill the following desiderata: (1) Construct **effective certificates** (proofs) of the defense. (2) Defend against **both trigger-less and backdoor attacks**. (3) Be **model-agnostic**. It is quite challenging to fulfill all three desiderata; indeed, existing techniques are forced to make tradeoffs. For instance, empirical approaches [10, 40, 20, 26, 36, 13, 33, 9] cannot construct certificates and are likely to be bypassed by new attack approaches [32, 37, 16]. Some certified approaches [39, 35] provide *ineffective* certificates for both trigger-less and backdoor attacks. Some certified approaches [15, 19, 4, 28, 22, 38] provide effective certificates for trigger-less attacks but not backdoor attacks. Other certification approaches [14, 7, 23] are restricted to specific learning algorithms, e.g., decision trees.

This paper proposes BagFlip, a *model-agnostic certified* approach that uses *randomized smoothing* [6, 8, 18] to effectively defend against both trigger-less and backdoor attacks (Section 4). BagFlip uses a

Submitted to 36th Conference on Neural Information Processing Systems (NeurIPS 2022). Do not distribute.

novel smoothing distribution that combines *bagging* of the training set and noising of the training data and the test input by randomly *flipping* features and labels.

Although both bagging-based and noise-based approaches have been proposed independently in the literature, combining them makes it challenging to compute the *certified radius*, i.e., the amount of poisoning a learning algorithm can withstand without changing its prediction.

To compute the certified radius precisely, we use the Neyman–Pearson lemma to the sample space of BagFlip's smoothing distribution. This lemma requires partitioning the outcomes in the sample space into subspaces such that the likelihood ratio in each subspace is a constant. However, because our sample space is exponential in the number of features, we cannot naively apply the lemma. To address this problem, we exploit properties of our smoothing distribution to design an efficient algorithm that partitions the sample space into polynomially many subspaces (Section 5). Furthermore, we present a *relaxation* of the Neyman–Pearson lemma that further speeds up the computation (Section 6). Our evaluation against existing approaches shows that BagFlip is comparable to them or more effective for trigger-less attacks and more effective for backdoor attacks (Section 7).

2   Related Work

Our *model-agnostic* approach BagFlip uses randomized smoothing to compute *effective certificates* for *both* trigger-less and backdoor attacks. BagFlip focuses on feature-and-label-flipping perturbation functions that modify training examples and test inputs. We discuss how existing approaches differ.

Some model-agnostic certified approaches can defend against both trigger-less and backdoor attacks but cannot construct effective certificates. Wang et al. [35], Weber et al. [39] defended feature-flipping poisoning attacks and l_2-norm poisoning attacks, respectively. However, their certificates are practically ineffective due to the curse of dimensionality [17].

Some model-agnostic certified approaches construct effective certificates but cannot defend against backdoor attacks. Jia et al. [15] proposed to defend against *general* trigger-less attacks, i.e., the attackers can add/delete/modify examples in the training dataset, by bootstrap aggregating (bagging). Chen et al. [4] extended bagging by designing other selection strategies, e.g., selecting without replacement and with a fixed probability. Rosenfeld et al. [28] defended against label-flipping attacks and instantiated their framework on linear classifiers. Differential privacy [22] can also provide probabilistic certificates for trigger-less attacks, but it cannot handle backdoor attacks, and its certificates are ineffective. Levine and Feizi [19] proposed a deterministic partition aggregation (DPA) to defend against general trigger-less attacks by partitioning the training dataset using a secret hash function. Wang et al. [38] further improved DPA by introducing a spread stage.

Other certified approaches construct effective certificates but are not model-agnostic. Jia et al. [14] provided deterministic certificates *only* for nearest neighborhood classifiers (kNN/rNN), but for both trigger-less and backdoor attacks. Meyer et al. [23], Drews et al. [7] provided deterministic certificates *only* for decision trees but against general trigger-less attacks.

3   Problem Definition

We take a holistic view of training and inference as a single deterministic algorithm A. Given a dataset $D = \{(\mathbf{x}_1, y_1), \ldots, (\mathbf{x}_n, y_n)\}$ and a (test) input $\mathbf{x}$, we write $A(D, \mathbf{x})$ to denote the prediction of algorithm A on the input $\mathbf{x}$ after being trained on dataset D.

We are interested in certifying that the algorithm will still behave "well" after training on a tampered dataset. Before describing what "well" means, to define our problem we need to assume a *perturbation space*—i.e., what possible changes the attacker could make to the dataset. Given a pair $(\mathbf{x}, y)$, we write $\pi(\mathbf{x}, y)$ to denote the set of perturbed examples that an attacker can transform the example $(\mathbf{x}, y)$ into. Given a dataset D and a *radius* $r \geq 0$, we define the *perturbation space* as the set of datasets that can be obtained by modifying up to r examples in D using the perturbation π:

$$S_r^\pi(D) = \left\{ \{(\widetilde{\mathbf{x}}_i, \widetilde{y}_i)\}_i \mid \forall i.\, (\widetilde{\mathbf{x}}_i, \widetilde{y}_i) \in \pi(\mathbf{x}_i, y_i),\ \sum i = 1^n \mathbb{1}_{(\widetilde{\mathbf{x}}_i, \widetilde{y}_i) \neq (\mathbf{x}_i, y_i)} \leq r \right\}$$

Threat models. We consider two attacks: one where an attacker can perturb only the training set (a *trigger-less* attack) and one where the attacker can perturb both the training set and the test input (a

backdoor attack). We assume a backdoor attack scenario where test input perturbation is the same as training one. If these two perturbation spaces can perturb examples differently, we can always over-approximate them by their union. In the following definitions, we assume that we are given a perturbation space π, a radius $r \geq 0$, and a benign training dataset D and test input $\mathbf{x}$.

We say that algorithm A is robust to a **trigger-less attack** on the test input $\mathbf{x}$ if A yields the same prediction on the input $\mathbf{x}$ when trained on any perturbed dataset $\widetilde{D}$ and the benign dataset D. Formally,

$$\forall \widetilde{D} \in S_r^\pi(D). \ A(\widetilde{D}, \mathbf{x}) = A(D, \mathbf{x}) \tag{1}$$

We say that algorithm A is robust to a **backdoor attack** on the test input $\mathbf{x}$ if the algorithm trained on any perturbed dataset $\widetilde{D}$ produces the prediction $A(D, \mathbf{x})$ on any perturbed input $\widetilde{\mathbf{x}}$. Let $\pi(\mathbf{x}, y)_1$ denote the projection of the perturbation space $\pi(\mathbf{x}, y)$ onto the feature space (the attack can only backdoor features of the test input). Robustness to a backdoor attack is defined as

$$\forall \widetilde{D} \in S_r^\pi(D), \ \widetilde{\mathbf{x}} \in \pi(\mathbf{x}, y)_1. \ A(\widetilde{D}, \widetilde{\mathbf{x}}) = A(D, \mathbf{x}) \tag{2}$$

Given a large enough radius r, an attacker can always change enough examples and succeed at breaking robustness for either kind of attack. Therefore, we will focus on computing the maximal radius r, for which we can prove that Eq 1 and 2 hold for a given perturbation function π. We refer to this quantity as the *certified radius*. It is infeasible to prove Eq 1 and 2 by enumerating all possible $\widetilde{D}$ because $S_r^\pi(D)$ can be ridiculously large, e.g., $|S_r^\pi(D)| > 10^{30}$ when $|D| = 1000$ and $r = 10$.

Defining the perturbation function. We have not yet specified the perturbation function π, i.e., how the attacker can modify examples. In this paper, we focus on the following perturbation spaces.

Given a bound $s \geq 0$, a **feature-and-label-flipping perturbation**, FL_s, allows the attacker to modify the values of up to s features and the label in an example $(\mathbf{x}, y)$, where $\mathbf{x} \in [K]^d$ (i.e., $\mathbf{x}$ is a d-dimensional feature vector with each dimension having $\{0, 1, \ldots, K\}$ categories). Formally,

$$\text{FL}_s(\mathbf{x}, y) = \{(\widetilde{\mathbf{x}}, y') \mid \|\mathbf{x} - \widetilde{\mathbf{x}}\|_0 + \mathbb{1}_{y \neq y'} \leq s\},$$

There are two special cases of FL_s, a **feature-flipping perturbation** F_s and a **label-flipping perturbation** L. Given a bound $s \geq 0$, F_s allows the attacker to modify the values of up to s features in the input $\mathbf{x}$ but not the label. L only allows the attacker to modify the label of a training example. Note that L cannot modify the test input's features, so it can only be used in trigger-less attacks.

Example 3.1. *If D is a binary-classification image dataset, where each pixel is either black or white, then the perturbation function F_1 assumes the attacker can modify up to one pixel per image.*

The goal of this paper is to design a certifiable algorithm that can defend against trigger-less and backdoor attacks (Section 4) by computing the certified radius (Sections 5 and 6). Given a benign dataset D, our algorithm certifies that an attacker can perturb D by some amount (the certified radius) without changing the prediction. Symmetrically, if we suspect that D is poisoned, our algorithm certifies that even if an attacker had not poisoned D by up to the certified radius, the prediction would have been the same. The two views are equivalent, and we use the former in the paper.

4 BagFlip: Dual-Measure Randomized Smoothing

Our approach, which we call BagFlip, is a model-agnostic certification technique. Given a learning algorithm A, we want to automatically construct a new learning algorithm $\bar{A}$ with certified poisoning-robustness guarantees (Eqs. (1) and (2)). To do so, we adopt and extend the framework of *randomized smoothing*. Initially used for test-time robustness, randomized smoothing robustifies a function f by carefully constructing a *noisy* version $\bar{f}$ and theoretically analyzing the guarantees of $\bar{f}$.

Our approach, BagFlip, constructs a noisy algorithm $\bar{A}$ by randomly perturbing the training set and the test input and invoking A on the result. Formally, if we have a set of output labels $\mathcal{C}$, we define the *smoothed* learning algorithm $\bar{A}$ as follows:

$$\bar{A}(D, \mathbf{x}) \triangleq \underset{y \in \mathcal{C}}{\text{argmax}} \ \underset{\dot{D}, \dot{\mathbf{x}} \sim \mu(D, \mathbf{x})}{\Pr} (A(\dot{D}, \dot{\mathbf{x}}) = y), \tag{3}$$

where μ is a carefully designed probability distribution—the *smoothing distribution*—over the training set and the test input ($\dot{D}, \dot{\mathbf{x}}$ are a sampled dataset and a test input from the smoothing distribution).

We present two key contributions that enable BagFlip to efficiently certify up to large poisoning radii. First, we define a smoothing distribution μ that combines *bagging* [15] of the training set D, and noising [35] of the training set and the test input $\mathbf{x}$ (done by randomly *flipping* features and labels). This combination, described below, allows us to defend against trigger-less and backdoor attacks. However, this combination also makes it challenging to compute the certified radius due to the combinatorial explosion of the sample space. Our second contribution is a partition strategy for the Neyman–Pearson lemma that results in an efficient certification algorithm (Section 5), as well as a relaxation of the Neyman–Pearson lemma that further speeds up certification (Section 6).

Smoothing distribution. We describe the smoothing distribution μ that defends against feature and label flipping. Given a dataset $D = \{(\mathbf{x}_i, y_i)\}$ and a test input $\mathbf{x}$, sampling from $\mu(D, \mathbf{x})$ generates a random dataset and test input in the following way: (1) Uniformly select k examples from D with replacement and record their indices as $w_1, \ldots, w_k$. (2) Modify each selected example $(\mathbf{x}_{w_i}, y_{w_i})$ and the test input $\mathbf{x}$ to $(\mathbf{x}'_{w_i}, y'_{w_i})$ and $\mathbf{x}'$, respectively, as follows: For each feature, with probability $1 - \rho$, uniformly change its value to one of the other categories in $\{0, \ldots, K\}$. Randomly modify labels y_{w_i} in the same way. In other words, each feature will be *flipped* to another value from the domain with probability $\gamma \triangleq \frac{1-\rho}{K}$, where $\rho \in [0, 1]$ is the parameter controlling the noise level.

For F_s (resp. L), we simply do not modify the labels (resp. features) of the k selected examples.

Example 4.1. *Suppose we defend against F_1 in a trigger-less setting, then the distribution μ will not modify labels or the test input. Let $\rho = \frac{4}{5}$, $k = 1$. Following Example 3.1, let the binary-classification image dataset $D = \{(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2)\}$, where each image contains only one pixel. Then, one possible element of $\mu(D, \mathbf{x})$ can be the pair $(\{(\mathbf{x}'_2, y_2)\}, \mathbf{x})$, where $\mathbf{x}'_2 = \mathbf{x}_2$. The probability of this element is $\frac{1}{2} \times \rho = 0.4$ because we uniformly select one out of two examples and do not flip any feature, that is, the single feature retains its original value.*

5   A Precise Approach to Computing the Certified Radius

In this section, we show how to compute the certified radius of the smoothed algorithm $\bar{A}$ given a dataset D, a test input $\mathbf{x}$, and a perturbation function π. We focus on binary classification and provide the multi-class case in Appendix B.

Suppose that we have computed the prediction $y^* = \bar{A}(D, \mathbf{x})$. We want to show how many examples we can perturb in D to obtain any other $\widetilde{D}$ so the prediction remains y^*. Specifically, we want to find the largest possible radius r such that

$$\forall \widetilde{D} \in S_r^\pi(D), \; \widetilde{\mathbf{x}} \in \pi(\mathbf{x}, y)_1. \Pr_{\dot{D}, \dot{\mathbf{x}} \sim \mu(\widetilde{D}, \widetilde{\mathbf{x}})} (A(\dot{D}, \dot{\mathbf{x}}) = y^*) > 0.5 \tag{4}$$

We first show how to *certify* that Eq 4 holds for a given r and then rely on binary search to compute the largest r, i.e., the certified radius.[1] In Section 5.1, we present the Neyman–Pearson lemma to certify Eq 4 as it is a common practice in randomized smoothing. In Section 5.2, we show how to compute the certified radius for the distribution μ and the perturbation functions F_s, FL_s, and L.

5.1   The Neyman–Pearson Lemma

Hereinafter, we simplify the notation and use o to denote the pair $(\dot{D}, \dot{\mathbf{x}})$ and $A(o)$ to denote the prediction of the algorithm training and evaluating on o. We further simplify the distribution $\mu(D, \mathbf{x})$ as μ and the distribution $\mu(\widetilde{D}, \widetilde{\mathbf{x}})$ as $\widetilde{\mu}$. We define the performance of the smoothed algorithm $\bar{A}$ on dataset D, i.e., the probability of predicting y^*, as $p^* = \Pr_{o \sim \mu}(A(o) = y^*)$.

The challenge of certifying Eq 4 is that we cannot directly estimate the performance of the smoothed algorithm on the perturbed data, i.e., $\Pr_{o \sim \widetilde{\mu}}(A(o) = y^*)$, because $\widetilde{\mu}$ is universally quantified. To address this problem, we use the Neyman–Pearson lemma to find a lower bound lb for $\Pr_{o \sim \widetilde{\mu}}(A(o) = y^*)$. We do so by constructing a worst-case algorithm $\bar{A}^?$ and distribution $\widetilde{\mu}$. Specifically, we minimize $\Pr_{o \sim \widetilde{\mu}}(A^?(o) = y^*)$ while maintaining the algorithm's performance on μ, i.e., keeping

---

[1]It is difficult to get a closed-form solution of the certified radius (as done in [15, 39]) in our setting because the distribution of BagFlip is complicated. We rely on binary search as it is also done in Wang et al. [35], Lee et al. [18]. Appendix D shows a *loose* closed-form bound on the certified radius by KL-divergence [8].

$\Pr_{o\sim\mu}(A^?(o)=y^*)=p^*$. We use $\mathcal{A}$ to denote the set of all possible algorithms. We formalize the computation of the lower bound lb as the following constrained minimization objective:

$$\text{lb} \triangleq \min_{A^? \in \mathcal{A}} \Pr_{o\sim\widetilde{\mu}}(A^?(o)=y^*) \quad s.t. \; \Pr_{o\sim\mu}(A^?(o)=y^*)=p^* \text{ and } \widetilde{D} \in S_r^\pi(D), \widetilde{\mathbf{x}} \in \pi(\mathbf{x},y)_1 \quad (5)$$

It is easy to see that lb is the lower bound of $\Pr_{o\sim\widetilde{\mu}}(A(o)=y^*)$ in Eq. 4 because $\bar{A} \in \mathcal{A}$ and $\bar{A}$ satisfies the minimization constraint. Thus, $\text{lb} > 0.5$ implies the correctness of Eq. 4.

We show how to construct $\bar{A}^?$ and $\widetilde{\mu}$ greedily. For each outcome $o = (\dot{D}, \dot{\mathbf{x}})$ in the sample space Ω—i.e., the set of all possible sampled datasets and test inputs—we define the likelihood ratio of o as $\eta(o) = p_\mu(o)/p_{\widetilde{\mu}}(o)$, where p_μ and $p_{\widetilde{\mu}}$ are the PMFs of μ and $\widetilde{\mu}$, respectively.

The key idea is as follows: We partition Ω into finitely many disjoint subspaces $\mathcal{L}_1, \ldots, \mathcal{L}_m$ such that the likelihood ratio in each subspace $\mathcal{L}_i$ is some constant $\eta_i \in [0, \infty]$, i.e., $\forall o \in \mathcal{L}_i . \eta(o) = \eta_i$. We can sort and reorder the subspaces by likelihood ratios such that $\eta_1 \geq \ldots \geq \eta_m$. We denote the probability mass of μ on subspace $\mathcal{L}_i$ as $p_\mu(\mathcal{L}_i)$.

Example 5.1. *Suppose* $p_\mu(o_1)=\frac{4}{10}, p_{\widetilde{\mu}}(o_1)=\frac{4}{10}, p_\mu(o_2)=\frac{1}{10}, p_{\widetilde{\mu}}(o_2)=\frac{1}{10}, p_\mu(o_3)=\frac{4}{10}, p_{\widetilde{\mu}}(o_3)=\frac{1}{10},$ $p_\mu(o_4)=\frac{1}{10}, p_{\widetilde{\mu}}(o_4)=\frac{4}{10}.$ *We can partition* o_1 *and* o_2 *into one subspace* $\mathcal{L}$ *because* $\eta(o_1)=\eta(o_2)=1.$

The construction of the $\bar{A}^?$ that minimizes Eq. 5 is a greedy process, which iteratively assigns $A^?(o) = y^*$ for $\mathcal{L}_1, \mathcal{L}_2, \ldots$ until the budget p^* is met. The worst-case $\widetilde{\mu}$ can also be constructed greedily by maximizing the top-most likelihood ratios, and we can prove that the worst-case happens when the difference between μ and $\widetilde{\mu}$ is maximized, i.e., $\widetilde{D}$ and $\widetilde{\mathbf{x}}$ are maximally perturbed. The following theorem adapts the Neyman–Pearson lemma to our setting.

Theorem 5.1 (Neyman–Pearson Lemma for $\text{FL}_s, \text{F}_s, \text{L}$). *Let* $\widetilde{D}$ *and* $\widetilde{\mathbf{x}}$ *be a maximally perturbed dataset and test input, i.e.,* $|\widetilde{D} \setminus D| = r$, $\|\widetilde{\mathbf{x}} - \mathbf{x}\|_0 = s$, *and* $\|\widetilde{\mathbf{x}}_i - \mathbf{x}_i\|_0 + \mathbb{1}_{\widetilde{y}_i \neq y_i} = s$, *for each perturbed example* $(\widetilde{\mathbf{x}}_i, \widetilde{y}_i)$ *in* $\widetilde{D}$. *Let* $i_{\text{lb}} \triangleq \arg\min_{i\in[1,m]} \sum_{j=1}^i p_\mu(\mathcal{L}_j) \geq p^*$. *Then,* $\text{lb} = \sum_{j=1}^{i_{\text{lb}}-1} p_{\widetilde{\mu}}(\mathcal{L}_j) + \left(p^* - \sum_{j=1}^{i_{\text{lb}}-1} p_\mu(\mathcal{L}_j)\right)/\eta_{i_{\text{lb}}}.$

We say that lb is *tight* due to the existence of the minimizer $\bar{A}^?$ and $\widetilde{\mu}$ (see Appendix B).

Example 5.2. *Following Example 5.1, suppose* $\Omega = \{o_1, \ldots, o_4\}$ *and we partition it into* $\mathcal{L}_1 = \{o_3\}, \mathcal{L}_2 = \{o_1, o_2\}, \mathcal{L}_3 = \{o_4\}$, *and* $\eta_1 = 4, \eta_2 = 1, \eta_3 = 0.25$. *Let* $p^* = 0.95$, *then we have* $i_{\text{lb}} = 3$ *and* $\text{lb} = 0.1 + 0.5 + (0.95 - 0.4 - 0.5)/0.25 = 0.8$.

5.2 Computing the Certified Radius of BagFlip

Computing the certified radius boils down to computing lb in Eq 5 using Theorem 5.1. To compute lb for BagFlip, we address the following two challenges: 1) The argmin of computing i_{lb} and the summation of lb in Theorem 5.1 depend on the number of subspaces $\mathcal{L}_j$'s. We design a **partition strategy** that partitions Ω into a polynomial number of subspaces ($O(k^2 d)$). Recall that k is the size of the bag sampled from D and d is the number of features. 2) In Theorem 5.1, lb depends on $p_\mu(\mathcal{L}_j)$, which according to its definition (Eq 8) can be computed in exponential time ($O(kd^{2k})$). We propose an **efficient algorithm** that computes these two quantities in polynomial time ($O(d^3 + k^2 d^2)$).

We first show how to address these challenges for F_s and then show how to handle FL_s and L.

Partition strategy. We partition the large sample space Ω into disjoint subspaces $\mathcal{L}_{c,t}$ that depend on c, the number of perturbed examples in the sampled dataset, and t, the number of features the clean data and the perturbed data differ on with respect to the sampled data. Intuitively, c takes care of the bagging distribution and t takes care of the feature-flipping distribution. Formally,

$$\mathcal{L}_{c,t} = \{(\{(\mathbf{x}'_{w_i}, y_{w_i})\}_i, \mathbf{x}') \mid$$

$$\sum_{i=1}^k \mathbb{1}_{\mathbf{x}_{w_i} \neq \widetilde{\mathbf{x}}_{w_i}} = c, \quad (6)$$

$$\underbrace{\left(\sum_{i=1}^k \|\mathbf{x}'_{w_i} - \mathbf{x}_{w_i}\|_0 + \|\mathbf{x}' - \mathbf{x}\|_0\right)}_{\Delta} - \underbrace{\left(\sum_{i=1}^k \|\mathbf{x}'_{w_i} - \widetilde{\mathbf{x}}_{w_i}\|_0 + \|\mathbf{x}' - \widetilde{\mathbf{x}}\|_0\right)}_{\widetilde{\Delta}} = t\} \quad (7)$$

Eq 6 means that there are c perturbed indices sampled in o. Δ (and $\widetilde{\Delta}$) in Eq 7 counts how many features the sampled and the clean data (the perturbed data) differ on. The number of possible subspaces $\mathcal{L}_{c,t}$, which are disjoint by definition, is $O(k^2 d)$ because $0 \le c \le k$ and $|t| \le (k+1)d$.

The next theorems show how to compute the likelihood ratio of $\mathcal{L}_{c,t}$ and $p_\mu(\mathcal{L}_{c,t})$.

Theorem 5.2 (Compute $\eta_{c,t}$). $\eta_{c,t} = p_\mu(\mathcal{L}_{c,t})/p_{\widetilde{\mu}}(\mathcal{L}_{c,t}) = (\gamma/\rho)^t$, where γ and ρ are parameters controlling the noise level in BagFlip's smoothing distribution μ.

Theorem 5.3 (Compute $p_\mu(\mathcal{L}_{c,t})$).

$$p_\mu(\mathcal{L}_{c,t}) = \Pr_{o \sim \mu}(o \text{ satisfies Eq 6}) \Pr_{o \sim \mu}(o \text{ satisfies Eq 7} \mid o \text{ satisfies Eq 6}),$$

where $\Pr_{o \sim \mu}(o \text{ satisfies Eq 6}) = \mathrm{Binom}(c; k, \frac{r}{n})$ is the PMF of the binomial distribution and

$$T(c, t) \triangleq \Pr_{o \sim \mu}(o \text{ satisfies Eq 7} \mid o \text{ satisfies Eq 6}) = \sum_{\substack{0 \le \Delta_i, \widetilde{\Delta}_i \le d, \forall i \in [0,d] \\ \Delta_0 - \widetilde{\Delta}_0 + \ldots + \Delta_c - \widetilde{\Delta}_c = t}} \prod_{i=0}^{c} L(\Delta_i, \widetilde{\Delta}_i; s, d) \gamma^{\Delta_i} \rho^{d - \Delta_i},$$

$$(8)$$

where $L(\Delta, \widetilde{\Delta}; s, d)$ is the same quantity defined in Lee et al. [18].

Remark 5.1. We can compute $p_{\widetilde{\mu}}(\mathcal{L}_{c,t})$ as $\eta_{c,t} p_\mu(\mathcal{L}_{c,t})$ by the definition of $\eta_{c,t}$.

Efficient algorithm to compute Eq 8. The following algorithm computes $T(c, t)$ efficiently in time $O(d^3 + k^2 d^2)$: 1) Computing $L(u, v; s, d)$ takes $O(d^3)$ (see Appendix C for details). 2) Computing $T(0, t)$ by the definition in Eq 8 takes $O(kd^2)$. 3) Computing $T(c, t)$ for $c \ge 1$ by the following equation takes $O(k^2 d^2)$.

$$T(c, t) = \sum_{t_1 = \max(-d, t-cd)}^{\min(d, t+cd)} T(c-1, t - t_1) T(0, t_1) \tag{9}$$

Theorem 5.4 (Correctness of the Algorithm). $T(c, t)$ in Eq 9 is the same as the one in Eq 8.

The above computation still applies to perturbation function FL_s and L. Intuitively, flipping the label can be seen as flipping another dimension in the input features (Details in Appendix B.1).

Practical perspective. For each test input $\mathbf{x}$, we need to estimate p^* for the smoothed algorithm $\bar{A}$ given the benign dataset D. We use Monte Carlo sampling to compute p^* by the Clopper-Pearson interval. We also reuse the trained algorithms for each test input in the test set by using *Bonferroni correction*. We memoize the certified radius by enumerating all possible p^* beforehand so that checking Eq 4 can be done in constant time in an online scenario (details in Appendix B.2).

6 An Efficient Relaxation for Computing a Certified Radius

Theorem 5.1 requires computing the likelihood ratio of a large number of subspaces $\mathcal{L}_1, \ldots, \mathcal{L}_m$. We propose a generalization of the Neyman–Pearson lemma that **underapproximates** the subspaces by a small subset and computes a lower bound lb_δ for lb. We then show how to choose a subset of subspaces that yields a tight underapproximation.

A relaxation of the Neyman–Pearson lemma. The key idea is to use Theorem 5.1 to examine only a *subset* of the subspaces $\{\mathcal{L}_i\}_i$. Suppose that we partition the subspaces $\{\mathcal{L}_i\}_i$ into two sets $\{\mathcal{L}_i\}_{i \in B}$ and $\{\mathcal{L}_i\}_{i \notin B}$, using a set of indices B. We define a new lower bound lb_δ by applying Theorem 5.1 to the first group $\{\mathcal{L}_i\}_{i \in B}$ and underapproximating p^* in Theorem 5.1 as $p^* - \sum_{i \notin B} p_\mu(\mathcal{L}_i)$. Intuitively, the underapproximation ensures that any subspace in $\{\mathcal{L}_i\}_{i \notin B}$ will not contribute to the lower bound, even though these subspaces may have contributed to the precise lb computed by Theorem 5.1.

The next theorem shows that lb_δ will be smaller than lb by an error term δ that is a function of the partition B. The gain is in the number of subspaces we have to consider, which is now $|B|$.

Theorem 6.1 (Relaxation of the Lemma). *Define* lb *for the subspaces* $\mathcal{L}_1, \ldots, \mathcal{L}_m$ *as in Theorem 5.1. Define* lb_δ *for* $\{\mathcal{L}_i\}_{i \in B}$ *as in Theorem 5.1 by underapproximating* p^* *as* $p^* - \sum_{i \notin B} p_\mu(\mathcal{L}_i)$, *then we have **Soundness:*** $\mathrm{lb}_\delta \le \mathrm{lb}$ *and* δ-**Tightness:** *Let* $\delta \triangleq \sum_{i \notin B} p_{\widetilde{\mu}}(\mathcal{L}_i)$, *then* $\mathrm{lb}_\delta + \delta \ge \mathrm{lb}$.

Example 6.1. *Consider $\mathcal{L}_1, \mathcal{L}_2$, and $\mathcal{L}_3$ from Example 5.2. If we set $B = \{1, 2\}$ and underapproximate p^* as $p^* - p_\mu(\mathcal{L}_3) = 0.85$, then we have $\text{lb}_\delta = 0.1 + (0.85 - 0.4)/1 = 0.55$, which can still certify Eq. 4. However, lb_δ is not close to the original $\text{lb} = 0.8$ because the error $\delta = 0.4$ is large in this example. Next, we will show how to choose B as small as possible while still keeping δ small.*

Speeding up radius computation. The total time for computing lb consists of two parts, (1) the efficient algorithm (Eq 9) computes $T(c, t)$ in $O(d^3 + k^2 d^2)$, and (2) Theorem 5.1 takes $O(k^2 d)$ for a given p^* because there are $O(k^2 d)$ subspaces. Even though we have made the computation polynomial by the above two techniques in Section 5.2, it can still be slow for large bag sizes k, which can easily be in hundreds or thousands.

We will apply Theorem 6.1 to replace the bag size k with a small constant κ. Observe that $p_\mu(\mathcal{L}_{c,t})$ and $p_{\widetilde{\mu}}(\mathcal{L}_{c,t})$, as defined in Section 5.2, are negligible for large c, i.e., the number of perturbed indices in the smoothed dataset. Intuitively, if only a small portion of the training set is perturbed, it is unlikely that we select a large number of perturbed indices in the smoothed dataset. We underapproximate the full subspaces to $\{\mathcal{L}_{c,t}\}_{(c,t) \in B}$ by choosing the set of indices $B = \{(c, t) \mid c \leq \kappa, |t| \leq (c + 1)d\}$, where κ is a constant controlling the error δ, which can be computed as $\delta = \sum_{i \notin B} p_{\widetilde{\mu}}(\mathcal{L}_i) = 1 - \sum_{c=0}^{\kappa} \text{Binom}(c; k, \frac{r}{n})$. Theorem 6.1 reduces the number of subspaces to $|B| = O(\kappa^2 d)$. As a result, all the k appearing in previous time complexity can be replaced with κ.

Example 6.2. *Suppose $k = 150$ and $\frac{r}{n} = 0.5\%$, choosing $\kappa = 6$ leads to $\delta = 1.23 \times 10^{-5}$. In other words, it speeds up almost 625X for computing $T(c, t)$ and for applying Theorem 5.1.*

7 Experiments

We provide the implementation in the supplementary materials. In this section, we evaluate BagFlip against trigger-less and backdoor attacks and compare BagFlip with existing work.

Datasets. We conduct experiments on MNIST, CIFAR10, EMBER [2], and Contagio (`http://contagiodump.blogspot.com`). MNIST and CIFAR10 are datasets for image classification. EMBER and Contagio are datasets for malware detection, where data poisoning can lead to disastrous security consequences. To align with existing work, we select subsets of MNIST and CIFAR10 as MNIST-17, MNIST-01, and CIFAR10-02 in some experiments, e.g., MNIST-17 is a subset of MNIST with classes 1 and 7. We discretize all the features when applying BagFlip. We use clean datasets except in the comparison with RAB [39], where we follow their experimental setup and use backdoored datasets generated by BadNets [12]. A detailed description of the datasets is in Appendix E.1.

Models. We train neural networks for MNIST, CIFAR10, EMBER and random forests for Contagio. Unless we specifically mention the difference in some experiments, whenever we compare BagFlip to an existing work, we will use the same network structures, hyper-parameters, and data augmentation as the compared work, and we train $N = 1000$ models and set the confidence level as 0.999 for each configuration. All the configurations we used can be found in the implementation.

Metrics. For each test input $(\mathbf{x}_i, y_i)$, algorithm $\bar{A}$ will predict a label and the certified radius r_i. Assuming that the attacker had poisoned $R\%$ of the examples in the training set, we define **certified accuracy** as the percentage of test inputs that are correctly classified and their certified radii are no less than R, i.e., $\sum_{i=1}^{m} \mathbb{1}_{\bar{A}(D,\mathbf{x}_i)=y_i \wedge \frac{r_i}{n} \geq R\%}$. We define **normal accuracy** as the percentage of test inputs that are correctly classified, i.e., $\sum_{i=1}^{m} \mathbb{1}_{\bar{A}(D,\mathbf{x}_i)=y_i}$.

7.1 Defending Against Trigger-less Attacks

Two model-agnostic certified approaches, Bagging [15] and LabelFlip [28], can defend against trigger-less attacks. BagFlip outperforms LabelFlip (comparison in Appendix E.2). We evaluate BagFlip on the perturbation F_s using MNIST, CIFAR10, EMBER, and Contagio and compare BagFlip to Bagging. We provide comparisons with Bagging on other perturbation spaces in Appendix E.2.

We present BagFlip with different noise levels (different probabilities of ρ when flipping), denoted as BagFlip-ρ. When comparing to Bagging, we use the same k, the size of sampled datasets, for a fair comparison. We provide the choices of k in Appendix E.2.

Figure 1: Comparison to Bagging on MNIST and EMBER, showing the certified accuracy at different poisoning amounts R. For MNIST: $a = 0.9$ and $b = 0.8$. For EMBER: $a = 0.95$ and $b = 0.9$.

Table 1: Certified accuracy on MNIST and EMBER with perturbations F_1 and F_∞. Note that the certified accuracies are the same poisoning amount $R = 0$ because we reuse the trained models.

				F_1						F_∞		
	R	0	0.5	1.0	1.5	2.0	2.5	0	0.13	0.27	0.40	0.53	0.67
MNIST	Bagging	**94.54**	66.84	0	0	0	0	**94.54**	**90.83**	**85.45**	**77.61**	**61.46**	0
	BagFlip-0.9	93.62	**75.95**	27.73	4.02	0	0	93.62	89.45	83.62	74.19	56.65	0
	BagFlip-0.8	90.72	73.94	**46.20**	**33.39**	**24.23**	**5.07**	90.72	84.11	74.50	60.56	39.21	0
	R	0	0.07	0.13	0.20	0.27	0.33	0	0.05	0.10	0.15	0.20	0.25
EMBER	Bagging	**82.65**	75.11	66.01	0	0	0	**82.65**	76.30	**72.94**	61.78	0	0
	BagFlip-0.95	79.17	**75.93**	**69.30**	**57.36**	0	0	79.17	**76.83**	72.41	**62.04**	**30.36**	0
	BagFlip-0.9	78.18	69.40	62.11	41.21	**13.89**	**1.70**	78.18	70.79	63.16	41.24	11.64	0

Comparison to Bagging. Bagging on a discrete dataset is a special case of BagFlip when $\rho = 1$, i.e., no noise is added to the dataset. (We present the results of bagging on the original dataset (undiscretized) in Appendix E.2). Table 1 and Fig 1 show the results of BagFlip and Bagging on perturbations F_1 and F_∞ over MNIST and EMBER. The results of CIFAR10 and Contagio are similar and shown in Appendix E.2. F_1 only allows the attacker to modify one feature in each training example, and F_∞ allows the attacker to modify each training example arbitrarily without constraint. For F_1 on MNIST, BagFlip-0.9 performs better than Bagging after $R = 0.19$ and BagFlip-0.8 still retains non-zero certified accuracy at $R = 2.5$ while Bagging's certified accuracy drops to zero after $R = 0.66$. We observe similar results on EMBER for F_1 in Table 1. **BagFlip achieves higher certified accuracy than Bagging when the poisoning amount is large for F_1.**

For F_∞ on MNIST, Bagging performs better than BagFlip across all R because the noise added by BagFlip to the training set hurts the accuracy. However, we find that the noise added by BagFlip helps it perform better for F_∞ on EMBER. Specifically, BagFlip achieves similar certified accuracy as Bagging at small radii and BagFlip performs better than Bagging after $R = 0.15$. **Bagging achieves higher certified accuracy than BagFlip for F_∞. Except in EMBER, BagFlip achieves higher certified accuracy than Bagging when the amount of poisoning is large.**

We also study the effect of different s in the perturbation function F_s. Figure 2(a) shows the result of BagFlip-0.8 on MNIST. BagFlip has the highest certified accuracy for F_1. As s increases, the result monotonically converges to the curve of BagFlip-0.8 in Figure 1(b). Bagging neglects the perturbation function and performs the same across all s. Bagging performs better than BagFlip-0.8 when $s \geq 8$. Other results for different datasets and different noise levels follow a similar trend (see Appendix E.2). **BagFlip performs best at F_1 and monotonically degenerates to F_∞ as s increases.**

7.2 Defending Against Backdoor Attacks

Two model-agnostic certified approaches, FeatFlip [35] and RAB [39], can defend against backdoor attacks. We compare BagFlip to FeatFlip on MNIST-17 perturbed using FL_1, and compare BagFlip to RAB on poisoned MNIST-01 and CIFAR10-02 (by BadNets [12]) perturbed using F_1. We further evaluate BagFlip on the perturbation F_1 using MNIST and Contagio, on which existing work is unable to compute a meaningful certified radius.

Comparison to FeatFlip. FeatFlip scales poorly compared to BagFlip because its computation of the certified radius is polynomial in the size of the training set. BagFlip's computation is polynomial in the size of the bag instead of the size of the whole training set, and it uses a relaxation of the Neyman–Pearson lemma for further speed up. **BagFlip is more scalable than FeatFlip.**

Figure 2: (a) BagFlip-0.8 on MNIST against F_s with different s. $s = 8$ almost overlaps with $s = \infty$. (b) BagFlip-0.8 on MNIST and BagFlip-0.9 on Contagio against backdoor attack with F_1. Dashed lines show normal accuracy.

We directly cite FeatFlip's results from Wang et al. [35] and note that FeatFlip is evaluated on a subset of MNIST-17. As shown in Table 2, BagFlip achieves similar normal accuracy, but much higher certified accuracy across all R (see full results in Appendix E.2) than FeatFlip. **BagFlip significantly outperforms FeatFlip against FL_1 on MNIST-17.**

Comparison to RAB. RAB assumes a perturbation function that perturbs the input within an l_2-norm ball of radius s. We compare BagFlip to RAB on F_1, where the l_0-norm ball (our perturbation function) and l_2-norm ball are the same because the feature values are within $[0, 1]$. Since RAB targets single-test-input prediction, we do not use Bonferroni correction for BagFlip as a fair comparison. We directly cite RAB's results from Weber et al. [39]. Table 2 shows

Table 2: Compared to FeatFlip on FL_1 and RAB F_1 with normal accuracy (Acc.) and certified accuracy at different poisoning amount R (CF@R).

		Acc.	CF@0	CF@0.5	CF@1.0
MNIST-17	FeatFlip	98	36	0	0
	BagFlip-0.95	97	**87**	**75.3**	**26.3**
	BagFlip-0.9	97	72	45.9	3.6
		Acc.	CF@0	CF@1.5	CF@3.0
MNIST-01	RAB	100.0	74.4	0	0
	BagFlip-0.8	99.6	**98.4**	**96.5**	84.6
	BagFlip-0.7	99.5	98.0	95.8	**91.9**
		Acc.	CF@0	CF@0.1	CF@0.2
CIFAR10-02	RAB	73.3	0	0	0
	BagFlip-0.8	73.1	16.8	11.0	6.8
	BagFlip-0.7	71.7	**41.0**	**34.6**	**29.2**

that on MNIST-01 and CIFAR10-02, BagFlip achieves similar normal accuracy, but a much higher certified accuracy than RAB for all values of R (detailed figures in Appendix E.2). **BagFlip significantly outperforms RAB against F_1 on MNIST-01 and CIFAR10-02.**

Results on MNIST and Contagio. Figure 2(b) shows the results of BagFlip on MNIST and Contagio. When fixing R, the certified accuracy for the backdoor attack is much smaller than the certified accuracy for the trigger-less attack (Figure 1 and Figure 3 in Appendix E.2) because backdoor attacks are strictly stronger than trigger-less attacks. BagFlip cannot provide effective certificates for backdoor attacks on the more complex datasets CIFAR10 and EMBER, i.e., the certified radius is almost zero. **BagFlip can provide certificates against backdoor attacks on MNIST and Contagio, while BagFlip's certificates are not effective for CIFAR10 and EMBER.**

8 Conclusion, Limitations, and Future Work

We presented BagFlip, a certified probabilistic approach that can effectively defend against both trigger-less and backdoor attacks. We foresee many future improvements to BagFlip. First, BagFlip treats both the data and the underlying machine learning models as closed boxes. Assuming a specific data distribution and training algorithm can further improve the computed certified radius. Second, BagFlip uniformly flips the features and the label, while it is desirable to adjust the noise levels for the label and important features for better normal accuracy according to the distribution of the data. Third, while probabilistic approaches need to retrain thousands of models after a fixed number of predictions, the deterministic approaches can reuse models for every prediction. Thus, it is interesting to develop a deterministic model-agnostic approach that can defend against both trigger-less and backdoor attacks.

References

[1] Hojjat Aghakhani, Dongyu Meng, Yu-Xiang Wang, Christopher Kruegel, and Giovanni Vigna. Bullseye polytope: A scalable clean-label poisoning attack with improved transferability. In *IEEE European Symposium on Security and Privacy, EuroS&P 2021, Vienna, Austria, September 6-10, 2021*, pages 159–178. IEEE, 2021. doi: 10.1109/EuroSP51992.2021.00021. URL https://doi.org/10.1109/EuroSP51992.2021.00021.

[2] Hyrum S. Anderson and Phil Roth. EMBER: an open dataset for training static PE malware machine learning models. *CoRR*, abs/1804.04637, 2018. URL http://arxiv.org/abs/1804.04637.

[3] Battista Biggio, Blaine Nelson, and Pavel Laskov. Poisoning attacks against support vector machines. In *Proceedings of the 29th International Conference on Machine Learning, ICML 2012, Edinburgh, Scotland, UK, June 26 - July 1, 2012*. icml.cc / Omnipress, 2012. URL http://icml.cc/2012/papers/880.pdf.

[4] Ruoxin Chen, Jie Li, Chentao Wu, Bin Sheng, and Ping Li. A framework of randomized selection based certified defenses against data poisoning attacks. *CoRR*, abs/2009.08739, 2020. URL https://arxiv.org/abs/2009.08739.

[5] Xinyun Chen, Chang Liu, Bo Li, Kimberly Lu, and Dawn Song. Targeted backdoor attacks on deep learning systems using data poisoning. *CoRR*, abs/1712.05526, 2017. URL http://arxiv.org/abs/1712.05526.

[6] Jeremy M. Cohen, Elan Rosenfeld, and J. Zico Kolter. Certified adversarial robustness via randomized smoothing. In Kamalika Chaudhuri and Ruslan Salakhutdinov, editors, *Proceedings of the 36th International Conference on Machine Learning, ICML 2019, 9-15 June 2019, Long Beach, California, USA*, volume 97 of *Proceedings of Machine Learning Research*, pages 1310–1320. PMLR, 2019. URL http://proceedings.mlr.press/v97/cohen19c.html.

[7] Samuel Drews, Aws Albarghouthi, and Loris D'Antoni. Proving data-poisoning robustness in decision trees. In Alastair F. Donaldson and Emina Torlak, editors, *Proceedings of the 41st ACM SIGPLAN International Conference on Programming Language Design and Implementation, PLDI 2020, London, UK, June 15-20, 2020*, pages 1083–1097. ACM, 2020. doi: 10.1145/3385412.3385975. URL https://doi.org/10.1145/3385412.3385975.

[8] Krishnamurthy (Dj) Dvijotham, Jamie Hayes, Borja Balle, J. Zico Kolter, Chongli Qin, András György, Kai Xiao, Sven Gowal, and Pushmeet Kohli. A framework for robustness certification of smoothed classifiers using f-divergences. In *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net, 2020. URL https://openreview.net/forum?id=SJlKrkSFPH.

[9] Yansong Gao, Change Xu, Derui Wang, Shiping Chen, Damith Chinthana Ranasinghe, and Surya Nepal. STRIP: a defence against trojan attacks on deep neural networks. In David Balenson, editor, *Proceedings of the 35th Annual Computer Security Applications Conference, ACSAC 2019, San Juan, PR, USA, December 09-13, 2019*, pages 113–125. ACM, 2019. doi: 10.1145/3359789.3359790. URL https://doi.org/10.1145/3359789.3359790.

[10] Jonas Geiping, Liam Fowl, Gowthami Somepalli, Micah Goldblum, Michael Moeller, and Tom Goldstein. What doesn't kill you makes you robust(er): Adversarial training against poisons and backdoors. *CoRR*, abs/2102.13624, 2021. URL https://arxiv.org/abs/2102.13624.

[11] Jonas Geiping, Liam H. Fowl, W. Ronny Huang, Wojciech Czaja, Gavin Taylor, Michael Moeller, and Tom Goldstein. Witches' brew: Industrial scale data poisoning via gradient matching. In *9th International Conference on Learning Representations, ICLR 2021, Virtual Event, Austria, May 3-7, 2021*. OpenReview.net, 2021. URL https://openreview.net/forum?id=01olnfLIbD.

[12] Tianyu Gu, Brendan Dolan-Gavitt, and Siddharth Garg. Badnets: Identifying vulnerabilities in the machine learning model supply chain. *CoRR*, abs/1708.06733, 2017. URL http://arxiv.org/abs/1708.06733.

[13] Jonathan Hayase, Weihao Kong, Raghav Somani, and Sewoong Oh. Defense against backdoor attacks via robust covariance estimation. In Marina Meila and Tong Zhang, editors, *Proceedings of the 38th International Conference on Machine Learning, ICML 2021, 18-24 July 2021, Virtual Event*, volume 139 of *Proceedings of Machine Learning Research*, pages 4129–4139. PMLR, 2021. URL http://proceedings.mlr.press/v139/hayase21a.html.

[14] Jinyuan Jia, Xiaoyu Cao, and Neil Zhenqiang Gong. Certified robustness of nearest neighbors against data poisoning attacks. *CoRR*, abs/2012.03765, 2020. URL https://arxiv.org/abs/2012.03765.

[15] Jinyuan Jia, Xiaoyu Cao, and Neil Zhenqiang Gong. Intrinsic certified robustness of bagging against data poisoning attacks. In *Thirty-Fifth AAAI Conference on Artificial Intelligence, AAAI 2021, Thirty-Third Conference on Innovative Applications of Artificial Intelligence, IAAI 2021, The Eleventh Symposium on Educational Advances in Artificial Intelligence, EAAI 2021, Virtual Event, February 2-9, 2021*, pages 7961–7969. AAAI Press, 2021. URL https://ojs.aaai.org/index.php/AAAI/article/view/16971.

[16] Pang Wei Koh, Jacob Steinhardt, and Percy Liang. Stronger data poisoning attacks break data sanitization defenses. *Mach. Learn.*, 111(1):1–47, 2022. doi: 10.1007/s10994-021-06119-y. URL https://doi.org/10.1007/s10994-021-06119-y.

[17] Aounon Kumar, Alexander Levine, Tom Goldstein, and Soheil Feizi. Curse of dimensionality on randomized smoothing for certifiable robustness. In *Proceedings of the 37th International Conference on Machine Learning, ICML 2020, 13-18 July 2020, Virtual Event*, volume 119 of *Proceedings of Machine Learning Research*, pages 5458–5467. PMLR, 2020. URL http://proceedings.mlr.press/v119/kumar20b.html.

[18] Guang-He Lee, Yang Yuan, Shiyu Chang, and Tommi S. Jaakkola. Tight certificates of adversarial robustness for randomly smoothed classifiers. In Hanna M. Wallach, Hugo Larochelle, Alina Beygelzimer, Florence d'Alché-Buc, Emily B. Fox, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pages 4911–4922, 2019. URL https://proceedings.neurips.cc/paper/2019/hash/fa2e8c4385712f9a1d24c363a2cbe5b8-Abstract.html.

[19] Alexander Levine and Soheil Feizi. Deep partition aggregation: Provable defenses against general poisoning attacks. In *9th International Conference on Learning Representations, ICLR 2021, Virtual Event, Austria, May 3-7, 2021*. OpenReview.net, 2021. URL https://openreview.net/forum?id=YUGG2tFuPM.

[20] Kang Liu, Brendan Dolan-Gavitt, and Siddharth Garg. Fine-pruning: Defending against backdooring attacks on deep neural networks. In Michael Bailey, Thorsten Holz, Manolis Stamatogiannakis, and Sotiris Ioannidis, editors, *Research in Attacks, Intrusions, and Defenses - 21st International Symposium, RAID 2018, Heraklion, Crete, Greece, September 10-12, 2018, Proceedings*, volume 11050 of *Lecture Notes in Computer Science*, pages 273–294. Springer, 2018. doi: 10.1007/978-3-030-00470-5_13. URL https://doi.org/10.1007/978-3-030-00470-5_13.

[21] Yingqi Liu, Shiqing Ma, Yousra Aafer, Wen-Chuan Lee, Juan Zhai, Weihang Wang, and Xiangyu Zhang. Trojaning attack on neural networks. In *25th Annual Network and Distributed System Security Symposium, NDSS 2018, San Diego, California, USA, February 18-21, 2018*. The Internet Society, 2018. URL http://wp.internetsociety.org/ndss/wp-content/uploads/sites/25/2018/02/ndss2018_03A-5_Liu_paper.pdf.

[22] Yuzhe Ma, Xiaojin Zhu, and Justin Hsu. Data poisoning against differentially-private learners: Attacks and defenses. In Sarit Kraus, editor, *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019, Macao, China, August 10-16, 2019*, pages 4732–4738. ijcai.org, 2019. doi: 10.24963/ijcai.2019/657. URL https://doi.org/10.24963/ijcai.2019/657.

[23] Anna P. Meyer, Aws Albarghouthi, and Loris D'Antoni. Certifying robustness to programmable data bias in decision trees. In Marc'Aurelio Ranzato, Alina Beygelzimer, Yann N.

Dauphin, Percy Liang, and Jennifer Wortman Vaughan, editors, *Advances in Neural Information Processing Systems 34: Annual Conference on Neural Information Processing Systems 2021, NeurIPS 2021, December 6-14, 2021, virtual*, pages 26276–26288, 2021. URL `https://proceedings.neurips.cc/paper/2021/hash/dcf531edc9b229acfe0f4b87e1e278dd-Abstract.html`.

[24] Blaine Nelson, Marco Barreno, Fuching Jack Chi, Anthony D. Joseph, Benjamin I. P. Rubinstein, Udam Saini, Charles Sutton, J. Doug Tygar, and Kai Xia. Exploiting machine learning to subvert your spam filter. In Fabian Monrose, editor, *First USENIX Workshop on Large-Scale Exploits and Emergent Threats, LEET '08, San Francisco, CA, USA, April 15, 2008, Proceedings*. USENIX Association, 2008. URL `http://www.usenix.org/events/leet08/tech/full_papers/nelson/nelson.pdf`.

[25] Fanchao Qi, Yuan Yao, Sophia Xu, Zhiyuan Liu, and Maosong Sun. Turn the combination lock: Learnable textual backdoor attacks via word substitution. In Chengqing Zong, Fei Xia, Wenjie Li, and Roberto Navigli, editors, *Proceedings of the 59th Annual Meeting of the Association for Computational Linguistics and the 11th International Joint Conference on Natural Language Processing, ACL/IJCNLP 2021, (Volume 1: Long Papers), Virtual Event, August 1-6, 2021*, pages 4873–4883. Association for Computational Linguistics, 2021. doi: 10.18653/v1/2021.acl-long.377. URL `https://doi.org/10.18653/v1/2021.acl-long.377`.

[26] Ximing Qiao, Yukun Yang, and Hai Li. Defending neural backdoors via generative distribution modeling. In Hanna M. Wallach, Hugo Larochelle, Alina Beygelzimer, Florence d'Alché-Buc, Emily B. Fox, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pages 14004–14013, 2019. URL `https://proceedings.neurips.cc/paper/2019/hash/78211247db84d96acf4e00092a7fba80-Abstract.html`.

[27] Goutham Ramakrishnan and Aws Albarghouthi. Backdoors in neural models of source code. *CoRR*, abs/2006.06841, 2020. URL `https://arxiv.org/abs/2006.06841`.

[28] Elan Rosenfeld, Ezra Winston, Pradeep Ravikumar, and J. Zico Kolter. Certified robustness to label-flipping attacks via randomized smoothing. In *Proceedings of the 37th International Conference on Machine Learning, ICML 2020, 13-18 July 2020, Virtual Event*, volume 119 of *Proceedings of Machine Learning Research*, pages 8230–8241. PMLR, 2020. URL `http://proceedings.mlr.press/v119/rosenfeld20b.html`.

[29] Aniruddha Saha, Akshayvarun Subramanya, and Hamed Pirsiavash. Hidden trigger backdoor attacks. In *The Thirty-Fourth AAAI Conference on Artificial Intelligence, AAAI 2020, The Thirty-Second Innovative Applications of Artificial Intelligence Conference, IAAI 2020, The Tenth AAAI Symposium on Educational Advances in Artificial Intelligence, EAAI 2020, New York, NY, USA, February 7-12, 2020*, pages 11957–11965. AAAI Press, 2020. URL `https://ojs.aaai.org/index.php/AAAI/article/view/6871`.

[30] Giorgio Severi, Jim Meyer, Scott E. Coull, and Alina Oprea. Explanation-guided backdoor poisoning attacks against malware classifiers. In Michael Bailey and Rachel Greenstadt, editors, *30th USENIX Security Symposium, USENIX Security 2021, August 11-13, 2021*, pages 1487–1504. USENIX Association, 2021. URL `https://www.usenix.org/conference/usenixsecurity21/presentation/severi`.

[31] Ali Shafahi, W. Ronny Huang, Mahyar Najibi, Octavian Suciu, Christoph Studer, Tudor Dumitras, and Tom Goldstein. Poison frogs! targeted clean-label poisoning attacks on neural networks. In Samy Bengio, Hanna M. Wallach, Hugo Larochelle, Kristen Grauman, Nicolò Cesa-Bianchi, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 31: Annual Conference on Neural Information Processing Systems 2018, NeurIPS 2018, December 3-8, 2018, Montréal, Canada*, pages 6106–6116, 2018. URL `https://proceedings.neurips.cc/paper/2018/hash/22722a343513ed45f14905eb07621686-Abstract.html`.

[32] Florian Tramèr, Nicholas Carlini, Wieland Brendel, and Aleksander Madry. On adaptive attacks to adversarial example defenses. In Hugo Larochelle, Marc'Aurelio Ranzato, Raia Hadsell, Maria-Florina Balcan, and Hsuan-Tien Lin, editors, *Advances in Neural Information Processing*

Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual, 2020. URL `https://proceedings.neurips.cc/pap er/2020/hash/11f38f8ecd71867b42433548d1078e38-Abstract.html`.

[33] Brandon Tran, Jerry Li, and Aleksander Madry. Spectral signatures in backdoor attacks. In Samy Bengio, Hanna M. Wallach, Hugo Larochelle, Kristen Grauman, Nicolò Cesa-Bianchi, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 31: Annual Conference on Neural Information Processing Systems 2018, NeurIPS 2018, December 3-8, 2018, Montréal, Canada*, pages 8011–8021, 2018. URL `https://proceedings.neurips. cc/paper/2018/hash/280cf18baf4311c92aa5a042336587d3-Abstract.html`.

[34] Alexander Turner, Dimitris Tsipras, and Aleksander Madry. Label-consistent backdoor attacks. *CoRR*, abs/1912.02771, 2019. URL `http://arxiv.org/abs/1912.02771`.

[35] Binghui Wang, Xiaoyu Cao, Jinyuan Jia, and Neil Zhenqiang Gong. On certifying robustness against backdoor attacks via randomized smoothing. *CoRR*, abs/2002.11750, 2020. URL `https://arxiv.org/abs/2002.11750`.

[36] Bolun Wang, Yuanshun Yao, Shawn Shan, Huiying Li, Bimal Viswanath, Haitao Zheng, and Ben Y. Zhao. Neural cleanse: Identifying and mitigating backdoor attacks in neural networks. In *2019 IEEE Symposium on Security and Privacy, SP 2019, San Francisco, CA, USA, May 19-23, 2019*, pages 707–723. IEEE, 2019. doi: 10.1109/SP.2019.00031. URL `https://doi.org/10.1109/SP.2019.00031`.

[37] Hongyi Wang, Kartik Sreenivasan, Shashank Rajput, Harit Vishwakarma, Saurabh Agarwal, Jy-yong Sohn, Kangwook Lee, and Dimitris S. Papailiopoulos. Attack of the tails: Yes, you really can backdoor federated learning. In Hugo Larochelle, Marc'Aurelio Ranzato, Raia Hadsell, Maria-Florina Balcan, and Hsuan-Tien Lin, editors, *Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual*, 2020. URL `https://proceedings.neurips. cc/paper/2020/hash/b8ffa41d4e492f0fad2f13e29e1762eb-Abstract.html`.

[38] Wenxiao Wang, Alexander Levine, and Soheil Feizi. Improved certified defenses against data poisoning with (deterministic) finite aggregation. *CoRR*, abs/2202.02628, 2022. URL `https://arxiv.org/abs/2202.02628`.

[39] Maurice Weber, Xiaojun Xu, Bojan Karlas, Ce Zhang, and Bo Li. RAB: provable robustness against backdoor attacks. *CoRR*, abs/2003.08904, 2020. URL `https://arxiv.org/abs/ 2003.08904`.

[40] Huang Xiao, Battista Biggio, Gavin Brown, Giorgio Fumera, Claudia Eckert, and Fabio Roli. Is feature selection secure against training data poisoning? In Francis R. Bach and David M. Blei, editors, *Proceedings of the 32nd International Conference on Machine Learning, ICML 2015, Lille, France, 6-11 July 2015*, volume 37 of *JMLR Workshop and Conference Proceedings*, pages 1689–1698. JMLR.org, 2015. URL `http://proceedings.mlr.press/v37/xiao15.html`.

[41] Yuanshun Yao, Huiying Li, Haitao Zheng, and Ben Y. Zhao. Latent backdoor attacks on deep neural networks. In Lorenzo Cavallaro, Johannes Kinder, XiaoFeng Wang, and Jonathan Katz, editors, *Proceedings of the 2019 ACM SIGSAC Conference on Computer and Communications Security, CCS 2019, London, UK, November 11-15, 2019*, pages 2041–2055. ACM, 2019. doi: 10.1145/3319535.3354209. URL `https://doi.org/10.1145/3319535.3354209`.

[42] Chen Zhu, W. Ronny Huang, Hengduo Li, Gavin Taylor, Christoph Studer, and Tom Goldstein. Transferable clean-label poisoning attacks on deep neural nets. In Kamalika Chaudhuri and Ruslan Salakhutdinov, editors, *Proceedings of the 36th International Conference on Machine Learning, ICML 2019, 9-15 June 2019, Long Beach, California, USA*, volume 97 of *Proceedings of Machine Learning Research*, pages 7614–7623. PMLR, 2019. URL `http://proceedings. mlr.press/v97/zhu19a.html`.