RETURN-BASED CONTRASTIVE REPRESENTATION LEARNING FOR REINFORCEMENT LEARNING

Anonymous authors
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ABSTRACT

Recently, various auxiliary tasks have been proposed to accelerate representation learning and improve sample efficiency in deep reinforcement learning (RL). Existing auxiliary tasks do not take the characteristics of RL problems into consideration and are unsupervised. By leveraging returns, the most important feedback signals in RL, we propose a novel auxiliary task that forces the learnt representations to discriminate state-action pairs with different returns. Our auxiliary loss is theoretically justified to learn representations that capture the structure of a new form of state-action abstraction, under which state-action pairs with similar return distributions are aggregated together. Empirically, our algorithm outperforms strong baselines on complex tasks in DeepMind Control suite and Atari games, and achieves even better performance when combined with existing auxiliary tasks.

1 INTRODUCTION

Deep reinforcement learning (RL) algorithms can learn representations from high-dimensional inputs, as well as learn policies based on such representations to maximize long-term returns simultaneously. However, deep RL algorithms typically require large numbers of samples, which can be quite expensive to obtain (Mnih et al., 2015). In contrast, it is usually much more sample efficient to learn policies with learned representations/extracted features (Srinivas et al., 2020). To this end, various auxiliary tasks have been proposed to accelerate representation learning in aid of the main RL task (Suddarth and Kergosien, 1990; Sutton et al., 2011; Gelada et al., 2019; Bellemare et al., 2019; François-Lavet et al., 2019; Shen et al., 2020; Zhang et al., 2020; Dabney et al., 2020; Srinivas et al., 2020). Representative examples of auxiliary tasks include predicting the future in either the pixel space or the latent space with reconstruction-based losses (e.g., Jaderberg et al., 2016; Hafner et al., 2019a;b).

Recently, contrastive learning has been introduced to construct auxiliary tasks and achieves better performance compared to reconstruction based methods in accelerating RL algorithms (Oord et al., 2018; Srinivas et al., 2020). Without the need to reconstruct inputs such as raw pixels, contrastive learning based methods can ignore irrelevant features such as static background in games and learn more compact representations. Oord et al. (2018) propose a contrastive representation learning method based on the temporal structure of state sequence. Srinivas et al. (2020) propose to leverage the prior knowledge from computer vision, learning representations that are invariant to image augmentation. However, existing works mainly construct contrastive auxiliary losses in an unsupervised manner, without considering feedback signals in RL problems as supervision.

In this paper, we take a further step to leverage the return feedback to design a contrastive auxiliary loss to accelerate RL algorithms. Specifically, we propose a novel method, called Return-based Contrastive representation learning for Reinforcement Learning (RCRL). In our method, given an anchor state-action pair, we choose a state-action pair with the same or similar return as the positive sample, and a state-action pair with different return as the negative sample. Then, we train a discriminator to classify between positive and negative samples given the anchor based on their representations as the auxiliary task. The intuition here is to learn state-action representations that capture return-relevant features while ignore return-irrelevant features.

From a theoretical perspective, RCRL is supported by a novel state-action abstraction, called Z^π-irrelevance. Z^π-irrelevance abstraction aggregates state-action pairs with similar return distribu-

tions under certain policy π. We show that Z^π-irrelevance abstraction can reduce the size of the state-action space (cf. Appendix A) as well as approximate the Q values arbitrarily accurately (cf. Section 4.1). We further propose a method called Z-learning that can calculate Z^π-irrelevance abstraction with sampled returns rather than the return distribution, which is hardly available in practice. Z-learning can learn Z^π-irrelevance abstraction provably efficiently. Our algorithm RCRL can be seen as the empirical version of Z-learning by making a few approximations such as integrating with deep RL algorithms, and collecting positive pairs within a consecutive segment in a trajectory of the anchors.

We conduct experiments on DeepMind Control suite (Tassa et al., 2018) and Atari games (Bellemare et al., 2013) in low data regime. The experiment results show that our auxiliary task combined with Rainbow (Hessel et al., 2017) for discrete control tasks or SAC (Haarnoja et al., 2018) for continuous control tasks achieves superior performance over other state-of-the-art baselines for this regime. Our method can be further combined with existing unsupervised contrastive learning methods to achieve even better performance. We also perform a detailed analysis on how the representation changes during training with/without our auxiliary loss. We find that a good embedding network assigns similar/dissimilar representations to state-action pairs with similar/dissimilar return distributions, and our algorithm can boost such generalization and speed up training.

Our contributions are summarized as follows:

- We propose RCRL by introducing a novel contrastive loss based on return, to learn return-relevant representation and speed up deep RL algorithms.
- We theoretically build the connection between the contrastive loss and a new form of state-action abstraction, which can reduce the size of the state-action space as well as approximate the Q values arbitrarily accurately.
- Our algorithm achieves superior performance against strong baselines in DeepMind Control suite and Atari games. Besides, the performance can be further enhanced when combined with existing auxiliary tasks.

2 Related Work

2.1 Auxiliary task

In reinforcement learning, the auxiliary task can be used for both the model-based setting and the model-free setting. In the model-based settings, world models can be used as auxiliary tasks and lead to better performance, such as CRAR (François-Lavet et al., 2019), Dreamer (Hafner et al., 2019a), and PlaNet (Hafner et al., 2019b). Due to the complex components (e.g., the latent transition or reward module) in the world model, such methods are empirically unstable to train and relies on different regularizations to converge. In the model-free settings, many algorithms construct various auxiliary tasks to improve performance, such as predicting the future (Jaderberg et al., 2016; Shelhamer et al., 2016; Guo et al., 2020; Lee et al., 2020; Mazoure et al., 2020), learning value functions with different rewards or under different policies (Veeriah et al., 2019; Schaul et al., 2015; Borsa et al., 2018; Bellemare et al., 2019; Dabney et al., 2020), learning from many-goals (Veeriah et al., 2018), or the combination of different auxiliary objectives (de Bruin et al., 2018). Moreover, auxiliary tasks can be designed based on the prior knowledge about the environment (Mirowski et al., 2016; Shen et al., 2020; van der Pol et al., 2020) or the raw state representation (Srinivas et al., 2020). Hessel et al. (2019) also apply auxiliary task to the multi-task RL setting.

Contrastive learning has seen dramatic progress recently, and been introduced to learn state representation (Oord et al., 2018; Sermanet et al., 2018; Dwibedi et al., 2018; Aytar et al., 2018; Anand et al., 2019; Srinivas et al., 2020). Temporal structure (Sermanet et al., 2018; Aytar et al., 2018) and local spatial structure (Anand et al., 2019) has been leveraged for state representation learning via contrastive losses. CPC (Oord et al., 2018) and CURL (Srinivas et al., 2020) adopt a contrastive auxiliary tasks to accelerate representation learning and speed up main RL tasks, by leveraging the temporal structure and image augmentation respectively. To the best of our knowledge, we are the first to leverage return to construct a contrastive auxiliary task for speeding up the main RL task.

2.2 ABSTRACTION

State abstraction (or state aggregation) aggregates states by ignoring irrelevant state information. By reducing state space, state abstraction can enable efficient policy learning. Different types of abstraction are proposed in literature, ranging from fine-grained to coarse-grained abstraction, each reducing state space to a different extent. Bisimulation or model irrelevance (Dean and Givan, 1997; Givan et al., 2003) define state abstraction under which both transition and reward function are kept invariant. By contrast, other types of state abstraction that are coarser than bisimulation such as Q^π irrelevance or Q^* irrelevance (see e.g., Li et al., 2006), which keep the Q function invariant under any policy π or the optimal policy respectively. There are also some works on state-action abstractions, e.g., MDP homomorphism (Ravindran, 2003; Ravindran and Barto, 2004), which is similar to bisimulation in keeping reward and transition invariant, but extending bisimulation from state abstraction to state-action abstraction.

In this paper, we consider a new form of state-action abstraction Z^π-irrelevance, which aggregates state-action pairs with the same return distribution and is coarser than bisimulation or homomorphism which are frequently used as auxiliary tasks (e.g., Gelada et al., 2019; Biza and Platt, 2018). However, it is worth noting that Z^π-irrelevance is only used to build the theoretical foundation of our algorithm, and show that our proposed auxiliary task is well-aligned with the main RL task. Representation learning in deep RL is in general very different from aggregating states in tabular case, though the latter may build nice theoretical foundation for the former. Here we focus on how to design auxiliary tasks to accelerate representation learning using contrastive learning techniques, and we propose a novel return-based contrastive method based on our proposed Z^π-irrelevance abstraction.

3 PRELIMINARY

We consider a Markov Decision Process (MDP) which is a tuple $(\mathcal{S}, \mathcal{A}, P, R, \mu, \gamma)$ specifying the state space $\mathcal{S}$, the action space $\mathcal{A}$, the state transition probability $P(s_{t+1}|s_t, a_t)$, the reward $R(r_t|s_t, a_t)$, the initial state distribution $\mu \in \Delta^{\mathcal{S}}$ and the discount factor γ. Also, we denote $x := (s, a) \in \mathcal{X} := \mathcal{S} \times \mathcal{A}$ to be the state-action pair. A (stationary) policy $\pi : \mathcal{S} \to \Delta^{\mathcal{A}}$ specifies the action selection probability on each state. Following the policy π, the discounted sum of future rewards (or return) is denoted by the random variable $Z^\pi(s, a) = \sum_{t=0}^{\infty} \gamma^t R(s_t, a_t)$, where $s_0 = s, a_0 = a, s_t \sim P(\cdot|s_{t-1}, a_{t-1})$, and $a_t \sim \pi(\cdot|s_t)$. We divide the range of return into K equal bins $\{R_0 = R_{\min}, R_1, \cdots, R_K = R_{\max}\}$ such that $R_k - R_{k-1} = (R_{\max} - R_{\min})/K, \forall k \in [K]$, where $R_{\min}$ (resp. $R_{\max}$) is the minimum (reps. maximum) possible return, and $[K] := \{1, 2, \cdots, K\}$. We use $b(R) = k \in [K]$ to denote the event that R falls into the kth bin, i.e., $R_{k-1} < R \leq R_k$. Hence, $b(R)$ can be viewed as the discretized version of the return, and the distribution of discretized return can be represented by a K-dimensional vector $\mathbb{Z}^\pi(x) \in \Delta^K$, where the k-th element equals to $\Pr[R_{k-1} < Z^\pi(x) \leq R_k]$. The Q function is defined as $Q^\pi(x) = \mathbb{E}[Z^\pi(x)]$, and the state value function is defined as $V^\pi(s) = \mathbb{E}_{a\sim\pi(\cdot|s)}[Z^\pi(s, a)]$. The objective for RL is to find a policy π that maximizes the expected cumulative reward $J(\pi) = \mathbb{E}_{s\sim\mu}[V^\pi(s)]$. We denote the optimal policy as π^* and the corresponding optimal Q function as $Q^* := Q^{\pi^*}$.

4 METHODOLOGY

In this section, we present our method, from both theoretical and empirical perspectives. First, we propose Z^π-irrelevance, a new form of state-action abstraction based on return distribution. We show that the Q functions for any policy (and therefore the optimal policy) can be represented under Z^π-irrelevance abstraction. Then we consider an algorithm, Z-learning, that enables us to learn Z^π-irrelevance abstraction from the samples collected using π. Z-learning is simple and learns the abstraction by only minimizing a contrastive loss. We show that Z-learning can learn Z^π-irrelevance abstraction provably efficiently. After that, we introduce return-based contrastive representation learning for RL (RCRL) that incorporates standard RL algorithms with an auxiliary task adapted from Z-learning. At last, we present our network structure for learning state-action embedding, upon which RCRL is built.

Algorithm 1: Z-learning

1: Given the policy π, the number of bins for the return K, a constant $N \geq N_{\pi,K}$, the function class $\mathcal{F}_N$ and a distribution $d \in \Delta^{\mathcal{X}}$ with $\text{supp}(d) = \mathcal{X}$
2: $\mathcal{D} = \emptyset$
3: **for** $i = 1, \cdots, n$ **do**
4: $\quad x_1, x_2 \sim d$
5: $\quad R_1 \sim Z^\pi(x_1), R_2 \sim Z^\pi(x_2)$
6: $\quad \mathcal{D} = \mathcal{D} \cup \{(x_1, x_2, y = \mathbb{I}[b(R_1) \neq b(R_2)])\}$
7: **end for**
8: $\hat{f} = \arg\min_{f \in \mathcal{F}_N} \mathcal{L}(f, \mathcal{D})$, where $\mathcal{L}(f, \mathcal{D})$ is defined in (1) and $\hat{f}(\cdot,\cdot) = \hat{w}(\hat{\phi}(\cdot), \hat{\phi}(\cdot))$
9: **return** the encoder $\hat{\phi}$

4.1 Z^π-IRRELEVANCE ABSTRACTION

A state-action abstraction aggregates the state-action pairs with similar properties, resulting in an abstract state-action space denoted as $[N]$, where N is the size of abstract state-action space. In this paper, we consider a new form of abstraction, Z^π-irrelevance, defined as follows: Given a policy π, Z^π-irrelevance abstraction is denoted as $\phi : \mathcal{X} \to [N]$ such that, for any $x_1, x_2 \in \mathcal{X}$ with $\phi(x_1) = \phi(x_2)$, we have $\mathbb{Z}^\pi(x_1) = \mathbb{Z}^\pi(x_2)$. Given a policy π and the parameter for return discretization K, we use $N_{\pi,K}$ to denote the minimum N such that a Z^π-irrelevance exists. It is true that $N_{\pi,K} \leq N_{\pi,\infty} \leq |\phi_B(\mathcal{S})|\,|\mathcal{A}|$ for any π and K, where $|\phi_B(\mathcal{S})|$ is the number of abstract states for the coarsest bisimulation (cf. Appendix A).

Proposition 4.1. *Given a policy π and any Z^π-irrelevance $\phi : \mathcal{X} \to [N]$, there exists a function $Q : [N] \to \mathbb{R}$ such that $|Q(\phi(x)) - Q^\pi(x)| \leq \frac{R_{\max} - R_{\min}}{K}, \forall x \in \mathcal{X}$.*

We provide a proof in Appendix A. Note that K controls the coarseness of the abstraction. When $K \to \infty$, Z^π-irrelevance can accurately represent the value function and therefore the optimal policy when $\pi \to \pi^*$. When using an auxiliary task to learn such abstraction, this proposition indicates that the auxiliary task (to learn a Z^π-irrelevance) is well-aligned with the main RL task (to approximate Q^*). However, large K results in a fine-grained abstraction which requires us to use a large N and more samples to learn the abstraction (cf. Theorem 4.1). In practice, this may not be a problem since we learn a state-action representation in a low-dimensional space $\mathbb{R}^d$ instead of $[N]$ and reuse the samples collected by the base RL algorithm. Also, we do not need to choose a K explicitly in the practical algorithm (cf. Section 4.3).

4.2 Z-LEARNING

We propose Z-learning to learn Z^π-irrelevance based on a dataset $\mathcal{D}$ with a contrastive loss (see Algorithm 1). Each tuple in the dataset is collected as follows: First, two state-action pairs are drawn i.i.d. from a distribution $d \in \Delta^{\mathcal{X}}$ with $\text{supp}(d) = \mathcal{X}$ (cf. Line 4 in Algorithm 1). In practice, we can sample state-action pairs from the rollouts generated by the policy π. In this case, a stochastic policy (e.g., using ϵ-greedy) with a standard ergodic assumption on MDP ensures $\text{supp}(d) = \mathcal{X}$. Then, we obtain returns for the two state-action pairs (i.e., the discounted sum of the rewards after x_1 and x_2) which can be obtained by rolling out using the policy π (cf. Line 5 in Algorithm 1). The binary label y for this state-action pair indicates whether the two returns belong to the same bin (cf. Line 6 in Algorithm 1). The contrastive loss is defined as follows:

$$\min_{f \in \mathcal{F}_N} \mathcal{L}(f, \mathcal{D}) := \mathbb{E}_{(x_1, x_2, y) \sim \mathcal{D}} \left[(w(\phi(x_1), \phi(x_2)) - y)^2 \right], \tag{1}$$

which is optimized over a function class $\mathcal{F}_N$. To define $\mathcal{F}_N$, we first define a tabular regressor class $\mathcal{W}_N := \{[N] \times [N] \to [0, 1]\}$ and a class of encoders that map the state-action pairs to N discrete abstractions $\Phi_N := \{\mathcal{X} \to [N]\}$. Then, we set $\mathcal{F}_N := \{(x_1, x_2) \to w(\phi(x_1), \phi(x_2)) : w \in \mathcal{W}_N, \phi \in \Phi_N\}$. Notice that we choose $N \geq N_{\pi,K}$ to ensure that a Z^π-irrelevance $\phi : \mathcal{X} \to [N]$ exists. However, to aggregate the state-action pairs, N should be smaller than $|\mathcal{S}|\,|\mathcal{A}|$ (otherwise we will obtain an identity mapping). In this case, mapping two state-action pairs with different return distributions to the same abstraction will increase the loss and therefore is avoided. The following theorem shows that Z-learning can learn Z^π-irrelevance provably efficiently.

Theorem 4.1. *Given the encoder $\hat{\phi}$ returned by Algorithm 1, the following inequality holds with probability $1-\delta$:*

$$\mathbb{E}_{x_1,x,x'\sim d}\Big[\mathbb{I}[\hat{\phi}(x)=\hat{\phi}(x')]\,\Big|\mathbb{Z}^\pi(x_1)^T\left(\mathbb{Z}^\pi(x)-\mathbb{Z}^\pi(x')\right)\Big|\Big] \leq \sqrt{\frac{8N}{n}\Big(3+4N^2\ln n+4\ln|\Phi_N|+4\ln(\frac{2}{\delta})\Big)}, \tag{2}$$

where $|\Phi_N|$ is the cardinality of encoder function class.

We provide the proof in Appendix B. The theorem shows that whenever $\hat{\phi}$ maps two state-actions x, x' to the same abstraction, $\mathbb{Z}^\pi(x) \approx \mathbb{Z}^\pi(x')$ up to an error proportional to $1/\sqrt{n}$ (ignoring the logarithm factor), where n is the size of the dataset. The following corollary shows that $\hat{\phi}$ becomes a Z^π-irrelevance when $n \to \infty$.

Corollary 4.1.1. *The encoder $\hat{\phi}$ returned by Algorithm 1 with $n \to \infty$ is a Z^π-irrelevance, i.e., for any $x_1, x_2 \in \mathcal{X}$, $\mathbb{Z}^\pi(x_1) = \mathbb{Z}^\pi(x_2)$ if $\hat{\phi}(x_1) = \hat{\phi}(x_2)$.*

4.3 Return-based contrastive learning for RL (RCRL)

We adapt Z-learning as an auxiliary task that helps the agent to learn a representation with meaningful semantics. The auxiliary task based RL algorithm is called RCRL and shown in Algorithm 2. Here, we use Rainbow (for discrete control) and SAC (for continuous control) as the base RL algorithm for RCRL. However, RCRL can also be easily incorporated with other model-free RL algorithms. While Z-learning relies on a dataset sampled by rolling out the current policy, RCRL constructs such a dataset using the samples collected by the base RL algorithm and therefore does not require additional samples, e.g., directly using the replay buffer in Rainbow or SAC (see Line 5 in Algorithm 2). For the compound function to optimize $f(x_1, x_2) = w(\phi(x_1), \phi(x_2))$ in Z-learning, we use the state-action embedding network that is shared with the base RL algorithm as $\phi : \mathcal{X} \to \mathbb{R}^d$, and use an additional discriminator trained by the auxiliary task as $w : \mathbb{R}^d \times \mathbb{R}^d \to [0, 1]$.

However, when implementing Z-learning as the auxiliary task, the labels in the dataset may be unbalanced. Although this does not cause problems in the theoretical analysis since we assume the Bayes optimizer can be obtained for the contrastive loss, it may prevent the discriminator from learning properly in practice (cf. Line 8 in Algorithm 1). To solve this problem, instead of drawing samples independently from the replay buffer $\mathcal{B}$ (analogous to sampling from the distribution d in Z-learning), we sample the pairs for $\mathcal{D}$ as follows: As a preparation, we cut the trajectories in $\mathcal{B}$ into segments, where each segment contains state-action pairs with similar returns. For each sample in $\mathcal{D}$, we first draw an anchor state-action pair from $\mathcal{B}$ randomly. Afterwards, we generate a positive sample by drawing a state-action pair from the same segment of the anchor state-action pair. Then, we draw another state-action pair randomly from $\mathcal{B}$ and use it as the negative sample. As the successive state-action pairs with different returns are harder to distinguish, we can also construct harder auxiliary tasks by drawing the negative state-action pair from a different segment or adjacent segments within the same trajectory of the anchor.

Algorithm 2: Return based Contrastive learning for RL (RCRL)

1 Initialize the embedding $\phi_\theta : \mathcal{X} \to \mathbb{R}^d$ and a discriminator $w_\vartheta : \mathbb{R}^d \times \mathbb{R}^d \to [0, 1]$
2 Initialize the parameters φ for the base RL algorithm (SAC or Rainbow) that uses the learned embedding
3 A replay buffer $\mathcal{B} = \emptyset$
4 **foreach** *iteration* **do**
5 Rollout the current policy and store the samples to the replay buffer $\mathcal{B}$
6 Update θ and φ using the base RL algorithm
7 Draw a batch of samples $\mathcal{D}$ from the replay buffer $\mathcal{B}$
8 Update θ and ϑ by minimizing $\mathcal{L}(f, \mathcal{D})$ where $f(x_1, x_2) := w_\vartheta(\phi_\theta(x_1), \phi_\theta(x_2))$
9 **end**
10 **return** *The learned policy*

We believe our auxiliary task may boost the learning, due to better return-induced representations that facilitate generalization across different state-action pairs. Learning on one state-action pair will

affect the value of all state-action pairs that share similar representations with this pair. When the embedding network assigns similar representations to similar state-action pairs (e.g., sharing similar distribution over returns), the update for one state-action pair is representative for the updates for other similar state-action pairs, which improves sample efficiency. However, such generalization may not be achieved by the base RL algorithm since, when trained by the algorithm with only a return-based loss, similar state-action pairs may have similar Q values but very different representations. One may argue that, we can adopt several temporal difference updates to propagate the values for state-action pairs with same sampled return, and finally all such pairs are assigned with similar Q values. However, since we adopt a learning algorithm with bootstrapping/temporal difference learning and frozen target network in deep RL, it could take longer time to propagate the value across different state-action pairs, compared with direct generalization over state-action pairs with contrastive learning.

Meanwhile, since we construct auxiliary tasks based on return, which is a very different structure from image augmentation or temporal structure, our method could be combined with existing methods to achieve further improvement.

4.4 Network Structure for State-action Embedding

In our algorithm, the auxiliary task is based on the state-action embedding, instead of the state embedding that is frequently used in the previous work (e.g., Srinivas et al., 2020). To facilitate our algorithm, we design two new structures for Atari 2600 Games (discrete action) and DMControl Suite (continuous action) respectively. We show the structure in Figure 1. For Atari, we learn an action embedding for each action and use the element-wise product of the state embedding and action embedding as the state-action embedding. For DMControl, the action embedding is a real-valued vector and we use the concatenation of the action embedding and the state embedding.

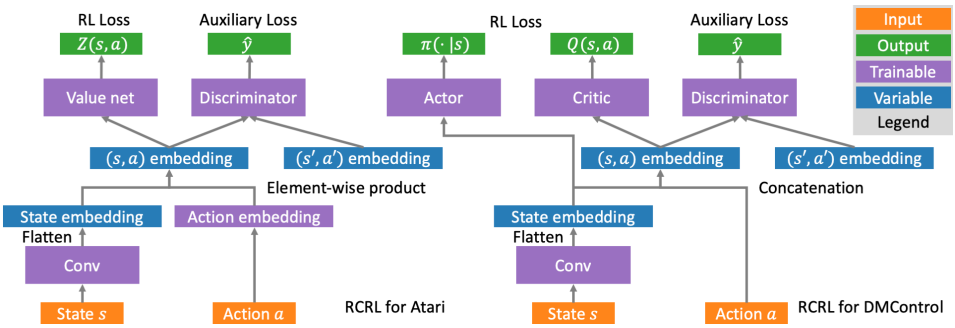

Figure 1: The network structure of RCRL for Atari (left) and DMControl Suite (right).

5 Experiment

In this section, we conduct the following experiments: 1) We implement RCRL on Atari 2600 Games (Bellemare et al., 2013) and DMControl Suite (Tassa et al., 2020), and compare with state-of-the-art model-based and model-free methods. Specifically, we compare with CURL (Srinivas et al., 2020), a top performing auxiliary task based RL algorithm for pixel-based control tasks that also uses a contrastive loss. In addition, we also combine RCRL with CURL to study whether our auxiliary task further boosts the learning when combined with other auxiliary tasks. 2) To further study the reason why our algorithm works, we analyze the generalization of the learned representation. Specifically, we compare the cosine similarity between the representations of different state-action pairs. We provide the implementation details in Appendix C.

5.1 Evaluation on Atari and DMControl

Experiments on Atari. Our experiments on Atari are conducted in low data regime of 100k interactions between the agent and the environment, which corresponds to two hours of human play. We show the performance of different algorithms/baselines, including the scores for average human

(Human), SimPLe (Kaiser et al., 2019) which is a strong model-based baseline for this regime, original Rainbow (Hessel et al., 2017), Data-Efficient Rainbow (ERainbow, van Hasselt et al., 2019), ERainbow with state-action embeddings (ERainbow-sa, cf. Figure 1 Left), CURL (Srinivas et al., 2020) that is based on ERainbow, RCRL which is based on ERainbow-sa, and the algorithm that combines the auxiliary loss for CURL and RCRL to ERainbow-sa (RCRL+CURL). We show the evaluation results of our algorithm and the baselines on Atari games in Table 1. First, we observe that using state-action embedding instead of state embedding in ERainbow does not lead to significant performance change by comparing ERainbow with ERainbow-sa. Second, built upon ERainbow-sa, the auxiliary task in RCRL leads to better performance compared with not only the base RL algorithm but also SimPLe and CURL in terms of the median human normalized score (HNS). Third, we can see that RCRL further boosts the learning when combined with CURL and achieves the best performance for 7 out of 26 games, which shows that our auxiliary task can be successfully combined with other auxiliary tasks that embed different sources of information to learn the representation.

	Human	SimPLe	Rainbow	ERainbow	ERainbow-sa	CURL	RCRL	RCRL+CURL
ALIEN	7127.7	616.9	318.7	739.9	813.8	558.2	854.2	**912.2**
AMIDAR	1719.5	88.0	32.5	**188.6**	154.2	142.1	157.7	125.1
ASSAULT	742.0	527.2	231.0	431.2	576.2	**600.6**	569.6	588.4
ASTERIX	8503.3	**1128.3**	243.6	470.8	697.0	734.5	799.0	683.0
BANK HEIST	753.1	34.2	15.6	51.0	96.0	**131.6**	107.2	99.0
BATTLE ZONE	37187.5	5184.4	2360.0	10124.6	13920.0	14870.0	14280.0	**17380.0**
BOXING	12.1	**9.1**	-24.8	0.2	2.2	1.2	2.7	6.7
BREAKOUT	30.5	**16.4**	1.2	1.9	3.4	4.9	4.3	4.0
CHOPPER COMMAND	7387.8	1246.9	120.0	861.8	1064.0	1058.5	**1262.0**	1008.0
CRAZY CLIMBER	35829.4	**62583.6**	2254.5	16185.3	21840.0	12146.5	15120.0	15032.0
DEMON ATTACK	1971.0	208.1	163.6	508.0	768.0	**817.6**	790.4	618.3
FREEWAY	29.6	20.3	0.0	**27.9**	26.5	26.7	26.6	25.4
FROSTBITE	4334.7	254.7	60.2	866.8	1472.0	1181.3	1337.6	**1516.6**
GOPHER	2412.5	**771.0**	431.2	349.5	384.8	669.3	429.6	458.8
HERO	30826.4	2656.6	487.0	6857.0	4787.9	6279.3	6454.1	**7647.4**
JAMESBOND	302.8	125.3	47.4	301.6	308.0	471.0	314.0	**503.0**
KANGAROO	3035.0	323.1	0.0	779.3	732.0	872.5	842.0	**932.0**
KRULL	2665.5	**4539.9**	1468.0	2851.5	2740.0	4229.6	2997.5	3905.8
KUNG FU MASTER	22736.3	**17257.2**	0.0	14346.1	11914.0	14307.8	9762.0	11856.0
MS PACMAN	6951.6	1480.0	67.0	1204.1	1384.5	1465.5	**1555.2**	1336.8
PONG	14.6	**12.8**	-20.6	-19.3	-18.3	-16.5	-16.9	-18.72
PRIVATE EYE	69571.3	58.3	0.0	97.8	80.0	218.4	102.6	**282.3**
QBERT	13455.0	**1288.8**	123.5	1152.9	893.5	1042.4	1121.0	942.0
ROAD RUNNER	7845.0	5640.6	1588.5	**9600.0**	5392.0	5661.0	6138.0	5392.0
SEAQUEST	42054.7	**683.3**	131.7	354.1	402.0	384.5	375.6	489.6
UP N DOWN	11693.2	3350.3	504.6	2877.4	3235.2	2955.2	**4210.2**	3127.8
Median HNS	100.0%	14.4%	0.0%	16.1%	16.7%	17.5%	18.5%	**19.6%**

Table 1: Scores of different algorithms/baselines on 26 games for Atari-100k benchmark. We show the mean score averaged over five random seeds.

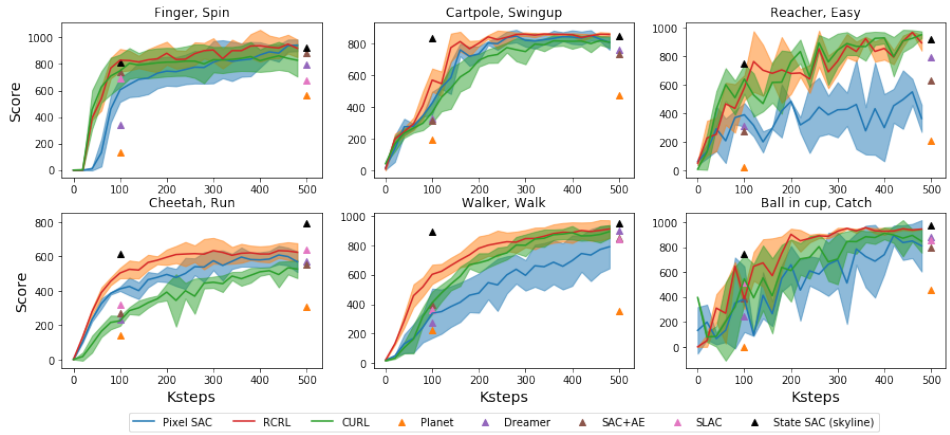

Figure 2: Scores achieved by RCRL and other baseline algorithms during the training for different tasks in DMControl suite. The line and the shaded area indicate the average and the standard deviation over 5 random seeds respectively.

Experiments on DMControl. For DMControl, we compare our algorithm with the following baselines: Pixel SAC (Haarnoja et al., 2018) which is the base RL algorithm that receives images as

the input; SLAC (Lee et al., 2019) that learns a latent variable model and then updates the actor and critic based on it; SAC+AE (Yarats et al., 2019) that uses a regularized autoencoder for reconstruction in the auxiliary task; PlaNet (Hafner et al., 2019b) and Dreamer (Hafner et al., 2019a) that learn a latent space world model and explicitly plan with the learned model. We also compare with a skyline, State SAC, the receives the low-dimensional state representation instead of the image as the input. Different from Atari games, tasks in DMControl yield dense reward. Consequently, we split the trajectories into segments using a threshold such that the difference of returns within each segment does not exceed this threshold. Similarly, we test the algorithms in low data regime of 500k interactions. We show the evaluation results in Figure 2. We can see that our auxiliary task not only brings performance improvement over the base RL algorithm but also outperforms CURL and other state-of-the-art baseline algorithms in different tasks. Moreover, we observe that our algorithm is more robust across runs with different seeds compared with Pixel SAC and CURL (e.g., for the task *Ball in cup, Catch*).

5.2 Analysis on the Learned Representation

We analyze on the learned representation of our model to demonstrate that our auxiliary task attains a representation with better generalization, which may explain why our algorithm succeeds. We use cosine similarity to measure the generalization from one state-action pair to another in the deep learning model. Given two state-action pairs $x_1, x_2 \in \mathcal{X}$, cosine similarity is defined as $\frac{\phi_\theta(x_1)^T \phi_\theta(x_2)}{||\phi_\theta(x_1)||\,||\phi_\theta(x_2)||}$, where $\phi_\theta(\cdot)$ is the learnt embedding network.

We show the cosine similarity of the representations between positive pairs (that are sampled within the same segment and therefore likely to share similar return distributions) and negative pairs (i.e., randomly sampled state-action pairs) during the training on the game *Alien* in Figure 3. First, we observe that when a good policy is learned, the representations of positive pairs are similar while those of negative pairs are dissimilar. This indicates that a good representation (or the representation that supports a good policy) aggregates the state-action pairs with similar return distributions. Then, we find that our auxiliary loss can accelerate such generalization for the representation, which makes RCRL learn faster.

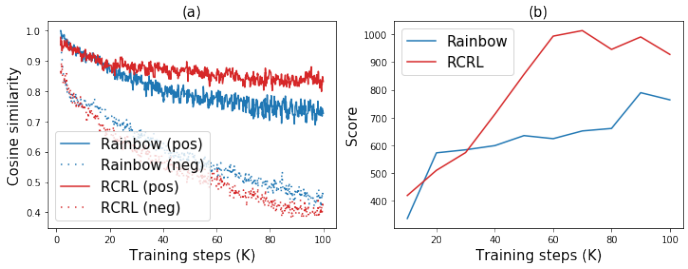

Figure 3: Analysis of the learned representation on *Alien*. (a) The cosine similarity between the representations of the positive/negative state-action pair and the anchor during the training of SAC and RCRL. (b) The scores of the two algorithms during the training.

6 Conclusion

In this paper, we propose return-based contrastive representation learning for RL (RCRL), which introduces a return-based auxiliary task to facilitate policy training with standard RL algorithms. Our auxiliary task is theoretically justified to learn representations that capture the structure of Z^π-irrelevance, which can reduce the size of the state-action space as well as approximate the Q values arbitrarily accurately. Experiments on Atari games and the DMControl suite demonstrate that our algorithm achieves superior performance not only when using our auxiliary task alone but also when combined with other auxiliary tasks.

As for future work, we are interested in how to combine different auxiliary tasks in a more sophisticated way, perhaps with a meta-controller. Another potential direction would be providing a theoretical analysis for auxiliary tasks and justifying why existing auxiliary tasks can speed up deep RL algorithms.

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