# QMAMBAEXTEND: IMPROVING LONG-CONTEXT EXTENSION OF MEMORY-EFFICIENT MAMBA MODELS

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## ABSTRACT

Despite its impressive sub-quadratic compute efficiency, Mamba's effectiveness is significantly limited by its pre-training context length, resulting in a pronounced degradation when the model is tasked with handling longer contexts. This may be attributed to the out-of-distribution (OOD) discretization steps of Mamba on longer contexts. To address this critical limitation, we introduce ***MambaExtend***, a novel framework designed to enhance the context extension capabilities of Mamba. Specifically, MambaExtend leverages a ***training-free*** approach to calibrate *only* the scaling factors of discretization modules for different layers. To further enhance the model efficiency while improving their long context understanding, we benchmark the performance of quantized model variants, namely *QMambaExtend*. With this, for the first time, we can enable a training-free context extension of up to $\mathbf{32}\times$ from 2k to 64k, that too requiring up to $\mathbf{2.1}\times$ reduced weight memory footprint. The codes is available here[1].

## 1 INTRODUCTION

Recently, state-space models (SSMs) (Gu et al., 2022; 2020) have emerged as an alternative to attention-based models (Vaswani, 2017), offering a different approach to handling long sequences at sub-quadratic complexity. Unlike transformers, SSMs offer the potential to handle much longer sequences without blowing out the memory and compute demand. Mamba (Gu & Dao, 2023; Dao & Gu, 2024), a popular SSM variant built leveraging the selective state-space layers (S6), has shown impressive performance on various NLP, image, and medical genomics benchmarks (Schiff et al., 2024). The key advantage of Mamba stems from the sub-quadratic compute complexity of theoretically grounded linear RNN layers. However, Mamba models, despite their theoretical ability to capture global interactions, fail to generalize to long sequence or context lengths (Ben-Kish et al., 2024). This phenomenon has been tied to the Mamba model's implicit bias to a limited effective receptive field (ERF) governed by the training data sequence length (Ben-Kish et al., 2024).

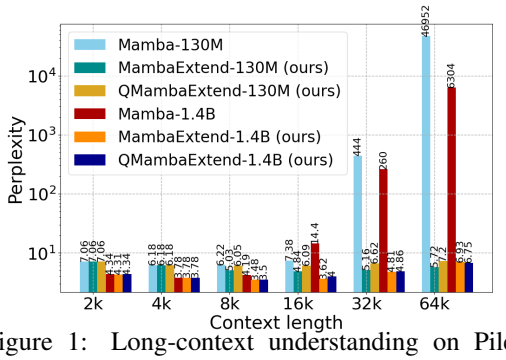


Figure 1: Long-context understanding on Pile. Compared to the pre-trained alternatives, MambaExtend provides up to $\sim\mathbf{8145}\times$ improvement in perplexity score, via a **training-free** calibration.

A contemporary work, namely DeciMamba (Ben-Kish et al., 2024), has presented a selective token *decimation* strategy to reduce the number of tokens to be processed per layer and increase the model's ERF. However, DeciMamba requires a memory- and compute-intensive fine-tuning of the model, resulting in significant time and effort to perform parameter updates of the pre-trained model.

[1]https://github.com/ArminAzizi98/LongContextMamba

**Our Contributions.** To mitigate the aforesaid issues, we first investigate the impact of OOD long-context extension on the discretization step of Mamba ($\Delta_t$ values, the step size that is used to transform continuous-time parameters to corresponding discrete state space variables). Interestingly, we have empirically found that *a scaled-down $\Delta_t$ can improve generalization on increased context length at inference time*. Based on this insight, we then present ***MambaExtend***, a framework designed to extend Mamba's context length **without any re-training** of the model weights. Specifically, MambaExtend employs a calibration function (`CF`) to optimize the discretization step sizes ($\Delta_t$) across various Mamba layers by introducing a learnable scaling factor associated with each layer's $\Delta_t$. The `CF` allows the proposed $\Delta_t$ scaling parameters to learn while freezing the model weights to their pre-trained values, reducing the required memory and updateable parameters by orders of magnitude. Notably, we present two `CF` approaches, namely, `CF` with back-propgation (BP), and that with zeroth-order (ZO) optimization (for fewer calibration parameters).

To further validate the effectiveness of MambaExtend on compressed models, we introduce *QMambaExtend*, applying MambaExtend to a quantized models. Our results demonstrate that QMambaExtend, with 8-bit and 6-bit model weights, achieves performance comparable to the full-precision counterparts, while having up to $\mathbf{2.1}\times$ weight memory saving. Fig. 1 demonstrates the ability of MambaExtend to improve the PPL by up to $\sim\mathbf{8145}\times$, as evaluated on context length of up to 64k.

## 2 Preliminaries and Motivation

**Mamba block.** One of the critical aspects of Mamba's architecture is how a Mamba block relates its input sequence $X = (x_1, x_2, \ldots, x_P)$ to its output sequence $Y = (y_1, y_2, \ldots, y_P)$ with $P$ corresponding to the sequence or context length. The relationship between the input and output of the Mamba block is expressed through a time-varying SSM described below:

$$G = \sigma(W_{gate_proj}X), Z = \texttt{Conv1D}(W_{in_proj}X) \tag{1}$$

$$O = \texttt{S6}(Z), Y = O \odot G \tag{2}$$

Here, $G$ is a gating function derived from a linear transformation of the input sequence $X$ followed by a SILU function, $\sigma$. The element-wise multiplication $\odot$ between $G$ and $O$ allows the model to selectively emphasize or attenuate parts of the input to focus on relevant input information. The input $Z$ to the `S6` is a linearly transformed version of the original input $X$ followed by a 1D convolution.

As demonstrated in these equations, the relationship between the last token $o_P$ and the first token is governed by the term $\alpha_{P,1} = C_P \prod_{k=2}^{P} \bar{A}_k \bar{B}_1 = C_P \texttt{exp}(A \sum_{k=2}^{P} \Delta_k) \bar{B}_1$. This means that the exponent of summed $\Delta_t$ determines the impact of the first token in the generation of the $P^{th}$ token. Consequently, the term $\texttt{exp}(-\sum \Delta_t)$ effectively governs the decay of influence from any previous token. A larger value of $\Delta_t$ results in greater forgetfulness, decreasing the model's reliance on earlier tokens. The increasing summation of $\Delta_t$ in longer context is referred to as the out-of-distribution (OOD) phenomenon in MambaExtend. Further details on the `S6` block, the role of $\Delta_t$, and its behavior in long-context scenarios can be found in Appendix A.1 and Appendix A.2.

**Motivation: Influence of scaled $\Delta_t$.** For transformer-based LLMs, a popular method for addressing the out-of-distribution (OOD) context length $P' > P$ (where $P$ represents the training context length) is positional interpolation (PI) (Chen et al., 2023). The PI method accomplishes this by multiplying the token index value in RoPE by $\frac{P}{P'}$. This rescaling ensures that the positional indices remain within a valid range, effectively mitigating the OOD problem associated with longer contexts without retraining.

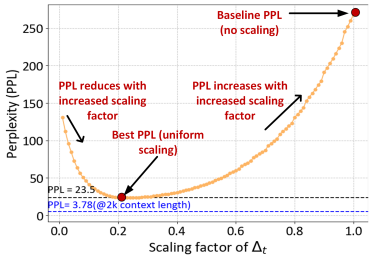


Figure 2: Impact of different values of uniform $\Delta_t$ scaling on the perplexity (PPL) metric.

Inspired by this, we propose a straightforward approach for Mamba to address the accumulated out-of-distribution (OOD) discretization steps by scaling the discretization matrix $\Delta_t$ by a fixed scalar value $s \leq 1$ across all model layers. This method aims to mitigate the OOD effects associated with longer context sizes. We utilized a pre-trained Mamba 1.4B to validate this approach, conducting a grid search over various values of $s$. We then evaluated the model's performance on the test set of the Pile dataset (Gao et al., 2020) for an evaluation context length of 32k tokens, reporting the average perplexity. The results, presented in Fig.

2, demonstrate that **scaling $\Delta_t$ can significantly reduce the model's PPL from approximately 268 to around 23.5**. However, the findings also indicate that the relationship between the choice of scaling value and performance improvement is not straightforward. As shown in Fig. 2, while increased scaling helps reduce the perplexity at lower values of (s), the PPL rises after reaching a certain threshold. This complex interplay encourages us to investigate the model's capacity to learn the optimal scaling. Additionally, **this uniform scaling factor cannot restore the model's performance for longer contexts to the level observed at its pre-trained context length**. For instance, the model achieves a PPL of 3.7 for a 2k context length, which remains significantly lower than the best PPL obtainable through uniform scaling.

## 3 MAMBAEXTEND METHODOLOGY

Motivated by the need to mitigate the $\sum \Delta_t$ OOD effects, we introduce MambaExtend, a *training-free* method for scaling the discretization steps of each layer. For an $L$-layer Mamba model, our primary objective is to determine the optimal scaling factors for each layer, denoted as $s_1, s_2, \ldots, s_L$, which will be used to adjust the discretization matrix $\Delta_t$. Note that for a layer $i$, $s_i \in \mathbb{R}^m$, where $m = 1$ indicates that $s_i$ is a scalar, and $m > 1$ indicates that $s_i$ is a vector. Without loss of generality, for $m = 1$, the discretization adjustment can be expressed as $\Delta'_{t_i} = s_i \Delta_{t_i}$, with $\Delta'_{t_i}$ applied during inference. The goal is to calibrate the newly introduced learnable parameters $s_i$ for all $i \in 1, \ldots, L$ in a way that is both memory- and compute-efficient, and *does not* involve any additional training or fine-tuning of the model parameters. These constraints will enable such calibration to be feasible on resource-limited edge devices.

In MambaExtend, we start from a pre-trained Mamba model as input, along with a small set of calibration samples from the target task and a specialized function known as the ***calibration function*** (`CF`). As its name implies, `CF` calibrates the learnable scaling factors. Importantly, unlike DeciMamba, which allows fine-tuning of the weights, MambaExtend keeps the model weights fixed to their pre-trained values (as indicated in Line 6 of Algorithm 1) throughout the calibration process. This approach makes MambaExtend significantly more compute- and memory-efficient compared to DeciMamba. Algorithm 1 (Appendix A.3) outlines the MambaExtend method.

**Calibration via back-propagation ($\texttt{CF}_{BP}$).** Gradient-based backpropagation is a widely used optimization method for updating the free (unfrozen) parameters on a calibration set. However, to minimize computational and memory overhead, we ensure parameter efficiency by restricting updates to the scaling factors **S** only. Algorithm 2 in the Appendix A.4 summarizes the $\texttt{CF}_{BP}$ algorithm for finding the optimal scaling factors. We utilize Adam as the optimizer for backpropagation (as noted in Line 4 of Algorithm 2). The `Evaluate()` function in Line 6 computes the loss of the model, which is parameterized by frozen weights and the learnable scaling factors **S**.

**Calibration via zeroth-order optimization ($\texttt{CF}_{ZO}$).** Zeroth-order optimization (Spall, 1992; Malladi et al., 2023) offers an efficient yet noisier method for calibration, as it relies solely on forward passes to approximate gradients. Algorithm 3 in Appendix A.4 outlines the process for optimizing the scaling factors **S** in $\texttt{CF}_{ZO}$. Specifically, this is a multi-iteration process in which, at each iteration, the scaling factors are randomly perturbed using a random variable $\delta$ sampled from a Rademacher distribution. The magnitude of the perturbation and the learning rate for the updates are controlled by the hyperparameters $c$ and $\eta$, respectively. We employ the two-sided variant of the simultaneous perturbation stochastic approximation method (SPSA) (Spall, 1992), which obtains gradient approximations by applying both positive and negative perturbations to the parameters simultaneously. The two-sided SPSA approach yields gradient estimates with lower variance than the one-sided version, thus enhancing accuracy, especially in noisy environments (Spall, 2005).

The convergence of the zeroth-order calibration method, $\texttt{CF}_{ZO}$, is affected by the number of parameters being optimized, specifically the size of **S**. Classical lower bounds indicate that convergence slows linearly as the number of parameters increases (Nemirovskij & Yudin, 1983; Duchi et al., 2015). Consequently, a natural strategy in our context is to employ the backpropagation-based method, $\texttt{CF}BP$, when optimizing a larger set of parameters in (**S**), while reserving $\texttt{CF}_{ZO}$ for smaller parameter sets. The details of the algorithm with the pseudo-code is outlined in the Appendix A.4.

Table 1: Perplexity for Mamba models over different evaluation context lengths on Pile dataset.

| | Mamba-130M | | | | | | Mamba-1.4B | | | | | | Mamba2-780M | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Context Length** | **2k** | **4k** | **8k** | **16k** | **32k** | **64k** | **2k** | **4k** | **8k** | **16k** | **32k** | **64k** | **2k** | **4k** | **8k** | **16k** | **32k** | **64k** |
| Pre-trained Model | 7.06 | 6.18 | 6.22 | 7.38 | 444 | 46592 | 4.34 | 3.78 | 4.19 | 14.4 | 260 | 6304 | 4.78 | 4.62 | 22.4 | 79 | 185 | 378 |
| MambaExtend | 7.06 | 6.18 | **5.03** | **4.84** | **5.16** | **5.72** | **4.31** | 3.78 | **3.48** | **3.62** | **4.81** | **6.93** | **4.59** | **3.95** | **3.89** | **4.25** | **5.56** | **5.00** |

Table 2: Mamba vs MambaExtend performance on representative LongBench tasks.

| **Model** | **Qasper** | **HotpotQA** | **2WikiMultihopQA** | **TREC** | **TriviaQA** | **LCC** | **RepoBench-P** | **Average** |
|---|---|---|---|---|---|---|---|---|
| Mamba-1.4B | 7.0 | 11.00 | 9.75 | 29.00 | 1.67 | 20.12 | 11.67 | 12.88 |
| MambaExtend-1.4B | **16.67** | **14.29** | **13.82** | **35.0** | **7.67** | **26.12** | **18.84** | **18.91** |
| Mamba2-780M | 7.50 | 6.06 | 9.48 | 17.0 | 0.1 | 22.1 | 14.01 | 10.89 |
| MambaExtend2-780M | **7.96** | **10.95** | **18.33** | **28.00** | **6.83** | **28.27** | **17.71** | **16.86** |

# 4 EXPERIMENTS

This section evaluates the performance and efficiency of our proposed MambaExtend. In specific, we first detail on the models and datasets used for our experiments. We then present extensive empirical results to outline our findings in terms of long-context performance of the Mamba model variants. We finally discuss on the compute, time, and memory requirements for MambaExtend.

## 4.1 EXPERIMENTAL SETUP

**Models and datasets**. To evaluate the performance of MambaExtend, we use both long-context understanding and long-context retrieval ability tasks. For long-context understanding, we use the Pile (Gao et al., 2020) and PG-19 (Rae et al., 2019) datasets and assess the performance of the MambaExtend in terms of perplexity scores at various context lengths. We use Mamba-130M, Mamba-1.4B (Gu & Dao, 2023), and Mamba2-780M (Dao & Gu, 2024) for these evaluations. Additionally, we use the LongBench benchmark (Bai et al., 2023) to evaluate the performance accuracy of the Mamba-1.4B and Mamba2-780M models. For the passkey retrieval task, we follow the setup described in (Ben-Kish et al., 2024) and evaluate the performance of the Mamba-130M and Mamba-1.4B models in retrieving a 5-digit code embedded at a random sequence depth within samples from the WikiText-103 dataset (Merity et al., 2016). In our retrieval setup, the input sequence lengths range from 1K to 64K tokens.

## 4.2 EXPERIMENTAL RESULTS

**Perplexity evaluations on PG-19 and Pile**. To evaluate perplexity (PPL) on the Pile and PG-19, we use twenty calibration samples from the corresponding training set for a given context length. We use these samples to learn the scaling factors in MambaExtend, then evaluate perplexity on the test set for a given context length. As stated earlier for the perplexity evaluation, for each layer $i$, we use a single scaling factor $s_i \in \mathbb{R}_+$ per layer[2], that scales the $\Delta_t$ tensor uniformly for that layer. Fig. 3 depicts the performance of MambaExtend compared to the pre-trained Mamba and DeciMamba. Specifically, at 70k context length, MambaExtend-130M yields a PPL of 30.62, a $\sim\mathbf{32506}\times$ improvement over the baseline that fails to provide a very high PPL of 995328. Compared to the DeciMamba, it shows consistent benefit with reduced PPL of up to $\sim\mathbf{40.6}\%$.

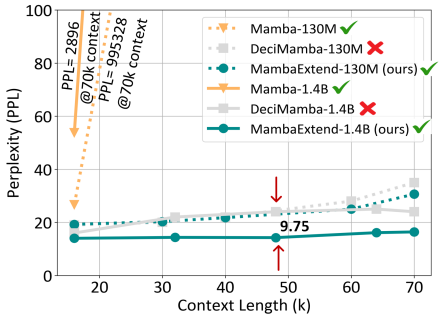


Figure 3: PPL comparison on PG-19. The ✓ and ✗ identify the fine-tuning requirements to be false and true, respectively.

Table 1 reports the PPL values of MambaExtend models and compares them to those of the pre-trained models on Pile. As shown in the table, MambaExtend through only minimal calibration, allows the models to maintain their performance even with increasing context lengths. Specifically, MambaExtend can improve the PPL by up to $\sim\mathbf{8145}\times$ at longer contexts.

[2]This may be attributed to the relatively simpler nature of long-context understanding as opposed to long-context retrieval, as for the later we need more fine-grain scaling increasing the number of calibration params.

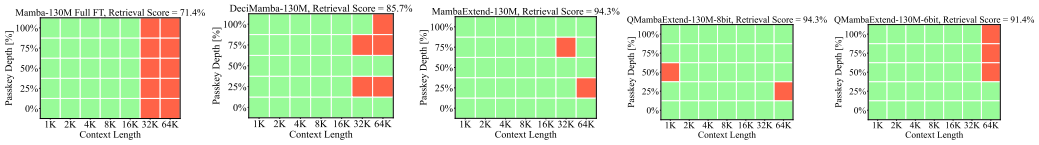


Figure 4: Passkey retrieval performance after fine-tuning (for Mamba and DeciMamba) or calibrating (for MambaExtend and QMambaExtend) on samples of 4k length.

Table 3: QMambaExtend perplexity results on Pile and PG-19 datasets.

| Model | Weight Memory (GB) | Pile Perplexity | | | | PG-19 Perplexity | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 8k | 16k | 32k | 64k | 8k | 16k | 32k | 64k |
| MambaExtend-130M (W16/A16) | 0.25 | 5.03 | 4.84 | **5.16** | 5.72 | 19.2 | 19.25 | 20.3 | 27.2 |
| QMambaExtend-130M (W8/A16) | **0.12** | 6.05 | 6.09 | 6.62 | 7.2 | 20.2 | 20.2 | 21.4 | 27.7 |
| MambaExtend-1.4B (W16/A16) | 2.61 | **3.48** | **3.62** | **4.81** | 6.93 | **13.78** | **14.0** | **13.34** | 16.12 |
| QMambaExtend-1.4B (W8/A16) | **1.31** | 3.50 | 4.0 | 4.85 | **6.75** | 13.78 | 14.0 | 15.1 | **16.10** |

**LongBench**. For MambaExtend, we use seven popular tasks from LongBenchBai et al. (2023). Due to the lack of training data, we used 10 samples from the 4K-8K split of each dataset as calibration data and the remaining samples from the same split to evaluate. We apply the $\texttt{CF}_{ZO}$ calibration function to learn the scaling factors. Similar to the calibration setup for perplexity evaluation, we calibrate one scaling factor per layer shared over the whole $\Delta_t$ tensor for that layer. As demonstrated in the Table 2, **MambaExtend can improve the average LongBench accuracy by up to** $6.03\%$.

**Passkey Retrieval**. Previous works have demonstrated that tasks requiring exact retrieval are more challenging than achieving low perplexity in longer context (Liu et al., 2024), so we use more fine-grained sharing of scaling factors to optimize. For $\Delta_t$ tensor of a layer $i$, we use one scaling factor per channel yielding total $D$ scaling factors per layer ($s_i \in \mathbb{R}_+^D$). Unless otherwise stated, we use $\texttt{CF}_{BP}$ for *one* epoch to calibrate on a dataset with 4k context length. For the baseline, we performed standard fine-tuning with the same context length for one epoch as we get significant failure in the retrieval. For DeciMamba to have a fair comparison, we fine-tune for the same epochs as ours[3].

The result for 130M model is showns in Fig. 4. Although it calibrates approximately $\mathbf{3500}\times$ **fewer parameters for Mamba-130M, it performs better then the two alternatives** The result for Mamba-1.4B is demonstrated in the Appendix Fig. 8.

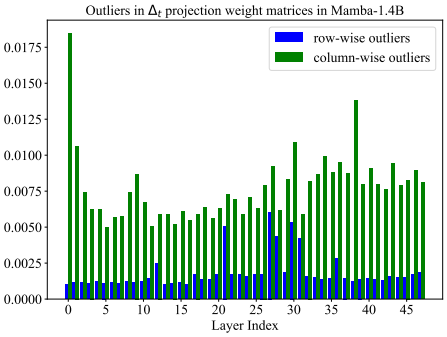


Figure 5: Normalized outlier count across different layers of a pre-trained Mamba. Outliers are identified as elements that deviate from the mean by more than $3\sigma$, with $\sigma$ as the standard deviation.

**Quantization.**The quantized variant of the context-extended model, QMambaExtend, is obtained by applying MambaExtend directly to a quantized model. Following Yao et al. (2022), we apply RTN linear integer quantization to the parameters of the pretrained Mamba model. Since quantization granularity significantly impacts quantization error, we analyze outlier counts in groups of 128 elements across both rows and columns of the $\Delta_{t,\text{proj}}$ weight matrix. We then plot the number of outliers normalized by the total number of elements in $\Delta_{t,\text{proj}}$ in Fig. 5. As the outlier counts in all layers are substantially higher in each column (channel), we opt for row-wise quantization.

After quantizing the model weights, we apply the MambaExtend scaling factors, which are retained in FP16 format to preserve performance. These scaling factors are calibrated using either $\texttt{CF}_{ZO}$ (for Pile and PG-19) or $\texttt{CF}_{BP}$ (for passkey retrieval). The perplexity results of QMambaExtend-130M and QMambaExtend-140B are presented in Table 3. The competitive performance of the quantized model with $\mathbf{2.1}\times$ weight memory reduction relative to the FP16 (floating-point) model demonstrates the effectiveness of MambaExtend, even when applied to a quantized model. Furthermore, Fig. 4 also demonstrates the effectiveness of MambaExtend for extending the context of INT8, and INT6 quantized models on this critical passkey retrieval task. In fact, QMambaExtend with INT8 quantization for model parameters, matches the performance of MambaExtend and surpasses DeciMamba.

[3]In the original paper (Ben-Kish et al., 2024) the model was fine-tuned for longer duration, however we focus on limited resource calibration and thus keep our experiments limited to fine-tuning for one epoch. Please see Appendix for fine-tuning results with longer epochs.

## 5 CONCLUSIONS

We addressed the limitations of Mamba in handling long-context tasks by introducing MambaExtend, a framework to extend the context length of Mamba models without model training. Through calibration of the discretization matrix ($\Delta_t$) scaling factors across different layers of the model, we enabled context extension by up to $32\times$ while maintaining similar perplexity levels that too at up to $2.1\times$ lower weight memory footprint.

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# A Appendix

## A.1 The S6 Layer and Mamba

At its core, each Mamba block utilizes the selective SSM (S6) layer (Gu & Dao, 2023), which is specifically designed to handle sequential data by preserving structured state dynamics across the input sequence.

**The S6 layer**: Using a **linear recurrent system** with the hidden state $h_t$, input $z_t$, and output $o_t$ at discrete time instant $t$, the S6 layer's sequence generation and state update can be simplified as:

$$h_t = \bar{A}h_{t-1} + \bar{B}z_t, o_t = Ch_t \tag{3}$$

The P-length sequence of a representative channel is given as $Z = \{z_1, z_2, \cdots, z_P\}$, $\bar{A} \in \mathbb{R}^{N\times N}$, $\bar{B} \in \mathbb{R}^{N\times 1}$, and $C \in \mathbb{R}^{1\times N}$ are discrete time-variant system, input, and output matrices, respectively, governing the discrete state transitions and output sequence generation. The S6 layer produces the 'per-time' ($t$) discrete time-variant matrices from input and "continuous parameters" as:

$$\bar{A}_t = \texttt{exp}(\Delta_t A), \bar{B}_t = \Delta_t B_t \text{ where } \Delta_t = \texttt{SFT}(\Delta_{t_{proj}}(z_t)), B_t = W_B(z_t), C_t = (W_C(z_t))^T \tag{4}$$

Here, $z_t \in \mathbb{R}^D$ with $D$ being channel dimension and $\Delta_t$ be the discretization step used at time $t$. $\Delta_{t_{proj}}$, $W_B$, and $W_C$ are linear projection layers. SFT and exp represent the *softplus* and pointwise *exponential* operation, respectively. After the discretization step, the S6 layer's input-output behavior via time-unrolling can be described as:

$$O = \alpha Z \text{ with } \alpha_{i,j} = C_i \left( \prod_{k=j+1}^{i} \bar{A}_k \right) \bar{B}_j \tag{5}$$

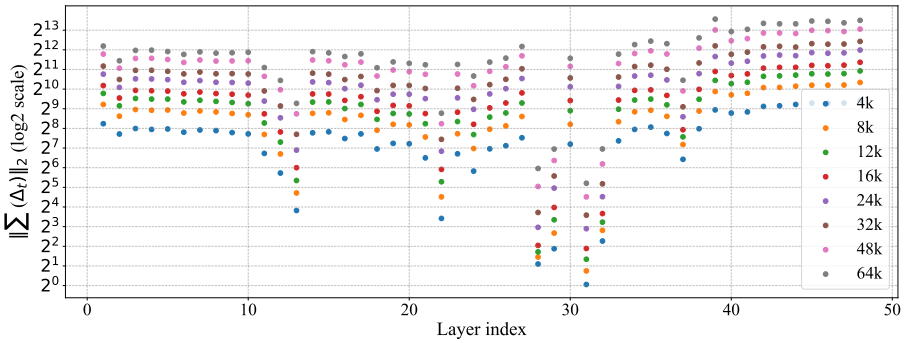



Figure 6: Layer-wise behavior of $\sum(\Delta_t)$ for different context length during test-time. We used the Pile dataset on Mamba 1.4B for the evaluation.

Thus, for a context length of $P$, the entire output $O = \{o_1, o_2, .., o_P\}$ is computed as follows:

$$\begin{pmatrix} o_1 \\ o_2 \\ \vdots \\ o_P \end{pmatrix} = \begin{pmatrix} C_1\bar{B}_1 & 0 & \cdots & 0 \\ C_2\bar{A}_2\bar{B}_1 & C_2\bar{B}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_P\prod_{k=2}^{P}\bar{A}_k\bar{B}_1 & C_P\prod_{k=3}^{P}\bar{A}_k\bar{B}_2 & \cdots & C_P\bar{B}_P \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_P \end{pmatrix} \tag{6}$$

This matrix formulation shows that each output $o_i$ is a weighted sum of the inputs $z_1, z_2, \ldots, z_P$, with the weights determined by the state-space matrices $\bar{A}$, $\bar{B}$, and $C$. The model can thus integrate information across different time steps while maintaining computational efficiency. This matrix resembles the attention score map in transformer-based models (Ali et al., 2024). In other words, S6 layers may be interpreted as data-controlled linear operators.

Notably, as these matrices are dynamically adjusted based on the input sequence, they enable the model to efficiently capture temporal dependencies across various time steps. This approach allows Mamba to maintain computational complexity that scales linearly with the context length.

### A.2 Motivational Case Studies

**The behavioral change of $\Delta_t$.** We first investigate the behavior of the accumulated discretization matrix $\Delta_t$ in the pre-trained Mamba-1.4B model when exposed to inputs of different context lengths. Using 100 samples from Pile, for each Mamba layer, we compute the $\|(\sum_{t=1}^{P'}\Delta_t)\|_2$ for different evaluation context lengths $P'$, where $||.||_2$ represents the $l_2$-norm of a tensor. We plot this analysis in Fig. 6, which reveals how the accumulation of $\Delta_t$ scales with increasing context length. Specifically, Fig. 6 discloses that for each layer of the model, the magnitude of $\sum\Delta_t$ increases with the increase in context lengths $P'$. According to Equations 5 and 6, and given that all entries of $A$ are always negative (Gu & Dao, 2023), we observe that the negative sum of $\Delta_t$ appears as the exponent in the $\texttt{exp}$ function. Consequently, the term $\texttt{exp}(-\sum\Delta_t)$ effectively governs the decay of influence from any previous token. A larger value of $\Delta_t$ results in greater forgetfulness, decreasing the model's reliance on earlier tokens. In contrast, smaller $\Delta_t$ values enable the model to retain information from more distant tokens. Therefore, $\texttt{exp}(-\sum_{t=n}^{P'}\Delta_t)$ can be interpreted as a parameter that potentially regulates the retention level for the $n$-th input to compute the token at $P'$.

**Variable impact of $\Delta_t$ on different layers.** Another important observation in Fig. 6 is that for a given test-time context length $P'$, different layers of the model produce significantly different $\sum\Delta_t$ values (even when viewed on a logarithmic scale). This underscores the point that **each layer should not employ the same scaling factor to reduce the impact of $\Delta_t$.** This observation motivates us to implement a heterogeneous (layer-specific) scaling mechanism across the various layers of the model to effectively address the OOD $\sum\Delta_t$.

**Algorithm 1** MambaExtend Algorithm

**Input:** An $L$-layer Mamba model parameterized by $\mathcal{M}$, set of calibration samples $\mathcal{C}$, calibration function `CF`
**Output:** Scaling factors $\mathbf{S} = [s_1, s_2, ..., s_L]$, where $s_i \in \mathbb{R}^m$
**for** $i \leq L$ **do**
    $s_i \leftarrow \texttt{init}(U(0,1))$
**end for**
$\texttt{freeze}(\mathcal{M})$
$\mathbf{S} \leftarrow \texttt{CF}(\mathbf{S}, \mathcal{C}, \mathcal{M})$
**return** $\mathbf{S}$

### A.3 MambaExtend Algorithm

### A.4 Calibration Functions

The algorithms 2 and 3 outline the detail o the $\texttt{CF}_{BP}$ and $\texttt{CF}_{BP}$, respectively. Our experiments show that long-context evaluation tasks, based on the perplexity measure, and the LongBench tasks require relatively fewer scaling factors. Specifically, for each layer $s_i \in \mathbb{R}+^m$, a setting of $m = 1$ is sufficient to improve PPL on long-context inputs. Here, $\mathbb{R}+$ represents the set of positive real numbers, as scaling factors cannot take negative values in our case. Any $s_i$ that updates to a negative value is clamped to a very small positive number to ensure this condition in our algorithm. We set $m = D$ for the passkey retrieval task, thereby increasing the number of parameters to be calibrated or updated. We empirically find that for the long-context tasks, $\texttt{CF}_{ZO}$ performs nearly as well as $\texttt{CF}_{BP}$. However, for the passkey retrieval task, we prefer $\texttt{CF}_{BP}$ due to its faster convergence trend compared to the zeroth-order method.

**Algorithm 2** $\texttt{CF}_{BP}$ Algorithm

**Input:** An $L$-layer Mamba model parameterized by frozen weights $\mathcal{M}$, set of calibration samples $\mathcal{C}$, the initialized scaling factors $\mathbf{S}$
**Input:** Learning rate $\eta$, number of iterations $K$
**Output:** Learned Scaling factors $\mathbf{S} = [s_1, s_2, ..., s_L]$, where $s_i \in \mathbb{R}_+^m$
`optimizer` = Adam($\mathbf{S}, \eta$)
**for** $k \leq K$ **do**
    $\mathcal{L}$ = $\texttt{Evaluate}(\mathcal{M}_{\Delta_t \times \mathbf{S}}, \mathcal{C})$
    $\mathcal{L}$.`backward()`
    `optimizer.step()`
    $\mathbf{S} \leftarrow \mathbf{S}.\texttt{clamp}\,(\min = 0.001)$ # make sure scaling factors remain positive
**end for**
return $\mathbf{S}$

### A.5 Detailed Hyperparameters

**$\texttt{CF}_{ZO}$ hyperparameters.** For Pile, PG-19, and LongBench dataset calibration, we set the ZO optimization hyperparameters to $\eta = 0.001$, $c = 0.1$, and $K = 50$.

**$\texttt{CF}_{BP}$ hyperparameters.** For the passkey retrieval task, we train the models for one epoch using Adam optimizer with learning-rate of 1e-1 for MambaExtend. For DeciMamba, and full fine-tuning we use the learning-rate to be 1e-4, as suggested by the authors Ben-Kish et al. (2024). For all three cases, we use a batch size of 32, a gradient clipping of 1.0, a weight decay of 0.1, and train on sequences of length 6144.

### A.6 Pre-trained Model Checkpoints Used

The pretrained model checkpoints of Mamba are taken from the Hugging Face model Hub[4]:

[4]https://github.com/state-spaces/mamba

**Algorithm 3** $\texttt{CF}_{ZO}$ Algorithm

**Input:** An $L$-layer Mamba model parameterized by $\mathcal{M}$, set of calibration samples $\mathcal{C}$, the initialized scaling factors $\mathbf{S}$
**Output:** Learned scaling factors $\mathbf{S} = [s_1, s_2, ..., s_L]$, where $s_i \in \mathbb{R}_+^m$
Specify learning rate $\eta$, perturbation magnitude $c$, number of iterations K
**for** $\mathrm{k} \leq \mathrm{K}$ **do**
    $\delta \in \mathbb{R}^{L \times m} \sim \texttt{Rademacher()}$
    $\mathbf{S}^+ = \mathbf{S} + c \times \delta, \quad \mathbf{S}^- = \mathbf{S} - c \times \delta$
    $\mathcal{L}^+ = \texttt{Evaluate}(\mathcal{M}_{\Delta_t \times \mathbf{S}^+}, \mathcal{C}), \quad \mathcal{L}^- = \texttt{Evaluate}(\mathcal{M}_{\Delta_t \times \mathbf{S}^-}, \mathcal{C})$
    $\hat{\nabla}_{\mathbf{S}} = (\mathcal{L}^+ - \mathcal{L}^-)/(2c\delta)$
    $\mathbf{S} \leftarrow \mathbf{S} - \eta \hat{\nabla}_{\mathbf{S}}$
    $\mathbf{S} \leftarrow \mathbf{S}.\texttt{clamp}\,(\min = 0.001)$ # make sure scaling factors remain positive
**end for**
return $\mathbf{S}$

- `state-spaces/mamba-130m`
- `state-spaces/mamba-1.4b`
- `state-spaces/mamba2-780m`

### A.7 More Results

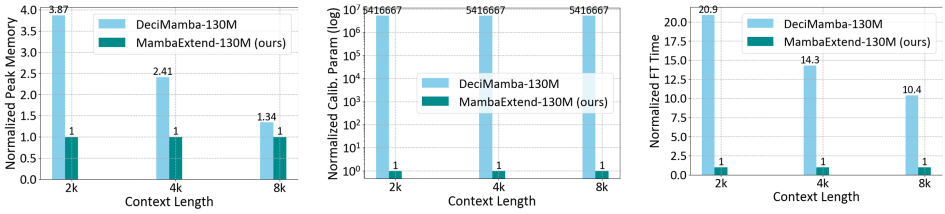



Figure 7: Comparison of normalized {*peak memory, number of parameter updates, and calibration/fine-tuning (FT) time (total)*} between DeciMamba, and MambaExtend for PG-19. We use Mamba-130M model for this evaluation.

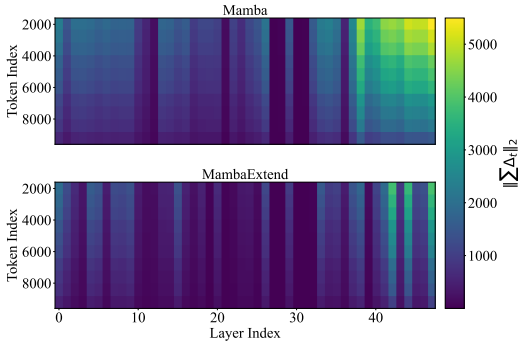
Table 4: Passkey retrieval performance of Mamba-130M with a different granularity of the scaling factor sharing, namely, per-channel, per-token, and per-tensor.

| Sharing granularity | # Params. ↓ | Retrieval Score (%) ↑ |
|---|---|---|
| Per-channel | 36.8K | **91.4** |
| Per-token | 98.3K | 62.8 |
| Per-tensor | **24** | 22.8 |

Fig. 7 demonstrates the performance comparison of DeciMamba and MambaExtend in terms of compute, memory, and time. For DeciMamba, we use the total training time of 5 epochs, to evaluate the normalized FT time. For MambaExtend, as we use ZO for the calibration, we report the time associated to the 50 iterations of calibrations. Notably, for MambaExtend we calibrate separately for each eval context length, while DeciMamba does one fine-tuning for 5 epochs with 2k context length. This causes the peak memory and fine-tuning time to increase for MambaExtend while keeping them constant for DeciMamba. For each evaluation metric, we performed the normalization by the corresponding value for MambaExtend at the context length under consideration.

Table 6: PPL comparison with transformer based LLM for long-context understanding on Pile.

| Model | 2K | 4K | 8K | 16K | 32K | 64K |
|---|---|---|---|---|---|---|
| TinyLLaMA1.1B (2K) | **4.6** | 62.6 | 426.6 | 1243.7 | 2684.6 | 3372.04 |
| TinyLLaMA1.1B-PI | **4.6** | 9.56 | 50.34 | 116.47 | 168.84 | 229.46 |
| MambaExtend-130M | 7.06 | **6.18** | **5.03** | **4.84** | **5.16** | **5.72** |




As Fig. 7 shows, MambaExtend requires $\sim\mathbf{5.42} * \mathbf{10^6}\times$ fewer parameter updates and costs up to $\mathbf{3.87}\times$ lower peak-memory. Additionally, MambaExtend provides up to $20.9\times$ faster calibration as opposed to the fine-tuning duration of DeciMamba.

### A.8 Discussion and Ablation Study

**Understanding the impact of learned scaling on $\Delta_t$.** To understand the impact of the learned scaling on the $\Delta_t$ discretization tensor, we compute the normalized sum of $\Delta_t$ $\|(\sum_{t=n}^{P'} \Delta_t)\|_2$. Here, $n$ refers to the token index whose impact we want to study on the output context length $P'$. $P'$ is set to 32k for this analysis. The Fig. A.7 demonstrates the heatmap of the $\|(\sum_{t=n}^{P'} \Delta_t)\|_2$ for different token index ($n$) at different layers of the model. Notably, as discussed earlier, high $\|(\sum_{t=n}^{P'} \Delta_t)\|_2$ value may be associated with a stronger decaying effect on the output token $P'$. As we can see, the original Mamba, particularly for later layers, induces a significant decaying effect for the earlier tokens (see the value for token index 2000 for layer index $> 40$). This finding aligns with that of Ben-Kish et al. (2024). MambaExtend, on the contrary, reduces this effect significantly, both overall and for later layers. This study highlights the benefit of the learned scaling in effectively controlling the $\Delta_t$. **Ablation on the granularity of scaling factor sharing.** In Table 4, we present the results with various levels of sharing of the scaling factor a layer's $\Delta_t$. Specifically, we allow per-channel, per-token, and per-tensor sharing where a scaling factor is shared over a channel, a token, and the whole tensor for a layer's $\Delta_t$, respectively. We calibrate for one epoch for three scenarios and measure the score on context. As we can see from the table, per-channel sharing can improve the retrieval score significantly. While per-tensor sharing requires considerably fewer calibration parameters, it fails to yield a good score, making per-channel sharing an optimal choice.

**Ablation on $\texttt{CF}_{BP}$ vs. $\texttt{CF}_{ZO}$.** For simpler long-context understanding tasks we demonstrated $\texttt{CF}_{ZO}$ to yield significantly improved PPL. In Table 5, we now demonstrate a direct comparison of the two calibration functions, namely, $\texttt{CF}_{ZO}$ and $\texttt{CF}_{BP}$ for Pile dataset. We used Mamba-130M for this experiment. As we can see, the perplexities for the three evaluation context lengths are similar for both of these methods. This experiment demonstrates the efficacy of $\texttt{CF}_{ZO}$ despite its efficient forward-pass-based gradient approximation approach, as opposed to the back-propagation-based alternative.

Table 5: Perplexity result with $\texttt{CF}_{ZO}$ v.s. $\texttt{CF}_{BP}$ on Pile dataset.

| CF\Context Length | 4K | 8K | 16K |
|---|---|---|---|
| $\texttt{CF}_{BP}$ | **6.10** | 5.11 | **4.79** |
| $\texttt{CF}_{ZO}$ | 6.18 | **5.03** | 4.84 |

### A.9 Comparison with Transformer-Based LLMs

Supporting longer context during inference is an equally important problem in transformer based LLMs, as compared to Mamba based LLMs. To have a broader picture on the long context extension results with Mamba models, in this section we compare our performance with that of the transformer based LLMs. In specific, we choose TinyLLaMA-1.1B model, trained on 2K context length as the

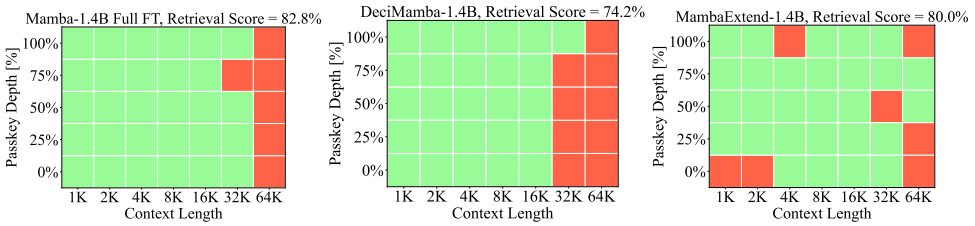



Figure 8: Passkey retrieval performance after fine-tuning (FT) (for Mamba-1.4B and DeciMamba-1.4B) or calibrating (for MambaExtend-1.4B) on samples of 4k context length. Although MambaExtend calibrates approximately $\mathbf{7100}\times$ fewer parameters, it performs better then the two alternatives.
.

baseline transformer model.Note, positional interpolation (PI) is a popular training-free method for the context extension of transformer models. As shown in the Table 6, the MambaExtend model despite being smaller, at longer context length consistently outperform the TinyLLaMA-1.1B both with and without PI. We additionally compare the results of TinyLLaMA1.1B (with and without PI) and MambaExtend on PG19, another popular benchmark for PPL evaluation on long context. As shown in Table 7, the results clearly shows the significant performance benefit of MambaExtend as opposed to the transformer based alternatives. Notably, with both smaller and similar sized models, MambaExtend significantly outperforms the TinyLLaMA model variants showcasing their benefits.

Table 7: PPL comparison with transformer based LLM for long-context understanding on PG19.

| Model | 16K | 32K | 64K |
|---|---|---|---|
| TinyLLaMA1.1B (2K) | 2236.98 | 4205.64 | 8664.11 |
| TinyLLaMA1.1B-PI | 226.69 | 300.46 | 375.49 |
| MambaExtend-130M | 19.25 | 20.3 | 25 |
| MambaExtend-1.4B | **14** | **14.34** | **16.12** |

### A.10 More Results and More Comparison with DeciMamba

Figure 8 illustrates the superior performance of MambaExtend compared to DeciMamba on the Mamba-1.4B model for the passkey retrieval task. The evaluation is conducted across context lengths of 1K, 2K, 4K, 8K, 16K, 32K, and 64K, with the target digit hidden at depths of 0%, 25%, 50%, 75%, and 100% of each of these sequence. Assuming that each correct retrieval receives a score of 1 and each incorrect retrieval receives a score of 0, we compute the *retrieval score* in percentage (%) as $\frac{\text{Total correct retrievals}}{\text{Total (correct + incorrect) retrievals}} * 100$, across all the depths overall context lengths. Also While in the main manuscript we demonstrate the benefits of MambaExtend over the baseline Mamba on Pile dataset, we now show comparison with DeciMamba (Ben-Kish et al., 2024) on the same. In specific, Table 8 demonstrates the efficacy of MambaExtend in maintaining the PPL better than DeciMamba, particularly at longer contexts with context length $\geq$ 8K. Additionally, we show results on LongBench to compare with that generated by DeciMamba in a zero-shot fashion. In specific, Table 9 shows that MambaExtend can yield reasonably improved performance as evaluated on HotpotQA and Qasper, respectively.

Table 8: PPL comparison between DeciMamba and MambaExtend on Pile.

| Model | 2K | 4K | 8K | 16K | 32K | 64K |
|---|---|---|---|---|---|---|
| DeciMamba-130M | **4.93** | **5.36** | 5.21 | 6.99 | 8.19 | 10.62 |
| MambaExtend-130M | 7.06 | 6.18 | **5.03** | **4.84** | **5.16** | **5.72** |

**Comparing the impact of learned scaling and full fine-tuning on $\Delta_t$.** MambaExtend applies a learned scaling policy to scale the discretization steps $\Delta_t$. On the contrary, DeciMamba (Ben-Kish et al., 2024) fine-tunes the full model for it to perform well on longer context. We now, visualize the impact of these two approaches on the Normalized sum of $\Delta_t$ per layer. In specific, in Fig. 9 we show a direct comparison of the impact on the same for MambaExtend (9(a)) and DeciMamba (9(b)). Interestingly, both the approaches has similar impact on the Normalized sum of $\Delta_t$, significantly reducing their values at the later layers. This experiment shows that both the approaches intend to

Table 9: F1 scores on HotpotQA and Qasper from LongBench on DeciMamba and MambaExtend, respectively. *Italicized* numbers identify the results taken from (Authors LongMamba, 2024) paper.

| Model | HotpotQA | Qasper |
|---|---|---|
| DeciMamba-1.4B | *13.88* | *14.24* |
| MambaExtend-1.4B | **14.29** | **16.67** |

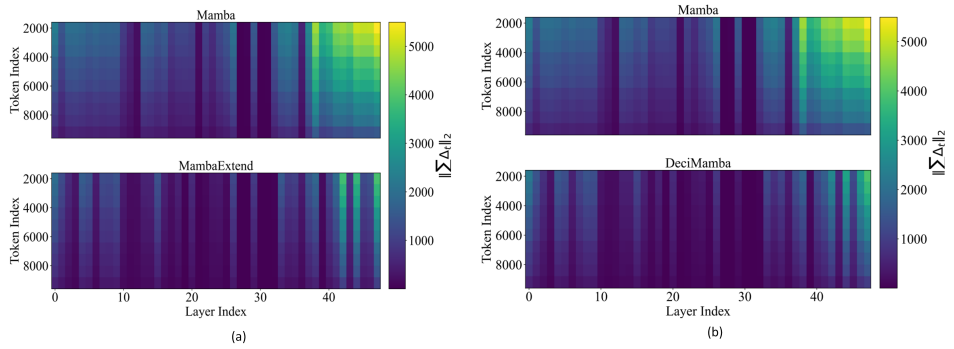



Figure 9: Impact of the calibrated scaling factors on $\Delta_t$ (a) Mamba vs. MambaExtend, and (b) Mamba vs. DeciMamba as evaluated on Pile 32K context length. (Top) of both (a) and (b) shows layer-wise Normalized sum of $\Delta_t$ for a pre-trained Mamba-1.4B. (Bottom) layer-wise Normalized sum of $\Delta_t$ for (a) MambaExtend-1.4B calibrated model, and (b) DeciMamba-1.4B fine-tuned model. To fine-tune DeciMamba 1.4B model we adhered to the setup described in (Ben-Kish et al., 2024).

recalibrate the $\Delta_t$s, while our approach yields similar benefit in more compute, memory, and latency efficient way.

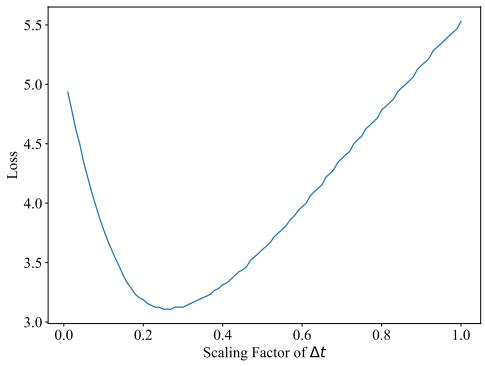



Figure 10: : Impact of different values of uniform $\Delta_t$ scaling on the loss landscaope of the model.

### A.11 Compute, Time, and Memory Cost Analysis

Fig. 11 demonstrates a comparison of full finetuning of baseline Mamba, DeciMamba, and calibration tuning with MambaExtend for the passkey retrieval task. Note here that to have a fair comparison and to demonstrate efficacy at extreme lost cost tuning, we set the epoch to one for all. For MambaExtend, we show results for fine-tuning with both 4k and 8k contexts, while for others, we only perform experiments with tuning with 4k contexts. Notably, **MambaExtend requires up to** $2.12\times$ **fewer memory for tuning with similar context; in other words, it can support calibration with higher context of up to** $2\times$. Regarding per epoch calibration time, MambaExtend can be faster by up to $1.69\times$ while requiring up to $3532.6\times$ fewer parameters to update. To measure the retrieval success, we compute the Interestingly, despite having significant calibration efficiency, 4k tuned MambaExtend provides up to $20\%$ improved accuracy. We yield even better efficiency for $\mathtt{CF}_{ZO}$ based calibration. In specific, compared to DeciMamba, MambaExtend requires up to $\sim\mathbf{5.42} * \mathbf{10^6}\times$ fewer parameter updates and costs up to $\mathbf{3.87}\times$ lower peak-memory (details provided in Appendix A.7).

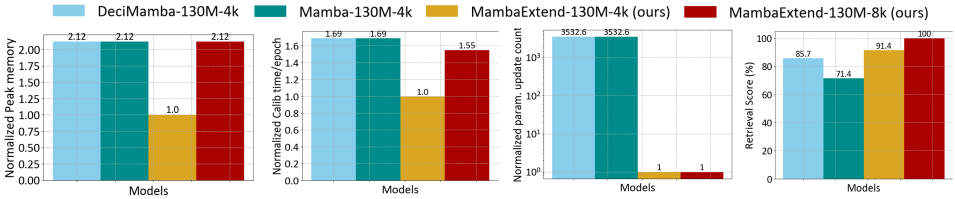



Figure 11: Comparison of {*peak memory, calibration time, and number of parameter updates*} between Mamba, DeciMamba, and MambaExtend for passkey retrieval. We use 130M model and for each method, we train for *one* epoch either with 4k or with 8k context length. For each of these three measurements, we normalize each value by the corresponding value of MambaExtend-130M-4k.

Table 10: PPL comparison with TBTT (Wang, 2024) fine-tuned model on Pile.

| Model | TBTT fine-tuned | 4K | 8K | 16K | 32K | 64K |
|---|---|---|---|---|---|---|
| Mamba2-780M (baseline) | No | **4.62** | 22.4 | 79 | 185 | 378 |
| Mamba2-780M | Yes | 4.62 | 4.34 | **3.89** | **4.92** | 5.16 |
| Mamba2Extend-780M | No | **3.95** | **3.89** | 4.25 | 5.56 | **5** |

Table 11: Comparison with TBTT (Wang, 2024) fine-tuned model on Passkey retrieval task.

| Model | TBTT fine-tuned | Avg. Accuracy (%) |
|---|---|---|
| Mamba2-780M (baseline) | No | 0 |
| Mamba2-780M | Yes | 5.7 |
| Mamba2Extend-780M | No | **91.34** |

### A.12 The Loss Landscape for Grid-Searched Scaling Factors

Fig. 2 in the main manuscript demonstrates the impact of uniform $\Delta_t$ scaling per layer in terms of PPL value. We now plot the loss landscape of the model with uniform scaling factor values in the same range as that of Fig. 2. In specific, 10 shows the loss landscape to have a convex nature as we sweep over the scale factors ($s$) in $0 < s \leq 1$.

### A.13 Comparison with Models Fine-Tuned via Truncated Backpropagation Through Time

Contemporary works on Mamba2 models trained via truncated backpropagation through time (TBTT) has shown promise to generalize well on longer contexts (Wang, 2024). TO compare MambaExtend with TBTT fine-tuned model, we perform a fine-tuning for three epochs based on TBTT approach on a pretrained Mamba2-780M with 0.8B tokens from the PG19 train split. We then measure performance on Pile and Passkey retrieval tasks, respectively and present the comparisons with MambaExtend in Table 10 and 11, respectively. Interestingly, for Pile, indeed we see a good performance boost on longer contexts, getting close to the performance of MambaExtend. However, on the critical benchmark of long context retrieval ( Table 11) TBTT fine-tuned model fails to provide any mentionable retrieval accuracy, while MambaExtend could provide significant accuracy boost by calibration of the scaling factors only.

**Important notes to highlight on TBTT training.** the approach of TBTT based fine-tuning has similarity with the approach of DeciMamba (Ben-Kish et al., 2024), which also suggests full fine-tuning to improve long context understanding (however, without TBTT). We thus would like to highlight that the key benefit of scaling based calibration of MambaExtend can still be considered as an orthogonal method to such full fine-tuning based approaches, not only yielding better accuracy but also providing high compute and memory advantage, potentially opening the door for limited resource calibration. Additionally, as illustrated in Figure 7 of the original LongSSM paper (Wang, 2024), particularly with relatively large models, training the 140M S5 model with previously-initialized state (TBTT policy), the model may severely suffer from stability issues. This raises a general concern on the scalability of such an approach as identified by the author(s).

### A.14 Fine-Tuning vs. Calibration for Longer Epochs

Table 12 shows results of fine-tuning with DeciMamba for five epochs on passkey retrieval. For a fair comparison we show results of MambaExtend with scaling factors calibrated for the same

Table 12: Comparison between fine-tuning and calibration for longer epochs on Passkey retrieval.

| Model | Passkey retrieval acc. (%) |
|---|---|
| DeciMamba-130M | 93.1 |
| MambaExtend-130M | **94.3** |

epochs. As we can see, MambaExtend can still retain improved performance over the other. However, please note, in this work we aim to achieve long context generalization with minimal compute and calibration overhead, thus we aim to focus on fine-tuning for only one epoch.

### A.15 Hardware and API Resources Used

For all the experiments we used an Nvidia A6000 GPU with 48 GB memory. To perform calibration and fine-tuning we used Pytorch API to write the corresponding code.