SPECTRANET: MULTIVARIATE FORECASTING AND IMPUTATION UNDER DISTRIBUTION SHIFTS AND MISSING DATA

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ABSTRACT

In this work, we tackle two widespread challenges in real applications for time-series forecasting that have been largely understudied: distribution shifts and missing data. We propose `SpectraNet`, a novel multivariate time-series forecasting model that dynamically infers a latent space spectral decomposition to capture current temporal dynamics and correlations on the recent observed history. A Convolution Neural Network maps the learned representation by sequentially mixing its components and refining the output. Our proposed approach can simultaneously produce forecasts and interpolate past observations and can, therefore, greatly simplify production systems by unifying imputation and forecasting tasks into a single model. `SpectraNet`achieves SoTA performance simultaneously on both tasks on five benchmark datasets, compared to forecasting and imputation models, with up to 92% fewer parameters and comparable training times. On settings with up to 80% missing data, `SpectraNet`has average performance improvements of almost 50% over the second-best alternative.

1 INTRODUCTION

Multivariate time-series forecasting is an essential task in a wide range of domains. Forecasts are a key input to optimize the production and distribution of goods (Böse et al., 2017), predict healthcare patient outcomes (Chen et al., 2015), plan electricity production (Olivares et al., 2022), build financial portfolios (Emerson et al., 2019), among other examples. Due to its high potential benefits, researchers have dedicated many efforts to improving the capabilities of forecasting models, with breakthroughs in model architectures and performance (Benidis et al., 2022).

The main focus of research in multivariate forecasting has been on accuracy and scalability, to which the present paper contributes. In addition, we identify two widespread challenges for real applications which have been largely understudied: *distribution shifts* and *missing data*.

We refer to *distribution shifts* as changes in the time-series behavior. In particular, we focus on discrepancies in distribution between the train and test data, which can considerably degrade the accuracy (Kuznetsov & Mohri, 2014; Du et al., 2021). This has become an increasing problem in recent years with the COVID-19 pandemic, which disrupted all aspects of human activities. *Missing values* is a generalized problem in applications. Some common causes include faulty sensors, the impossibility of gathering data, corruption, and misplacement of information. As we demonstrate in our experiments, these challenges hinder the performance of current state-of-the-art (SoTA) models, limiting their use and potential benefits in applications where these problems are predominant.

In this work, we propose `SpectraNet`, a novel multivariate forecasting model that achieves SoTA performance in benchmark datasets and is also *intrinsically* robust to distribution shifts and extreme cases of missing data. `SpectraNet` achieves its high accuracy and robustness by dynamically inferring a latent vector projected on a temporal basis, a process we name *latent space spectral decomposition* (LSSD). A series of convolution layers then synthesize both the reference window, which is used to infer the latent vectors and the forecast window.

To the best of our knowledge, `SpectraNet` is also the first solution that can simultaneously forecast the future values of a multivariate time series and accurately impute the past missing data. In

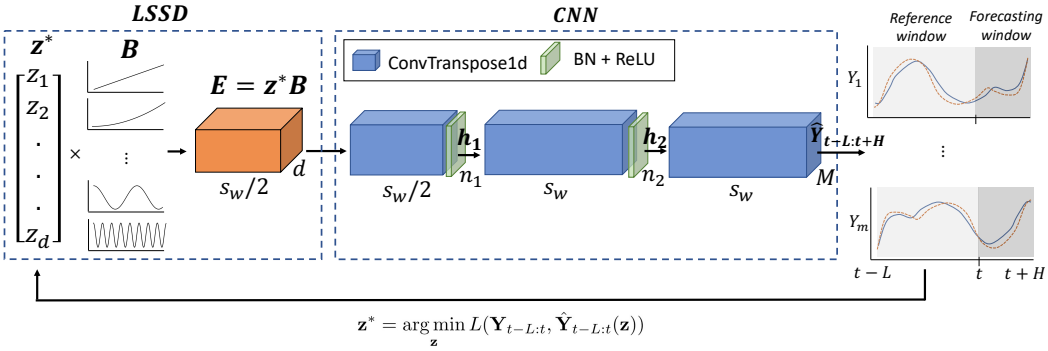


Figure 1: `SpectraNet` architecture. The Latent Space Spectral Decomposition (LSSD) encodes shared temporal dynamics of the target window into Fourier waves and polynomial functions. Latent vector $\mathbf{z}$ is inferred with Gradient Descent minimizing reconstruction error on the *reference window*. The Convolution Network (CNN) generates the time-series window by sequentially mixing the components of the embedding and refining the output.

practice, imputation models are first used to fill the missing information for all downstream tasks, including forecasting. `SpectraNet` can greatly simplify production systems by unifying imputation and forecasting tasks into a single model.

The main contributions are:

- **Latent Vector Inference**: methodology to dynamically capture current dynamics of the target time-series into a latent space, replacing parametric encoders.
- **Latent Space Spectral Decomposition:** representation of a multivariate time-series window on a shared latent space with temporal dynamics.
- **`SpectraNet`**: novel multivariate forecasting model that simultaneously imputes missing data and forecasts future values, with SoTA performance on several benchmark datasets and demonstrated robustness to distribution shifts and missing values. We will make our code publicly available upon acceptance.

The remainder of this paper is structured as follows. Section 2 introduces notation and the problem definition, Section 3 presents our method, Section 4 describes and presents our empirical findings. Finally, Section 5 concludes the paper. The literature review is included in A.1.

2 Notation and Problem definition

We introduce a new notation that we believe is lighter than the standard notation while being intuitive and formally correct. Let $\mathbf{Y} \in \mathbb{R}^{M \times T}$ be a multivariate time-series with M features and T timestamps. Let $\mathbf{Y}_{a:b} \in \mathbb{R}^{M \times (b-a)}$ be the observed values for the interval $[a, b)$, that is, $\mathbf{Y}_{0:t}$ is the set of t observations of $\mathbf{Y}$ from timestamp 0 to timestamp $t-1$ while $\mathbf{Y}_{t:t+H}$ is the set of H observations of $\mathbf{Y}$ from timestamp t to timestamp $t+H-1$. Let $y_{m,t} \in \mathbb{R}$ be the value of feature m at timestamp t.

In this work we consider the multivariate point forecasting task, which consists of predicting the future values of a multivariate time-series sequence based on past observations. The main task of a model F_Θ with parameters Θ at a timestamp t, is to produce forecasts for the future H values, denoted by $\hat{\mathbf{Y}}_{t:t+H}$, based on the previous history $\mathbf{Y}_{0:t}$.

$$\hat{\mathbf{Y}}_{t:t+H} = F_\Theta(\mathbf{Y}_{0:t}) \tag{1}$$

For the imputation task, to impute a missing value $y_{m,t}$, models are not constrained to only use past observations. Moreover, they are evaluated on how well they approximate only the missing values. We evaluate the performance with two common metrics used in the literature, mean squared error (MSE) and mean absolute error (MAE), given by equation 2 (Hyndman & Athanasopoulos, 2018).

$$\text{MSE} = \frac{1}{MH}\sum_{h=0}^{H-1}\sum_{m=1}^{M}(y_{m,t+h} - \hat{y}_{m,t+h})^2 \qquad \text{MAE} = \frac{1}{MH}\sum_{h=0}^{H-1}\sum_{m=1}^{M}|y_{m,t+h} - \hat{y}_{m,t+h}|. \quad (2)$$

3 SpectraNet

We start the description of our approach with a general outline of the model and explain each major component in detail in the following subsections. The overall architecture is illustrated in Figure 1. `SpectraNet` is a top-down model that *generates* a multivariate time-series window of fixed size $s_w = L+H$, where L is the length of the *reference window* and H is the forecast horizon, from a latent vector $\mathbf{z} \in \mathbb{R}^d$. To produce the forecasts at timestamp t, the model first *infers* the optimal latent vector on the *reference window*, consisting of the last L values, $\mathbf{Y}_{t-L:t}$, by minimizing the *reconstruction error*. This inference step is the main difference between our approach to existing models, which map the input into an embedding or latent space using an encoder network. The model generates the full time-series window s_w, which includes the *forecast window* $\mathbf{Y}_{t:t+H}$, with a *spectral decomposition* and a Convolutional Neural Network (CNN). The main steps of `SpectraNet` are given by

$$\mathbf{z}^* = \arg\min_{\mathbf{z}} L(\mathbf{Y}_{t-L:t}, \hat{\mathbf{Y}}_{t-L:t}(\mathbf{z})) \quad (3)$$

where $\mathbf{z}^*$ is the *inferred* latent vector, L is a reconstruction error metric, and $\hat{\mathbf{Y}}_{t-L:t}(\mathbf{z})$ is given by,

$$\mathbf{E} = \text{LSSD}(\mathbf{z}, \mathbf{B}) \quad (4)$$

$$\hat{\mathbf{Y}}_{t-L:t}, \hat{\mathbf{Y}}_{t:t+H} = \text{CNN}_{\Theta}(\mathbf{E}) \quad (5)$$

where LSSD (*latent space spectral decomposition*) is a basis expansion operation of $\mathbf{z}$ over the predefined temporal basis $\mathbf{B}$ to produce a temporal embedding $\mathbf{E} \in \mathbb{R}^{d \times d_t}$, and CNN is a Top-Down Convolutional Neural Network with learnable parameters Θ. The CNN simultaneously produces both the *reconstruction* of the past reference window $\hat{\mathbf{Y}}_{t-L:t}$, used to find the optimal latent vector for the full window and the forecast $\hat{\mathbf{Y}}_{t:t+H}$.

3.1 Latent vectors inference

The main difference between our approach and existing models is the inference of the latent vectors instead of relying on encoders. The latent vectors are inferred by minimizing the MSE between the observed values $\mathbf{Y}_{t-L:t}$ and the reconstruction $\hat{\mathbf{Y}}_{t-L:t}(\mathbf{z})$ on the reference window. By doing this, the model dynamically captures the temporal dynamics and correlations between features that best explain the target time series' current behavior. Figure 2 demonstrates how `SpectraNet`'s output evolves during the inference of $\mathbf{z}$, adapting to current behaviours on the *reference* window.

This optimization problem is non-convex, as the reconstruction follows equations 4 and 5. However, given that all operations are differentiable, we can compute the gradient of the objective function w.r.t to $\mathbf{z}$, allowing the use of gradient-based methods. In particular, we rely on gradient descent (GD), randomly initializing the latent vector with independent and identically distributed Gaussian distributions and fixing the learning rate and the number of iterations as hyperparameters.

3.2 Latent Space Spectral Decomposition

The second component of `SpectraNet` is the mapping from the latent vectors $\mathbf{z}$ into the temporal embedding $\mathbf{E}$. Each element of the latent vector $\mathbf{z}$ corresponds to the coefficient of one element of

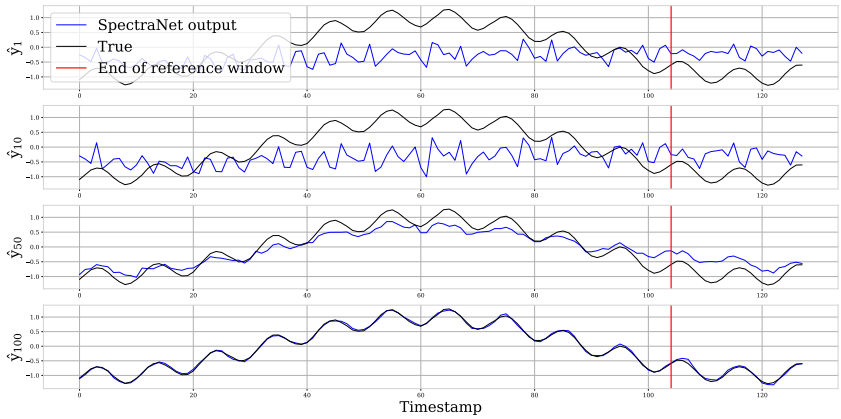


Figure 2: `SpectraNet`'s output evolution during latent vector inference with Gradient Descent. The model maps the latent vector to the complete window, including both *reference* (of size 104) and *forecasting* windows (of size 24), using only information from the former. The temporal basis $\mathbf{B}$ imposes strict dependencies between both windows. This inference process allows `SpectraNet`to dynamically adapt to new behaviours and forecast with missing data.

the temporal basis $\mathbf{B} \in \mathbb{R}^{d \times d_t}$, where d is the number of elements and d_t is the temporal length. The i-th row of $\mathbf{E}$ is given by,

$$\mathbf{E}_{\mathbf{i},:} = z_i^* \mathbf{B}_{\mathbf{i},:} \tag{6}$$

Each element of the basis consists of a predefined template function. Similarly to the N-BEATS model (Oreshkin et al., 2019), we split the basis into two types of patterns commonly found in time series: trends, represented by polynomial functions, and seasonal, represented by harmonic functions. The final basis matrix $\mathbf{B}$ is the row-wise concatenation of the three following matrices.

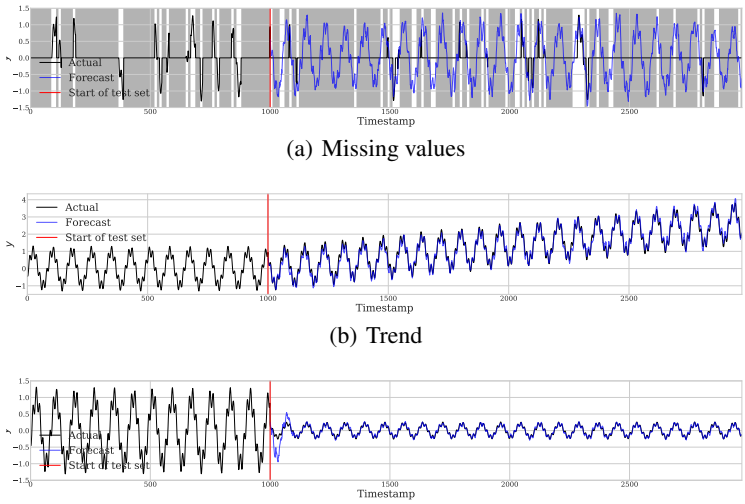
$$\begin{aligned}
\mathbf{B}_{i,t}^{trd} &= t^i && \text{, for } i \in \{0, ..., p\}, t \in \{0, ..., d_t\} \\
\mathbf{B}_{i,t}^{cos} &= \cos(2\pi i t) && \text{, for } i \in \{0, ..., s_w/2\}, t \in \{0, ..., d_t\} \\
\mathbf{B}_{i,t}^{sin} &= \sin(2\pi i t) && \text{, for } i \in \{0, ..., s_w/2\}, t \in \{0, ..., d_t\}
\end{aligned} \tag{7}$$

The temporal embedding $\mathbf{E}$ corresponds to a *latent space spectral decomposition* that encodes shared temporal dynamics of all features in the target window, as the latent vector $\mathbf{z}^*$ selects the relevant trend and frequency bands. Another crucial reason behind using a predefined basis is to impose strict temporal dependencies between the reference and forecasting windows. While inferring the latent vector, the forecasting window does not provide information (gradients). If all the elements of $\mathbf{E}$ are inferred (without a basis), given that we use a CNN with short kernels, the last values of the temporal embedding that determines the forecasting window cannot be optimized.

3.3 Top-Down Convolution Network

The last component of the model's architecture is a top-down CNN, which produces the final forecast and reconstruction of the reference window for the M features simultaneously, $\hat{\mathbf{Y}}_{t-L:t+H}$, from the temporal embedding $\mathbf{E}$. The full CNN architecture is presented in Figure 1, and can be formalized as:

$$\begin{aligned}
\mathbf{h_1} &= \text{ReLU}(\text{BN}(\text{ConvTranspose1d}(\mathbf{E}))) \\
\mathbf{h_2} &= \text{ReLU}(\text{BN}(\text{ConvTranspose1d}(\mathbf{h_1}))) \\
\hat{\mathbf{Y}}_{t-L:t+H} &= \text{ConvTranspose1d}(\mathbf{h_2})
\end{aligned} \tag{8}$$



(a) Missing values

(b) Trend

(c) Magnitude

Figure 3: Forecasts for the first feature of **Simulated7** dataset using `SpectraNet` with (a) 80% of missing data (missing regions in grey), (b) change in trend, and (c) change in magnitude. Forecasts are produced every 24 timestamps in a rolling window strategy.

where ConvTranspose1d is a transpose convolutional layer on the temporal dimension, BN is a batch normalization layer, and ReLU activations introduce non-linearity. The number of filters at each layer, kernel size, and stride are given in Appendix A.4.

The first two layers learn a common representation for the window from the temporal embedding $\mathbf{E}$. The second layer *refines* the temporal resolution to the final size of the window s_w. The last layer produces the final output for all M features from $\mathbf{h_2}$, with a single convolutional filter for each feature. Note that the convolutional filters are not *causal* as in a TCN, meaning that the values for a temporal position of $\mathbf{h}$ and $\hat{\mathbf{Y}}$ also depend on the next immediate temporal values (within the kernel size) from the previous layer. The *causality* of `SpectraNet` that allows the model to forecast from previous values is given by the fact that the latent vector $\mathbf{z}^*$ is only inferred with information on the reference window.

3.4 Training procedure

Each training iteration consists of two steps: the *inference step* and the *learning step*. During the *inference step*, the optimal latent vector $\mathbf{z}^*$ is inferred using Gradient Descent, solving the optimization problem given in equation 3 and with the current parameters Θ. During the *learning step*, the latent vector is used as the input to the model, and the parameters Θ are updated using ADAM optimizer (Kingma & Ba, 2014) and MSE loss. For each iteration, we sample a small batch (with replacement) of multivariate time-series windows $\mathbf{Y}_{t:t+s_w}$ from the training data, each starting at a random timestamp t. Each sampled observation is first *normalized* with the mean and standard deviation on the *reference* window, to decouple the scale and patterns (the output is scaled back before evaluation). The full training procedure is presented in Appendix A.5.

One potential drawback of inferring the latent vectors instead of using an encoder is the computational cost. We tackle this in several ways. First, by solving the optimization problem using Gradient Descent, we can rely on automatic differentiation libraries such as PyTorch to efficiently compute the gradients. Second, backpropagation on CNN is parallelizable in GPU since it does not require sequential computation. The forecasts are independent between windows, so they can be computed in parallel, unlike RNN models, which must produce the forecasts sequentially. Finally, during training, we *persist* the optimal latent vectors to future iterations on the same window. For a given observation t, the final latent vector $\mathbf{z_t^*}$ is stored and used as a warm start when observation t is sampled again. This considerably reduces the number of iterations during the *inference step*. We discuss training times and memory complexity in section 4.

3.5 ROBUSTNESS TO MISSING VALUES AND DISTRIBUTIONAL SHIFTS

The *inference* of latent vectors is the primary driver of the superior robustness of `SpectraNet` to handle both distributional changes in the temporal dynamics and missing values. During this step, our model will infer the latent vectors that best describe the dynamics on the reference window, even if they follow a behavior never seen on the training data. `SpectraNet` can, therefore, dynamically adjust the temporal embedding to capture the current behavior on the reference window, giving more flexibility to extrapolate. To deal with missing values, we change the optimization on the reference window to only include the available information.

`SpectraNet` is intrinsically robust to missing data; it does not require any adaptation to the architecture, training procedure, or interpolation models to fill the missing gaps. Our approach is even robust to the case where the *missing data regime* (how much or for how long missing data occurs) differs between the training and test sets. On the contrary, current SotA models require either modifications to the architecture or an interpolation model to first impute the missing values. Another advantage of our method is that it cannot only forecast directly on the raw data but by generating the entire window, it is simultaneously interpolating the past missing values and forecasting the future values.

We first test our previous claims with a controlled experiment on synthetic data, which we refer to as **Simulated7**. We first generate seven time series of length 20,000, each generated as the sum of two cosines with random frequencies and small Gaussian noise. We then inject missing values on both train and test sets, following the procedure described in section 4. For the distribution shifts experiments, we modify only the test set with two common distribution shifts: changes in *trend* and in *magnitude*. Figure 3 presents the forecasts for `SpectraNet` for the three settings considered. We only include the first feature in this figure, and we show the complete data in Appendix A.6. As seen in panel (a), our method can accurately forecast the ground truth even with 80% of missing values, a challenging setting since intervals where consecutive timestamps are available are short. This setting also demonstrates how `SpectraNet`can accurately learn correlations between features, as in many *reference* windows only few features contain information. Panels (b) and (c) demonstrate how `SpectraNet` can dynamically adapt to new data behaviors, even if the model never observed them during training. Table 1 presents a quantitative comparison between the performance of `SpectraNet` and baseline models.

Table 1: Forecasting accuracy on **Simulated7** dataset with missing values and distribution shifts between the train and test sets, forecasting horizon of 24 timestamps. Lower scores are better. Metrics are averaged over five runs, best model highlighted in bold.

		`SpectraNet`		N-HiTS		Informer		Autoformer		StemGNN		RNN	
	p_o	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Normal	0.0	0.004	0.041	**0.001**	**0.015**	0.008	0.079	0.017	0.096	0.036	0.122	0.001	0.022
Trend	0.0	**0.025**	**0.130**	0.427	0.443	2.821	1.410	0.060	0.19	1.354	0.952	1.214	0.859
Magnitude	0.0	**0.005**	**0.038**	0.009	0.059	0.020	0.094	0.043	0.168	0.060	0.192	0.089	0.248
M. Values	0.2	**0.009**	**0.081**	0.012	0.176	0.071	0.237	0.319	0.430	0.097	0.266	0.069	0.148
	0.4	**0.011**	**0.073**	0.022	0.213	0.095	0.242	0.591	0.612	0.181	0.385	0.172	0.308
	0.6	**0.018**	**0.100**	0.133	0.338	0.231	0.380	0.752	0.994	0.362	0.488	0.357	0.462
	0.8	**0.023**	**0.127**	0.365	0.439	0.372	0.478	0.925	1.106	0.436	0.614	0.500	0.571

4 EXPERIMENTAL RESULTS

We base our experimental setting, benchmark datasets, train/validation/test splits, hyperparameter tuning, and data processing on previous works on multivariate forecasting (Zhou et al., 2021; Wu et al., 2021; Challu et al., 2022a).

4.1 DATASETS

We evaluate our model on synthetic data and five popular benchmark datasets commonly used in the forecasting literature, comprising various applications and domains. All datasets are normalized with the mean and standard deviation computed on the train set. We split each dataset into train,

validation, and test sets, following different proportions based on the number of timestamps. We present summary statistics for each dataset in Appendix A.2.

The **Influenza-like illness (ILI)** dataset contains the weekly proportion of patients with influenza-like symptoms in the US, reported by the Centers for Disease Control and Prevention, from 2002 to 2021. **Exchange** reports the daily exchange rates of eight currencies relative to the US dollar from 1990 to 2016. The **Solar** dataset contains the hourly photo-voltaic production of 32 solar stations in Wisconsin during 2016. Finally, the **Electricity Transformer Temperature (ETTm2)** dataset is a stream of eight sensor measurements of an electricity transformer in China from July 2016 to July 2018. **Weather** contains 21 meteorological conditions recorded at the German Max Planck Biogeochemistry Institute in 2021.

4.2 Training and Evaluation setup

We train models on the training set comprising the earliest history of each dataset. We select the optimal hyperparameters for all models (including the baselines to ensure fair comparisons) on each dataset and occlusion percentage based on the performance (measured by MSE) on the validation set, using a Bayesian optimization algorithm, HYPEROPT (Bergstra et al., 2011). For `SpectraNet` we tune the number of convolutional filters, temporal size of the latent vectors, window normalization, and optimization hyperparameters for both the inferencial and learning steps. The complete list of hyperparameters is included in Appendix A.4. Finally, we use the model with the optimal configuration to forecast the test set. For the main results, we use a multi-step forecast with a horizon of size 24, and forecasts are produced in a rolling window strategy. We repeat the experiment five times with different random seeds and report average performances. We run the experiments on an AWS `g3.4xlarge` EC2 instance with NVIDIA Tesla M60 GPUs.

We compare our proposed model against several univariate and multivariate state-of-the-art baselines based on different architectures: Transformers, feed-forward networks (MLP), Graph Neural Network (GNN), and Recurrent Neural Network (RNN). For Transformer-based models we include the Informer (Zhou et al., 2021) and Autoformer (Wu et al., 2021), two recent models for multi-step long-horizon forecasting; for MLP we consider the univariate N-HiTS (Challu et al., 2022a) and N-BEATS (Oreshkin et al., 2019) models; for GNN models we include the StemGNN (Cao et al., 2020), and lastly we include a multi-step univariate RNN with dilations (Chang et al., 2017).

4.3 Missing data setup

We present a new experimental setting for testing the forecasting models' robustness to missing data. We parameterize the experiments with two parameters: the size of missing segments s and the probability that each segment is missing with p_o. First, the original time series is divided into disjoint segments of length s. Second, each feature m in each segment is occluded with probability p_o. We repeat the experiment with different probabilities p_o: 0% (no missing values), 20%, 40%, 60%, and 80%, to test models' performance under different proportions of missing values. The size of segments s is fixed at ten timestamps for ILI and at 100 for all other datasets. Figure 3 shows an example of the setting for the **Simulated7** dataset with 80% missing data (occluded timestamps in grey). All models are trained with the training loss only on available values since we observed this improves the forecasting accuracy.

4.4 Key results

Forecasting accuracy. Table 2 presents the forecasting accuracy. First, `SpectraNet` achieved SoTA performance even on full data, showing that the advantages of our method are not limited to the robustness to missing data and distribution shifts. `SpectraNet` outperformed N-HiTS/N-BEATS in two of the five datasets and placed in the Top-3 models across all datasets, outperforming Transformer-based models, StemGNN, and DilRNN. `SpectraNet` also achieves the best performance among all multivariate models.

The superior robustness of `SpectraNet` to missing values is evident in all datasets and proportions. In datasets with strong seasonalities, such as **Simulated7** and **Solar**, the accuracy of `SpectraNet` marginally degrades with even 80% of missing values. The relative performance of our method improves with the proportion of missing data. For example, for 80%, the average MSE

Table 2: Main forecasting accuracy results on benchmark datasets with different proportion of missing values (p_o), forecasting horizon of 24 timestamps, lower scores are better. Metrics are averaged over five runs, best model highlighted in bold.

		SpectraNet		N-HiTS/NBEATS		Informer		Autoformer		StemGNN		RNN	
	p_o	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
ILI	0.0	**0.724**	**0.557**	1.379	0.762	4.265	1.329	2.249	0.967	4.013	1.437	3.852	1.340
	0.2	**1.153**	**0.662**	2.028	0.933	3.624	1.127	3.250	1.292	4.372	1.503	3.647	1.321
	0.4	**1.646**	**0.793**	3.248	1.134	3.908	1.351	4.308	1.419	4.752	1.694	4.360	1.455
	0.6	**2.453**	**1.012**	3.871	1.257	3.991	1.277	4.571	1.515	5.170	1.863	5.561	1.614
	0.8	**3.136**	**1.139**	5.102	1.451	4.180	1.363	4.745	1.536	5.337	1.836	5.063	1.622
Exchange	0.0	0.048	0.157	**0.031**	**0.102**	0.472	0.534	0.049	0.167	0.102	0.251	0.086	0.210
	0.2	**0.091**	**0.223**	0.295	0.342	1.013	0.772	0.758	0.523	1.021	0.599	0.871	0.529
	0.4	**0.158**	**0.292**	0.321	0.534	1.162	0.905	0.782	0.606	0.826	0.608	0.694	0.550
	0.6	**0.367**	**0.418**	0.549	0.745	1.564	0.932	1.346	0.849	1.991	1.002	1.432	0.872
	0.8	**1.125**	**0.785**	1.792	1.188	2.530	1.281	2.520	1.231	2.778	1.318	2.791	1.312
Solar	0.0	**0.007**	**0.054**	0.008	0.061	0.011	0.065	0.018	0.095	0.015	0.077	0.012	0.060
	0.2	**0.012**	**0.058**	0.016	0.063	0.016	0.083	0.025	0.116	0.016	0.084	0.015	0.065
	0.4	**0.012**	**0.062**	0.017	0.068	0.020	0.102	0.023	0.118	0.023	0.103	0.017	0.064
	0.6	**0.013**	**0.068**	0.022	0.075	0.021	0.107	0.027	0.130	0.031	0.115	0.026	0.083
	0.8	**0.014**	**0.072**	0.025	0.089	0.035	0.156	0.036	0.127	0.065	0.192	0.028	0.113
ETTm2	0.0	0.136	0.212	**0.116**	**0.203**	0.366	0.462	0.171	0.275	0.154	0.273	0.179	0.238
	0.2	**0.155**	**0.260**	0.656	0.420	0.895	0.704	0.512	0.450	0.963	0.581	0.574	0.421
	0.4	**0.228**	**0.316**	1.191	0.637	0.923	0.679	1.078	0.683	1.542	0.795	0.807	0.584
	0.6	**0.500**	**0.442**	1.976	0.918	1.784	0.999	1.643	0.891	2.151	0.978	1.163	0.741
	0.8	**0.725**	**0.528**	2.019	1.230	1.982	1.115	1.992	1.062	2.595	1.162	2.085	1.007
Weather	0.0	0.112	0.193	0.109	**0.127**	0.218	0.283	0.186	0.263	**0.103**	0.152	0.124	0.201
	0.2	**0.154**	0.218	0.181	**0.205**	0.287	0.350	0.324	0.389	0.179	0.223	0.189	0.240
	0.4	**0.205**	**0.269**	0.264	0.279	0.320	0.385	3.105	1.456	0.296	0.311	0.291	0.325
	0.6	**0.329**	**0.387**	0.420	0.414	0.761	0.628	3.772	1.660	0.495	0.569	0.435	0.433
	0.8	**0.439**	**0.463**	0.517	0.503	0.915	0.889	4.204	1.859	0.760	0.702	0.480	0.48

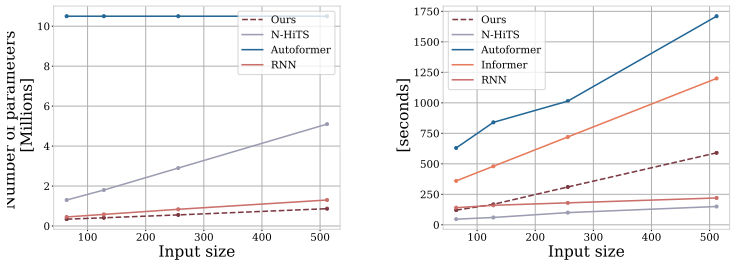
across datasets is 48% lower than the N-HiTS and 65% lower than the Autoformer. The improvement on **ILI** with complete data of almost 50% against the second best model is explained by the *distribution shift* between the train and test set, where the latter presents larger spikes and a clear positive trend on some features. Appendix A.7 presents a plot of the ILI dataset.

Imputation accuracy. Table 3 presents the results for the imputation task. We include a SoTA imputation diffusion model, CSDI (Tashiro et al., 2021), and three simple baselines: (i) imputation with the mean of each feature, (ii) imputation with the last available value (Naive), and (iii) linear interpolation between past and future available values. SpectraNet consistently achieves the best performance across all datasets. CSDI outperformes all simpler baselines on Simulated7 and Solar, but its performance degrades on datasets with distribution shifts such as Simulated7-Trend and ILI.

Table 3: Imputation accuracy on the test set for SpectraNet and baselines with different proportion of missing data (p_o). Lower scores are better, best model highlighted in bold.

		SpectraNet		CSDI		Mean		Naive		Linear	
	p_o	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE	MSE	MAE
Simulated7	0.2	**0.008**	**0.070**	0.062	0.184	0.535	0.642	0.188	0.344	0.069	0.213
	0.6	**0.022**	**0.116**	0.218	0.334	0.551	0.656	0.468	0.518	0.225	0.341
Simulated7-Trend	0.2	**0.029**	**0.136**	0.546	0.536	2.067	1.196	0.238	0.364	0.083	0.229
	0.6	**0.063**	**0.185**	0.757	1.009	1.862	1.127	0.470	0.518	0.228	0.343
Simulated7-Magnitude	0.2	**0.005**	**0.039**	0.009	0.053	0.033	0.139	0.024	0.087	0.010	0.055
	0.6	**0.011**	**0.053**	0.051	0.108	0.036	0.144	0.064	0.146	0.033	0.094
Solar	0.2	**0.004**	**0.042**	0.022	0.096	0.038	0.182	0.052	0.139	0.032	0.116
	0.6	**0.006**	**0.048**	0.034	0.103	0.038	0.182	0.056	0.145	0.039	0.128
Exchange	0.2	**0.018**	0.058	0.045	0.093	1.560	1.002	0.071	0.105	0.023	**0.055**
	0.6	**0.035**	**0.084**	0.101	0.196	1.573	1.010	0.099	0.138	0.042	0.089
ILI	0.2	**0.592**	**0.603**	2.153	0.903	4.085	1.412	1.878	0.871	0.630	0.642
	0.6	**2.568**	**1.349**	5.131	1.508	7.351	1.880	7.840	1.895	7.245	1.605

Memory and time complexity. We compare the training time and memory usage (as the number of learnable parameters) of SpectraNet and baseline models as a function of the input size on the ETTm2 dataset in Figure 4, using the optimal hyperparameter configuration. Panel (a) shows our method has the lowest memory footprint, with up to **85%** fewer parameters than the N-HiTS (the second best alternative in most datasets) and **92%** than the Autoformer. Despite the fact that



(a) Memory Efficiency (b) Time Efficiency

Figure 4: Memory efficiency and train time analysis on ETTm2. Memory efficiency measured as the number of parameters, train time includes the complete training procedure. We use the best hyperparameter configuration for each model, based on model accuracy.

`SpectraNet` performs the inference step in each iteration, thanks to the improvements discussed in section 3 and the reduced number of parameters, training times are comparable to other baseline models.

Ablation studies. Finally, we test the contribution of the two major components proposed in this work, the dynamic inference of latent vectors and the LSSD. We compare `SpectraNet`against two versions without these components. In `SpectraNet`$_1$ we add a CNN encoder to map inputs to a standard embedding, and in `SpectraNet`$_2$ we keep the inference step but remove the LSSD. Table 4 presents the average performance across the five benchmark datasets. As expected, `SpectraNet`$_1$ with the parametric encoder is not robust to missing data, with similar performance to other SoTA models. On top of improving the performance over `SpectraNet`$_2$, the LSSD reduces the number of parameters by 70% since fewer layers are needed to generate the complete window.

	`SpectraNet`$_1$		`SpectraNet`$_2$		`SpectraNet`	
p_o	MSE	MAE	MSE	MAE	MSE	MAE
0.0	0.691	0.461	0.237	0.283	**0.213**	**0.251**
0.2	0.839	0.623	0.382	0.328	**0.310**	**0.284**
0.4	1.412	0.666	0.492	0.427	**0.449**	**0.341**
0.6	1.952	0.974	0.916	0.620	**0.752**	**0.467**
0.8	2.541	1.082	1.547	0.859	**1.126**	**0.579**

Table 4: Average forecasting accuracy across five benchmark datasets for `SpectraNet` and two versions without main components. Lower scores are better, best highlighted in bold.

5 Discussions and Conclusion

This work proposes a novel multivariate time-series forecasting model that uses a Top-Down CNN to generate time-series windows from a novel latent space spectral decomposition. It dynamically infers the latent vectors that best explain the current dynamics, considerably improving the robustness to distribution shifts and missing data. We compare the accuracy of our method with SoTA models based on several architectures for forecasting and interpolation tasks. We demonstrate `SpectraNet` does not only achieve SoTA on benchmark datasets but can also produce accurate forecasts under some forms of distribution shifts and extreme cases of missing data.

Our experiments provide evidence that, as expected, the performance of current SoTA forecasting models significantly degrades under the presence of missing data and distribution shifts. These challenges are commonly present in high stake settings such as healthcare (for instance, cases with up to 80% missing data are common) and are becoming more predominant in many domains with the recent global events such as the COVID-19 pandemic. Designing robust algorithms and solutions that tackle these challenges can significantly improve their adoption and increase their benefits.

We believe that the technique of dynamically inferring latent vectors can have multiple other applications in time-series forecasting. For instance, it can have applications in transfer learning and few-shot learning. A pre-trained CNN (or any decoder) could forecast completely unseen time series, while the inference step will allow the model to adapt to different temporal patterns. Moreover, the inference step can be adapted to produce multiple samples and calibrated to learn the target time series distribution, allowing the model to produce probabilistic forecasts.

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A APPENDIX

A.1 RELATED WORK

Time-series forecasting. Time-series forecasting has long been an active area of research in academia and industry due to its broad range of high-impact applications. The latest developments in machine learning have been combined with classical methods or used to develop new models (Januschowski et al., 2020) which have been successful in recent large-scale forecasting competitions (Makridakis et al., 2020; 2022). The earlier breakthroughs in modeling time-series with Deep-learning (Januschowski et al., 2018) include the widely used Recurrent Neural Networks (RNN) (Chang et al., 2017; Salinas et al., 2020; 2019) and Temporal Convolution Networks (TCN) (Bai et al., 2018). The success of Transformers (Vaswani et al., 2017) in sequential data, such as natural language processing (NLP) and audio processing, inspired many recent models with attention mechanisms. The Informer (Zhou et al., 2021) introduces a Prob-sparse self-attention to reduce the quadratic complexity of vanilla Transformers; the Autoformer (Wu et al., 2021) proposed a decomposition architecture in trend and seasonal components, and the Auto-correlation mechanism.

Simpler approaches based on feed-forward networks have also shown remarkable performance. The N-BEATS model (Oreshkin et al., 2019) uses a deep stack of fully-connected layers to decompose the forecast into both learnable and predefined basis functions; the NBEATSx extension incorporates interpretable exogenous variable decomposition (Olivares et al., 2022); the N-HiTS model (Challu et al., 2022a) generalizes the N-BEATS by introducing mixed-data sampling and hierarchial interpolation to decompose the forecast in different frequencies. Finally, Graph Neural Networks (GNN) has been used to incorporate complex relations between a large number of time series, including the GraphWaveNet (Wu et al., 2019), and StemGNN (Cao et al., 2020) models.

Time-series imputation. The standard practice to handle missing data is filling the missing information, a process called *interpolation*. Simple interpolation alternatives include replacing missing values with zeros, the mean, most-recent value (naive), and linear interpolation. Most recent Deep-learning approaches consist of Generative Adversarial Networks (GANs) and RNN-based architectures. Some notable examples are E2gan (Luo et al., 2019), Brits (Cao et al., 2018), and NAOMI (Liu et al., 2019). More recent approaches include the CDSI (Tashiro et al., 2021) model, a score-based diffusion auto-regressive architecture that produces a distribution for the imputed values.

Alternating back-propagation. The method we propose to infer latent vectors is inspired by the Alternating back-propagation algorithm (ABP) for Generative models (Han et al., 2017). The key idea of this algorithm is to sample latent vectors from the posterior distribution with MCMC methods and train a Generative model by maximizing the observed likelihood directly. Generative models trained with ABP do not need an encoder, such as Variational Autoencoders (VAE), or Discriminator networks, such as GANs. More recent work extend the original architecture with energy-based models for CV (Pang et al., 2020), LSTM networks for text generation (Pang et al., 2021), and a hierarchical latent space for anomaly detection (Challu et al., 2022b).

A.2 BENCHMARK DATASETS

Table 5 presents summary information for the benchmark datasets, including the granularity (frequency), number of features and timestamps, and train/validation/test proportions.

Table 5: Summary of benchmark datasets

DATASET	GRANULARITY	# OF FEATURES	# OF TIMESTAMPS	TRAIN/VAL/TEST SPLIT
SIMULATED7	NA	7	20,000	80/10/10
ILI	WEEKLY	7	966	70/10/20
ETTM2	15 MINUTE	7	57,600	60/20/20
EXCHANGE	DAILY	8	7,588	70/20/10
SOLAR	HOURLY	32	8,760	60/20/20
WEATHER	10 MINUTE	21	52,695	70/10/20

All datasets are public, and are available in the following links:

- **ILI**: `https://gis.cdc.gov/grasp/fluview/fluportaldashboard.html`
- **Exchange**: `https://github.com/laiguokun/multivariate-time-series-data`
- **Solar**: `https://www.nrel.gov/grid/solar-power-data.html`
- **ETTm2**: `https://github.com/zhouhaoyi/ETDataset`
- **Weather**: `https://www.bgc-jena.mpg.de/wetter/`

A.3 SIMULATED7 DATASET

Simulated7 is a synthetic dataset that we design to evaluate forecasting models robustness to missing data and distribution shifts in a controlled environment. It consists of 7 time-series, each generated independently as the sum of two cosine functions with different frequencies and a small Gaussian noise. In particular, it is composed as the sum of the three following elements:

$$\begin{aligned} y_t^{\text{low}} &= \cos(it) \qquad , i \sim U(5, 50) \quad t \in [0, 5] \\ y_t^{\text{high}} &= \cos(it) \qquad , i \sim U(100, 300) \quad t \in [0, 5] \\ y_t^{\text{noise}} &\sim N(0, 0.001) \quad , t \in [0, 5] \end{aligned} \tag{9}$$

Each time-series consists of 20,000 timestamps, with regular intervals between $[0, 5]$. For the *missing data* we obfuscate random timestamps following the procedure described in section 4, using $s = 10$ and $p_o \in \{0, 0.2, 0.4, 0.6, 0.8\}$. For simulating *distribution shifts* we perturb **only the test set** with two transformations: adding a linear *trend* with slope 6, and scaling the *magnitude* by 0.5.

A.4 HYPERPARAMETER OPTIMIZATION

`SpectraNet` hyperparameters are tuned on the validation set of each dataset using HYPEROPT algorithm with 30 iterations, Table 6 presents the hyperparameter grid. To ensure a fair comparison with baseline models, we also tuned their respective hyperparameters with the same procedure, as the default configuration in their implementations might not perform well in the different settings we explore. We detail the hyperparameter grid for each baseline model next.

Table 6: Hyperparameters grid for `SpectraNet`.

HYPERPARAMETER	VALUES
Learning rate	{0.0001, 0.0005, 0.001, 0.005}
Training steps	{500, 1000}
Batch size	{8, 16, 32}
Random seed	[1,10]
GD iterations during training	{25, 50}
GD iterations during inference	{300, 1000}
GD step size	{0.2, 1, 2}
Max degree trend polynomial	{3}
Window size (s_w)	{128, 256, 512}
Kernel size	{4, 8}
Stride	{4}
Dilation	{1}
Convolution filters of two hidden layers (n_1,n_2)	{(256,128), (128, 64)}

For N-HiTS, we use the hyperparameter grid described in their paper (Challu et al., 2022a). The N-HiTS is a generalization of the N-BEATS, which is already included in the hyperparameter grid as a posible configuration. For the Informer and Autoformer we explore different values for the dropout probability, number of heads, learning rate (same as `SpectraNet`), batch size (same as `SpectraNet`), size of embedding, and sequence length. For StemGNN we explore different optimization parameters, including the learning rate, batch size, and epochs. Finally, for RNN we explore different dilations, number of layers, hidden size, and optimization parameters. For the N-HiTS, transformers, and RNN models we used the implementation of the `Neuralforecast` library (available in PyPI and Conda).

A.5 SPECTRANET TRAINING ALGORITHM

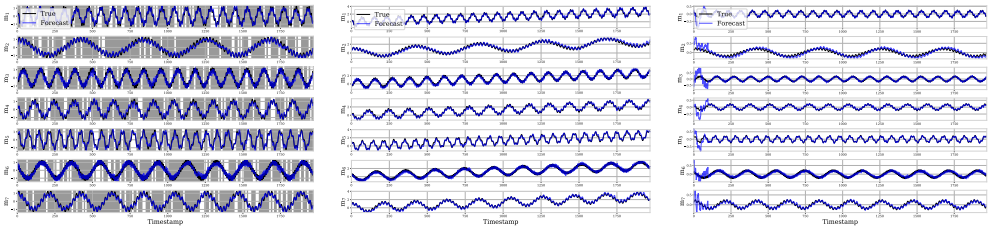
Algorithm 1 presents the training procedure of `SpectraNet`. The model is trained for a fixed number of iterations n_{iters}, randomly sampling b windows in each iteration. Parameters $\boldsymbol{\theta}$ are optimized with Adam optimizer.

Algorithm 1: Training procedure

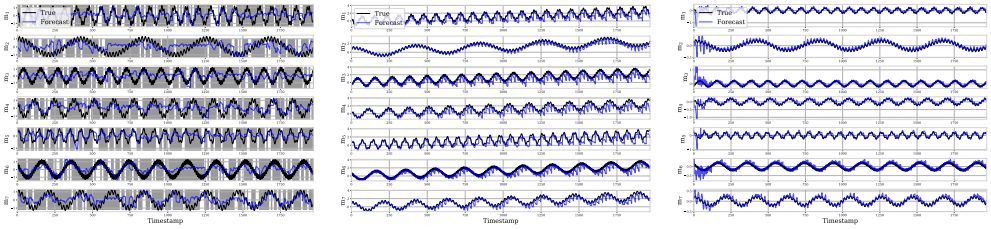
input : multivariate time-series $\mathbf{Y} \in \mathbb{R}^{m \times T}$, model $F_{\boldsymbol{\theta}}$, learning iterations n_{iters}
output: $F_{\boldsymbol{\theta}^*}$, inferred latent vectors $\{\mathbf{z}_t^*, t = 0, ..., T\}$
Let $i \leftarrow 0$, initialize $\boldsymbol{\theta}$
Initialize $\mathbf{z}_t$, for $t = 0, ..., T$
while $i < n_{iters}$ **do**
  Take a random mini-batch of windows $\{\mathbf{Y}_{j_k - L:j_k+H}, j_k \sim U(L, T-H), k\{1, ..., b\}$.
  $\mathbf{z}_{j_k}^* \leftarrow \arg\min_{\mathbf{z}} L(\mathbf{Y}_{j_k-L:j_k}, \hat{\mathbf{Y}}_{j_k-L:j_k}(\mathbf{z})), k \in \{1, ..., b\}$
  Update $\boldsymbol{\theta}$ with Adam using $\mathbf{z}^*$ as input.
  Store $\mathbf{z}_{j_k}^*, k \in \{1, ..., b\}$
  $i \leftarrow i + 1$
end

A.6 FORECASTS ON **SIMULATED7**

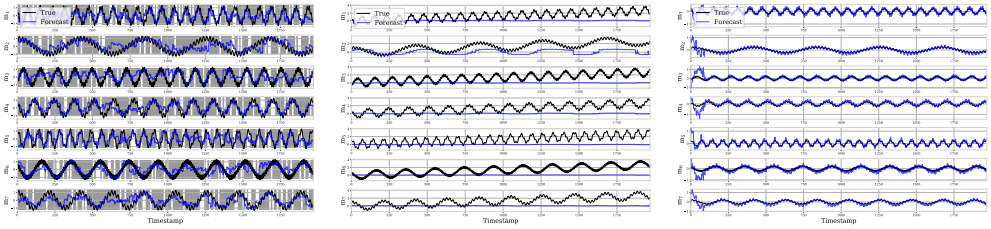
Figure 1 presents the complete forecasts on the test set for `SpectraNet`, N-HiTS, Informer, and RNN models. `SpectraNet` is the only model that can accurately forecast with up to 80% missing data and changes in distribution.



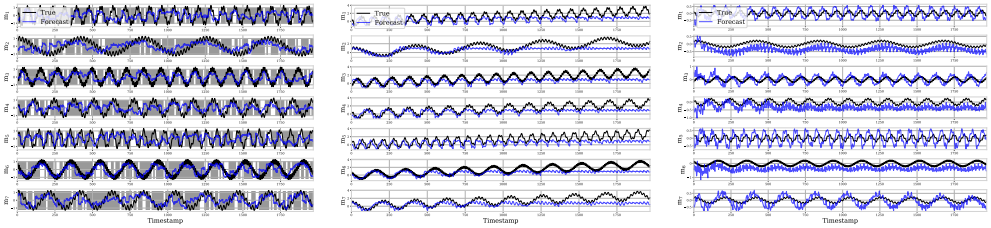
(a) Missing values - `SpectraNet` (b) Trend - `SpectraNet` (c) Magnitude - `SpectraNet`



(d) Missing values - N-HiTS (e) Trend - N-HiTS (f) Magnitude - N-HiTS



(g) Missing values - Informer (h) Trend - Informer (i) Magnitude - Informer



(j) Missing values - RNN (k) Trend - RNN (l) Magnitude - RNN

Figure 1: Forecasts on the test set for all features of **Simulated7** dataset with 80% of missing data (missing regions in grey), change in trend, and change in magnitude. Forecasts are produced every 24 timestamps in a rolling window strategy.

A.7 ILI DATASET

Figure 2 shows the time series of the ILI dataset and the start of the test set (vertical red line). The first five features present considerably larger and longer spikes on the test set. This is a particularly challenging task, as most models tend to underestimate the larger spikes.

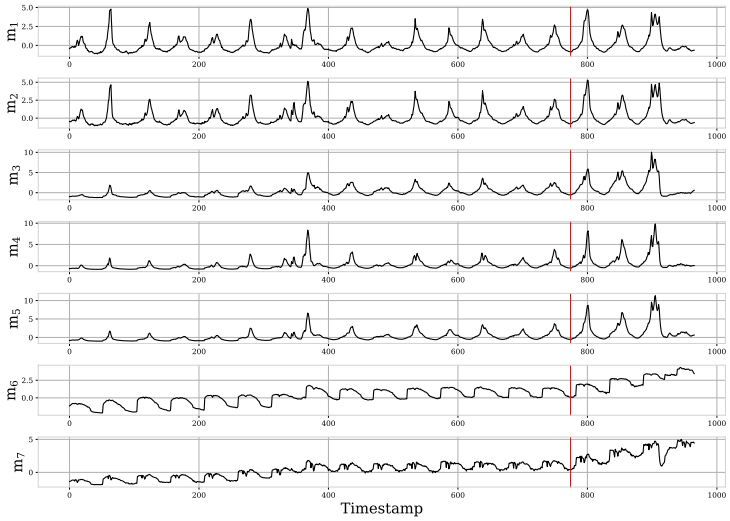


Figure 2: ILI dataset, start of the test set marked with vertical red line.