UNSUPERVISED PROGRESSIVE LEARNING AND THE STAM ARCHITECTURE

ABSTRACT

We first pose the Unsupervised Progressive Learning (UPL) problem: an online representation learning problem in which the learner observes a non-stationary and unlabeled data stream, learning a growing number of features that persist over time even though the data is not stored or replayed. To solve the UPL problem we propose the Self-Taught Associative Memory (STAM) architecture. Layered hierarchies of STAM modules learn based on a combination of online clustering, novelty detection, forgetting outliers, and storing only prototypical features rather than specific examples. We evaluate STAM representations using classification and clustering tasks. While there are no existing learning scenarios that are directly comparable to UPL, we compare the STAM architecture with two recent continual learning models, Memory Aware Synapses (MAS) and Gradient Episodic Memories (GEM), after adapting them in the UPL setting.

1 INTRODUCTION

The *Continual Learning (CL)* problem is predominantly addressed in the supervised context with the goal being to learn a sequence of tasks without "catastrophic forgetting" (Goodfellow et al., 2013; Parisi et al., 2019; van de Ven & Tolias, 2019). There are several CL variations but a common formulation is that the learner observes a set of examples $\{(x_i, t_i, y_i)\}$, where x_i is a feature vector, t_i is a task identifier, and y_i is the target vector associated with (x_i, t_i) (Chaudhry et al., 2019a;b; Lopez-Paz & Ranzato, 2017). Other CL variations replace task identifiers with task boundaries that are either given (Hsu et al., 2018) or inferred (Zeno et al., 2018). Typically, CL requires that the learner either stores and replays some previously seen examples (Aljundi et al., 2019a;b; Gepperth & Karaoguz, 2017; Hayes et al., 2019; Kemker et al., 2018; Rebuffi et al., 2017) or generates examples of earlier learned tasks (Kemker & Kanan, 2018; Liu et al., 2020; Shin et al., 2017).

The *Unsupervised Feature (or Representation) Learning (UFL)* problem, on the other hand, is unsupervised but mostly studied in the *offline context*: given a set of examples $\{x_i\}$, the goal is to learn a *feature vector* $h_i = f(x_i)$ of a given dimensionality that, ideally, makes it easier to identify the explanatory factors of variation behind the data (Bengio et al., 2013), leading to better performance in tasks such as classification or clustering. UFL methods differ in the prior $P(h)$ and the loss function. Autoencoders, for instance, aim to learn features of a lower dimensionality than the input that enable a sufficiently good reconstruction (Bengio, 2014; Kingma & Welling, 2013; Tschannen et al., 2018; Zhou et al., 2012). A similar approach is self-supervised methods, which learn representations by optimizing an auxiliary task (Berthelot et al., 2019; Doersch et al., 2015; Gidaris et al., 2018; Kuo et al., 2019; Oord et al., 2018; Sohn et al., 2020).

In this work, we focus on a new and pragmatic problem that adopts some elements of CL and FL but is also different than them – we refer to this problem as *single-pass unsupervised progressive learning* or *UPL* for short. UPL can be described as follows:
1. the data is observed as a non-IID stream (e.g., different portions of the stream may follow different distributions and there may be strong temporal correlations between successive examples),
2. the features should be learned exclusively from unlabeled data,
3. each example is "seen" only once and the unlabeled data are not stored for iterative processing,
4. the number of learned features may need to increase over time, in response to new tasks and/or changes in the data distribution,
5. to avoid catastrophic forgetting, previously learned features need to persist over time, even when the corresponding data are no longer observed in the stream.

The UPL problem is encountered in important AI applications, such as a robot learning new visual features as it explores a time-varying environment. Additionally, we argue that UPL is closer to how animals learn, at least in the case of *perceptual learning* (Goldstone, 1998). We believe that in order to mimic that, ML methods should be able to learn in a streaming manner and in the absence of supervision. Animals do not "save off" labeled examples to train in parallel with unlabeled data, they do not know how many "classes" exist in their environment, and they do not have to replay/dream periodically all their past experiences to avoid forgetting them.

To address the UPL problem, we describe an architecture referred to as STAM ("Self-Taught Associative Memory"). STAM learns features through *online clustering* at a hierarchy of increasing receptive field sizes. We choose online clustering, instead of more complex learning models, because it can be performed through a single pass over the data stream. Further, despite its simplicity, clustering can generate representations that enable better classification performance than more complex FL methods such as sparse-coding or some deep learning methods (Coates et al., 2011; Coates & Ng, 2012). STAM allows the number of clusters to increase over time, driven by a *novelty detection* mechanism. Additionally, STAM includes a brain-inspired *dual-memory hierarchy* (short-term versus long-term) that enables the conservation of previously learned features (to avoid catastrophic forgetting) that have been seen multiple times at the data stream, while forgetting outliers. To the extent of our knowledge, the UPL problem has not been addressed before. The closest prior work is CURL ("Continual Unsupervised Representation Learning") (Rao et al., 2019). CURL however does not consider the single-pass, online learning requirement. We further discuss this difference with CURL in Section 6.

2 STAM Architecture

In the following, we describe the STAM architecture as a sequence of its major components: a hierarchy of increasing receptive fields, online clustering (centroid learning), novelty detection, and a dual-memory hierarchy that stores prototypical features rather than specific examples. The notation is summarized for convenience in the Supplementary Material (SM) (section SM-A).

I. Hierarchy of increasing receptive fields: An input vector $\mathbf{x_t} \in \mathbb{R}^\mathbf{n}$ (an image in all subsequent examples) is analyzed through a hierarchy of Λ layers. Instead of neurons or hidden-layer units, each layer consists of STAM units – in its simplest form a STAM unit functions as an online clustering module. Each STAM unit processes one $\rho_l \times \rho_l$ *patch* (subvector) of the input at that layer. The patches are overlapping, with a small stride (set to one pixel in our experiments) to accomplish translation invariance (similar to CNNs). The patch dimension ρ_l increases in higher layers – the idea is that the first layer learns the smallest and most elementary features while the top layer learns the largest and most complex features.

II. Centroid Learning: Every patch of each layer is clustered, in an online manner, to a set of centroids. These time-varying centroids form the *features* that the STAM architecture gradually learns at that layer. All STAM units of layer l share the same set of centroids $C_l(t)$ – again for translation invariance.[1] Given the m'th input patch $\mathbf{x_{l,m}}$ at layer l, the nearest centroid of C_l selected for $\mathbf{x_{l,m}}$ is

$$\mathbf{c_{l,j}} = \arg \min_{c \in C_l} d(\mathbf{x_{l,m}}, \mathbf{c}) \tag{1}$$

where $d(\mathbf{x_{l,m}}, \mathbf{c})$ is the Euclidean distance between the patch $\mathbf{x_{l,m}}$ and centroid $\mathbf{c}$.[2] The selected centroid is updated based on a learning rate parameter α, as follows:

$$\mathbf{c_{l,j}} = \alpha \, \mathbf{x_{l,m}} + (1 - \alpha)\mathbf{c_{l,j}}, \quad 0 < \alpha < 1 \tag{2}$$

A higher α value makes the learning process faster but less predictable. We do not use a decreasing value of α because the goal is to keep learning in a non-stationary environment rather than convergence to a stable centroid.

III. Novelty detection: When an input patch $\mathbf{x_{l,m}}$ at layer l is significantly different than all centroids at that layer (i.e., its distance to the nearest centroid is a statistical outlier), a new centroid is created in C_l based on $\mathbf{x_{l,m}}$. We refer to this event as *Novelty Detection (ND)*. This function is necessary so that the architecture can learn novel features when the data distribution changes.

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[1] We drop the time index t from this point on but it is still implied that the centroids are dynamically learned over time.

[2] We have also experimented with the L1 metric with only minimal differences. Different distance metrics may be more appropriate for other types of data.

To do so, we estimate in an online manner the distance distribution between input patches and their nearest centroid (separately for each layer). The novelty detection threshold at layer l is denoted by $\hat{D}_l$ and it is defined as the 95-th percentile ($\beta = 0.95$) of this distance distribution.

Figure 1: A hypothetical pool of STM and LTM centroids visualized at seven time instants. From t_a to t_b, a centroid is moved from STM to LTM after it has been selected θ times. At time t_b, unlabeled examples from classes '2' and '3' first appear, triggering novelty detection and new centroids are created in STM. These centroids are moved into LTM by t_d. From t_d to t_g, the pool of LTM centroids remains the same because no new classes are seen. The pool of STM centroids keeps changing when we receive "outlier" inputs of previously seen classes. Those centroids are later replaced (Least-Recently-Used policy) due to the limited capacity of the STM pool.

IV. Dual-memory organization: New centroids are stored temporarily in a *Short-Term Memory (STM)* of limited capacity Δ, separately for each layer. Every time a centroid is selected as the nearest neighbor of an input patch, it is updated based on (2). If an STM centroid $\mathbf{c_{l,j}}$ is selected more than θ times, it is copied to the *Long-Term Memory (LTM)* for that layer. We refer to this event as *memory consolidation*. The LTM has (practically) unlimited capacity and the learning rate is much smaller (in our experiments the LTM learning rate is set to zero).

This memory organization is inspired by the Complementary Learning Systems framework (Kumaran et al., 2016), where the STM role is played by the hippocampus and the LTM role by the cortex. This dual-memory scheme is necessary to distinguish between infrequently seen patterns that can be forgotten ("outliers"), and new patterns that are frequently seen after they first appear ("novelty").

When the STM pool of centroids at a layer is full, the introduction of a new centroid (created through novelty detection) causes the removal of an earlier centroid. We use the Least-Recently Used (LRU) policy to remove atypical centroids that have not been recently selected by any input. Figure 1 illustrates this dual-memory organization.

V. Initialization: We initialize the pool of STM centroids at each layer using randomly sampled patches from the first few images of the unlabeled stream. The initial value of the novelty-detection threshold is calculated based on the distance distribution between each of these initial STM centroids and its nearest centroid.

3 CLASSIFICATION USING STAM

Given a small amount of labeled data, STAM can be used in classification tasks. We emphasize that the labeled data is not used for representation learning – it is only used to associate previously learned features with a given set of classes.

I. Associating centroids with classes: Suppose we are given some labeled examples $X_L(t)$ from a set of classes $L(t)$ at time t. We can use these labeled examples to associate existing LTM centroids at time t (learned strictly from unlabeled data) with the set of classes in $L(t)$.

Given a labeled example of class k, suppose that there is a patch $\mathbf{x}$ in that example for which the nearest centroid is $\mathbf{c}$. That patch contributes the following association between centroid $\mathbf{c}$ and class k:

$$f_{\mathbf{x},\mathbf{c}}(k) = e^{-d(\mathbf{x},\mathbf{c})/\bar{D}_l} \tag{3}$$

where $\bar{D}_l$ is a normalization constant (calculated as the average distance between input patches and centroids). The *class-association vector* $\mathbf{g_c}$ between centroid $\mathbf{c}$ and any class k is computed aggregating all such associations, across all labeled examples in X_L:

$$g_{\mathbf{c}}(k) = \frac{\sum_{\mathbf{x} \in X_L(k)} f_{\mathbf{x},\mathbf{c}}(k)}{\sum_{k' \in L(t)} \sum_{\mathbf{x} \in X_L(k')} f_{\mathbf{x},\mathbf{c}}(k')}, \quad k = 1 \dots L(t) \tag{4}$$

where $X_L(k)$ refers to labeled examples belonging to class k. Note that $\sum_k g_{\mathbf{c}}(k)=1$.

II. Class informative centroids: If a centroid is associated with only one class k ($g_{\mathbf{c}}(k) = 1$), only labeled examples of that class select that centroid. At the other extreme, if a centroid is equally likely to be selected by examples of any labeled class, ($g_{\mathbf{c}}(k) \approx 1/|L(t)|$), the selection of that centroid does not provide any significant information for the class of the corresponding input. We identify the centroids that are *Class INformative (CIN)* as those that are associated with at least one class significantly more than expected by chance. Specifically, a centroid $\mathbf{c}$ is CIN if

$$\max_{k \in L(t)} g_{\mathbf{c}}(k) > \frac{1}{|L(t)|} + \gamma \qquad (5)$$

where $1/|L(t)|$ is the chance term and γ is the significance term.

Figure 2: An example of the classification process. Every patch (at any layer) that selects a CIN centroid votes for the single class that has the highest association with. These patch votes are first averaged at each layer. The final inference is the class with the highest cumulative vote across all layers.

III. Classification using a hierarchy of centroids: At test time, we are given an input $\mathbf{x}$ of class $k(\mathbf{x})$ and infer its class as $\hat{k}(\mathbf{x})$. The classification task is a "biased voting" process in which every patch of $\mathbf{x}$, at any layer, votes for a single class as long as that patch selects a CIN centroid.

Specifically, if a patch $\mathbf{x_{l,m}}$ of layer l selects a CIN centroid $\mathbf{c}$, then that patch votes $v_{l,m} = \max_{k \in L(t)} g_{\mathbf{c}}(k)$ for the class k that has the highest association with $\mathbf{c}$, and zero for all other classes. If c is *not* a CIN centroid, the vote of that patch is zero for all classes.

The vote of layer l for class k is the average vote across all patches in layer l (as illustrated in Figure 2):

$$v_l(k) = \frac{\sum_{m \in M_l} v_{l,m}}{|M_l|} \qquad (6)$$

where M_l is the set of patches in layer l. The final inference for input $\mathbf{x}$ is the class with the highest cumulative vote across all layers:

$$\hat{k}(\mathbf{x}) = \arg\max_{k'} \sum_{l=1}^{\Lambda} v_l(k) \qquad (7)$$

4 CLUSTERING USING STAM

We can also use the STAM features in unsupervised tasks, such as offline clustering. For each patch of input $\mathbf{x}$, we compute the nearest LTM centroid. The set of all such centroids, across all patches of $\mathbf{x}$, is denoted by $\Phi(\mathbf{x})$. Given two inputs $\mathbf{x}$ and $\mathbf{y}$, their pairwise distance is the Jaccard distance of $\Phi(\mathbf{x})$ and $\Phi(\mathbf{y})$. Then, given a set of inputs that need to be clustered, and a target number of clusters, we apply a spectral clustering algorithm on the pairwise distances between the set of inputs. We could also use other clustering algorithms, as long as they do not require Euclidean distances.

5 EVALUATION

To evaluate the STAM architecture in the UPL context, we consider a data stream in which small groups of classes appear in successive *phases*, referred to as **Incremental UPL**. New classes are introduced two at a time in each phase, and they are only seen in that phase. STAM must be able to both recognize new classes when they are first seen in the stream, and to also remember all previously learned classes without catastrophic forgetting. Another evaluation scenario is **Uniform UPL**, where all classes appear with equal probability throughout the stream – the results for Uniform UPL are shown in SM-G. We include results on four datasets: MNIST (Lecun et al., 1998), EMNIST (balanced split with 47 classes) (Cohen et al., 2017), SVHN (Netzer et al., 2011), and CIFAR-10 (Krizhevsky

et al., 2014). For each dataset we utilize the standard training and test splits. We preprocess the images by applying per-patch normalization (instead of image normalization), and SVHN is converted to grayscale. More information about preprocessing can be found in SM-H.

We create the training stream by randomly selecting, with equal probability, N_p data examples from the classes seen during each phase. N_p is set to 10000, 10000, 2000, and 10000 for MNIST, SVHN, EMNIST, and CIFAR-10 respectively. More information about the impact of the stream size can be found in SM-E. In each task, we average results over three different unlabeled data streams. During testing, we select 100 random examples of each class from the test dataset. This process is repeated five times for each training stream (i.e., a total of fifteen results per experiment). The following plots show mean $\pm$ std-dev.

For all datasets, we use a 3-layer STAM hierarchy. In the classification task, we select a small portion of the training dataset as the labeled examples that are available only to the classifier. In the clustering task, we form the set $\Phi(\mathbf{x})$ considering only Layer-3 patches of the input $\mathbf{x}$. The hyperparameter values are tabulated in SM-A. The robustness of the results with respect to these values is examined in SM-F.

Baseline Methods: We evaluate the STAM architecture comparing its performance to two state-of-the-art baselines for continual learning: GEM and MAS. We emphasize that there are no prior approaches which are directly applicable to UPL. However, we have taken reasonable steps to adapt these two baselines in the UPL setting. Please see SM-B for additional details about our adaptation of GEM and MAS.

Gradient Episodic Memories (GEM) is a recent supervised continual learing model that expects known task boundaries (Lopez-Paz & Ranzato, 2017). To turn GEM into an unsupervised model, we combined it with a self supervised method for rotation prediction (Gidaris et al., 2018). Additionally, we allow GEM to know the boundary between successive phases in the data stream. This makes the comparison with STAM somehow unfair, because STAM does not have access to this information. The results show however that STAM performs better even without knowing the temporal boundaries of successive phases.

Memory Aware Synapse (MAS) is another supervised continual learning model that expects known task boundaries (Aljundi et al., 2018). As in GEM, we combined MAS with a rotation prediction self-supervised task, and provided the model with information about the start of each new phase in the data stream.

To satisfy the stream requirement of UPL, the number of training epochs for both GEM and MAS is set to one. Deep learning methods become weaker in this streaming scenario because they cannot train iteratively over several epochs on the same dataset. For all baselines, the classification task is performed using a $K = 1$ Nearest-Neighbor (KNN) classifier – we have experimented with various values of K and other single-pass classifiers, and report only the best performing results here. We have also compared the memory requirement of STAM (storing centroids at STM and LTM) with the memory requirement of the two baselines. The results of that comparison appear in SM-C.

Classification Task: We focus on an *expanding classification task*, meaning that in each phase we need to classify *all* classes seen so far. The results for the classification task are given in Figure 3. Note that we use only 10 labeled examples per class for MNIST and EMNIST, and 100 examples per class for SVHN and CIFAR-10. We emphasize that the two baselines, GEM and MAS, have access to the temporal boundaries between successive phases, while STAM does not.

As we introduce new classes in the stream, the average accuracy per phase decreases for all methods in each dataset. This is expected, as the task gets *more difficult* after each phase. In MNIST, STAM performs consistently better than GEM and MAS, and STAM is less vulnerable to catastrophic forgetting. For SVHN, the trend is similar after the first phase but the difference between STAM and both baselines is smaller. With CIFAR-10, we observe that all models including STAM perform rather poorly – probably due to the low resolution of these images. STAM is still able to maintain comparable accuracy to the baselines with a smaller memory footprint. Finally, in EMNIST, we see a consistently higher accuracy with STAM compared to the two baselines. We would like to emphasize that these baselines are allowed extra information in the form of known tasks boundaries and STAM is still performs better both on all datasets. For additional analysis and discussion of these results, please also see SM-D.

Figure 3: Classification accuracy for MNIST (left), SVHN (center), CIFAR-10 (right-center), and EMNIST (right). The task is expanding classification for incremental UPL, i.e., recognize all classes seen so far. Note that the number of labeled examples is 10 per class for MNIST and EMNIST and 100 per class for SVHN and CIFAR-10.

Clustering Task: Given that we have the same number of test vectors per class we utilize the *purity* measure for clustering accuracy. In MNIST, STAM performs consistently better than the two other models, and its accuracy stays almost constant throughout the stream, only dropping slightly in the final phase. In SVHN, STAM performs better than both deep learning baselines with the gap being much smaller in the final phase. In CIFAR-10 and EMNIST, on the other hand, we see similar performance between all three models. Again, we emphasize that STAM is not provided task boundary information while the baselines are and is still able to perform better, significantly in some cases.

Figure 4: Clustering accuracy for MNIST (left), SVHN (left-center), CIFAR-10 (right-center), and EMNIST (right). The task is expanding clustering for incremental UPL. The number of clusters is equal to 2 times the number of classes in the data stream seen up to that point in time.

Ablation studies: Several STAM ablations are presented in Figure 5. On the left, we remove the LTM capability and only use STM centroids for classification. During the first two phases, there is little (if any) difference in classification accuracy. However, we see a clear dropoff during phases 3-5. This suggests that, without the LTM mechanisms, features from classes that are no longer seen in the stream are forgotten over time, and STAM can only successfully classify classes that have been recently seen. We also investigate the importance of having static LTM centroids rather than dynamic centroids (Fig. 5-middle). Specifically, we replace the static LTM with a dynamic LTM in which

Figure 5: Ablation study: A STAM architecture without LTM (left), a STAM architecture in which the LTM centroids are adjusted with the same learning rate α as in STM (center), and a STAM architecture with removal of layers (right)

the centroids are adjusted with the same learning rate parameter α, as in STM. The accuracy suffers drastically because the introduction of new classes "takes over" LTM centroids of previously learned classes, after the latter are removed from the stream. Similar to the removal of LTM, we do not see the effects of "forgetting" until phases 3-5. Note that the degradation due to a dynamic LTM is less severe than that from removing LTM completely.

Finally, we look at the effects of removing layers from the STAM hierarchy (Fig. 5-right). We see a small drop in accuracy after removing layer 3, and a large drop in accuracy after also removing layer 2. The importance of having a deeper hierarchy would be more pronounced in datasets with higher-resolution images or videos, potentially showing multiple objects in the same frame. In such cases, CIN centroids can appear at any layer, starting from the lowest to the highest.

Additional results: The reader can find additional experimental results in the Supp-Material section that focus on the questions: how does the number of LTM centroids increase with time and what fraction of them are "Class Informative" (in classification tasks), how does the accuracy of STAM vary with the number of labeled examples per class, and how do the various hyperparameters of the STAM architecture affect classification accuracy?

6 RELATED WORK

The UPL problem has some similarities with several recent approaches in the machine learning literature but it is also different in important aspects we describe in this section. Each paragraph highlights the most relevant prior work and explains how it is different from UPL.

I: Continual learning: In addition to CL models cited in the introduction, other *supervised* CL methods include regularization-based approaches (Aljundi et al., 2018; Golkar et al., 2019; Hayes & Kanan, 2019; Kirkpatrick et al., 2017; Yoon et al., 2018; Zenke et al., 2017), expanding architectures (Lomonaco & Maltoni, 2017; Maltoni & Lomonaco, 2019; Rusu et al., 2016), and distillation-based methods (Lee et al., 2019; 2020; Li & Hoiem, 2017). Their main difference with UPL and STAM is that they are designed for supervised learning, and it is not clear how to adapt them for non-stationary and unlabeled data streams.

II. Offline unsupervised learning: Additional *offline* representation learning methods include clustering (Caron et al., 2018; Jiang et al., 2017a; Xie et al., 2016; Yang et al., 2016), generative models (Eslami et al., 2016; Jiang et al., 2017b; Kosiorek et al., 2018; 2019), information theory (Hjelm et al., 2019; Ji et al., 2019), among others. These methods require prior information about the number of classes present in a given dataset (to set the number of cluster centroids or class outputs) and iterative training (i.e. data replay), and therefore cannot be directly applied in the UPL setting.

III. Semi-supervised learning (SSL): SSL methods require labeled data during the representation learning stage and so they are not compatible with UPL (Kingma et al., 2014; Lee, 2013; Miyato et al., 2018; Oliver et al., 2018; Rasmus et al., 2015; Springenberg, 2015; Tarvainen & Valpola, 2017).

IV. Few-shot learning (FSL) and Meta-learning: These methods recognize object classes not seen in the training set with only a single (or handful) of labeled examples (Fei-Fei et al., 2006; Finn et al., 2017; Ren et al., 2018; Snell et al., 2017). Similar to SSL, FSL methods require labeled data to learn representations and therefore are not applicable in the UPL context. Centroid networks (Huang et al., 2019) do not require labeled examples at inference time but require labeled examples for training. Few-Shot Incremental Learning (Ayub & Wagner, 2020) includes a similar evaluation protocol to our work, but includes pre-training and learns centroids using data labels.

V. Multi-Task Learning (MTL): Any MTL method that involves separate heads for different tasks is not compatible with UPL because task boundaries are not known a priori in UPL (Ruder, 2017).

MTL methods that require pre-training on a large labeled dataset are also not applicable to UPL (Pan & Yang, 2010; Yosinski et al., 2014).

VI. Online and Progressive Learning: Many earlier methods learn in an online manner, meaning that data is processed in fixed batches and discarded afterwards. This includes progressive learning (Venkatesan & Er, 2016) and streaming with limited supervision (Chiotellis et al., 2018; Li et al., 2018; Loo & Marsono, 2015), both of which require labeled data in the training stream. (Caccia et al., 2019) is another very recent work which focuses on online continual compression using a discrete auto encoder and self-replay.

VII. Continual Unsupervised Representation Learning (CURL): Similar to the UPL problem, CURL (Rao et al., 2019) focuses on continual unsupervised learning from non-stationary data with unknown task boundaries. Like STAM, CURL also includes a mechanism to trigger dynamic capacity expansion as the data distribution changes. However, a major difference is that CURL is *not* a streaming method – it processes each training example multiple times. We have experimented with CURL but we found that its performance collapses in the UPL setting due to mostly three reasons: the small batch size required by UPL, the availability of a single-pass through the training data, and the fact that we can have more than one new classes per phase. For these reasons, we choose not to compare STAM with CURL because such a comparison would not be fair for the latter.

VIII. Data dimensionality and clustering-based representation learning: As mentioned earlier in this section, clustering has been used successfully in the past for offline representation learning (e.g., (Coates et al., 2011; Coates & Ng, 2012)). Its effectiveness, however, gradually drops as the input dimensionality increases (Beyer et al., 1999; Hinneburg et al., 2000). In the STAM architecture, we avoid this issue by clustering smaller subvectors (patches) of the input data. If those subvectors are still of high dimensionality, another approach is to reduce the *intrinsic dimensionality* of the input data at each layer by reconstructing that input using representations (selected centroids) from the previous layer.

VIII. Related work to other STAM components: STAM relies on online clustering. This algorithm can be implemented with a rather simple recurrent neural network of excitatory and inhibitory spiking neurons, as shown recently (Pehlevan et al., 2017). The novelty detection component of STAM is related to the problem of anomaly detection in streaming data (Dasgupta et al., 2018) — and the simple algorithm currently in STAM can be replaced with more sophisticated methods (e.g., (Cui et al., 2016; Yong et al., 2012)). Finally, brain-inspired dual-memory systems have been proposed before for memory consolidation (e.g., (Kemker & Kanan, 2018; Parisi et al., 2018; Shin et al., 2017)).

7 DISCUSSION

The STAM architecture aims to address the following desiderata that is often associated with Lifelong Learning (Parisi et al., 2019):

I. Online learning: STAMs update the learned features with every observed example. There is no separate training stage for specific tasks, and inference can be performed in parallel with learning.

II. Transfer learning: The features learned by the STAM architecture in earlier phases can be also encountered in the data of future tasks (forward transfer). Additionally, new centroids committed to LTM can also be closer to data of earlier tasks (backward transfer).

III. Resistance to catastrophic forgetting: The STM-LTM memory hierarchy of the STAM architecture mitigates catastrophic forgetting by committing to "permanent storage" (LTM) features that have been often seen in the data during any time period of the training period.

IV. Expanding learning capacity: The unlimited capacity of LTM allows the system to gradually learn more features as it encounters new classes and tasks. The relatively small size of STM, on the other hand, forces the system to forget features that have not been recalled frequently enough after their creation.

V. No direct access to previous experience: STAM only needs to store data centroids in a hierarchy of increasing receptive fields – there is no need to store previous exemplars or to learn a generative model that can produce such examples.

REFERENCES

Rahaf Aljundi, Francesca Babiloni, Mohamed Elhoseiny, Marcus Rohrbach, and Tinne Tuytelaars. Memory aware synapses: Learning what (not) to forget. In *ECCV*, 2018.

Rahaf Aljundi, Eugene Belilovsky, Tinne Tuytelaars, Laurent Charlin, Massimo Caccia, Min Lin, and Lucas Page-Caccia. Online continual learning with maximal interfered retrieval. In *Advances in Neural Information Processing Systems*, pp. 11849–11860, 2019a.

Rahaf Aljundi, Min Lin, Baptiste Goujaud, and Yoshua Bengio. Gradient based sample selection for online continual learning. In *Advances in Neural Information Processing Systems*, pp. 11816–11825, 2019b.

Ali Ayub and Alan R Wagner. Cognitively-inspired model for incremental learning using a few examples. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops*, pp. 222–223, 2020.

Yoshua Bengio. How auto-encoders could provide credit assignment in deep networks via target propagation. *arXiv preprint arXiv:1407.7906*, 2014.

Yoshua Bengio, Aaron Courville, and Pascal Vincent. Representation learning: A review and new perspectives. *IEEE Trans. Pattern Anal. Mach. Intell.*, 35(8):1798–1828, August 2013. ISSN 0162-8828. doi: 10.1109/TPAMI.2013.50.

David Berthelot, Nicholas Carlini, Ian Goodfellow, Nicolas Papernot, Avital Oliver, and Colin A Raffel. Mixmatch: A holistic approach to semi-supervised learning. In *Advances in Neural Information Processing Systems*, pp. 5050–5060, 2019.

Kevin S. Beyer, Jonathan Goldstein, Raghu Ramakrishnan, and Uri Shaft. When is "nearest neighbor" meaningful? In *Proceedings of the 7th International Conference on Database Theory*, ICDT '99, pp. 217–235, London, UK, UK, 1999. Springer-Verlag. ISBN 3-540-65452-6.

Lucas Caccia, Eugene Belilovsky, Massimo Caccia, and Joelle Pineau. Online learned continual compression with adaptive quantization modules. *arXiv*, pp. arXiv–1911, 2019.

Mathilde Caron, Piotr Bojanowski, Armand Joulin, and Matthijs Douze. Deep clustering for unsupervised learning of visual features. In *The European Conference on Computer Vision (ECCV)*, September 2018.

Arslan Chaudhry, Marc'Aurelio Ranzato, Marcus Rohrbach, and Mohamed Elhoseiny. Efficient lifelong learning with a-GEM. In *International Conference on Learning Representations*, 2019a.

Arslan Chaudhry, Marcus Rohrbach, Mohamed Elhoseiny, Thalaiyasingam Ajanthan, Puneet K Dokania, Philip HS Torr, and Marc'Aurelio Ranzato. Continual learning with tiny episodic memories. *arXiv preprint arXiv:1902.10486*, 2019b.

Ioannis Chiotellis, Franziska Zimmermann, Daniel Cremers, and Rudolph Triebel. Incremental semi-supervised learning from streams for object classification. In *2018 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, pp. 5743–5749. IEEE, 2018.

Adam Coates and Andrew Y Ng. *Learning feature representations with k-means*, pp. 561–580. Springer, 2012.

Adam Coates, Andrew Ng, and Honglak Lee. An analysis of single-layer networks in unsupervised feature learning. In *Proceedings of the fourteenth international conference on artificial intelligence and statistics*, pp. 215–223, 2011.

Gregory Cohen, Saeed Afshar, Jonathan Tapson, and André van Schaik. Emnist: an extension of mnist to handwritten letters. *ArXiv*, abs/1702.05373, 2017.

Yuwei Cui, Subutai Ahmad, and Jeff Hawkins. Continuous online sequence learning with an unsupervised neural network model. *Neural Comput.*, 28(11):2474–2504, November 2016. ISSN 0899-7667. doi: 10.1162/NECO_a_00893.

Sanjoy Dasgupta, Timothy C Sheehan, Charles F Stevens, and Saket Navlakha. A neural data structure for novelty detection. *Proceedings of the National Academy of Sciences*, 115(51):13093–13098, 2018.

Carl Doersch, Abhinav Gupta, and Alexei A. Efros. Unsupervised visual representation learning by context prediction. *2015 IEEE International Conference on Computer Vision (ICCV)*, pp. 1422–1430, 2015.

S. M. Ali Eslami, Nicolas Heess, Theophane Weber, Yuval Tassa, David Szepesvari, Koray Kavukcuoglu, and Geoffrey E. Hinton. Attend, infer, repeat: Fast scene understanding with generative models. In *Proceedings of the 30th International Conference on Neural Information Processing Systems*, NIPS'16, pp. 3233–3241, USA, 2016. Curran Associates Inc. ISBN 978-1-5108-3881-9.

Li Fei-Fei, R. Fergus, and P. Perona. One-shot learning of object categories. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 28(4):594–611, April 2006. ISSN 0162-8828. doi: 10.1109/TPAMI.2006.79.

Chelsea Finn, Pieter Abbeel, and Sergey Levine. Model-agnostic meta-learning for fast adaptation of deep networks. In *Proceedings of the 34th International Conference on Machine Learning - Volume 70*, ICML'17, pp. 1126–1135. JMLR.org, 2017.

Alexander Gepperth and Cem Karaoguz. Incremental learning with self-organizing maps. *2017 12th International Workshop on Self-Organizing Maps and Learning Vector Quantization, Clustering and Data Visualization (WSOM)*, pp. 1–8, 2017.

Spyros Gidaris, Praveer Singh, and Nikos Komodakis. Unsupervised representation learning by predicting image rotations. In *International Conference on Learning Representations*, 2018.

Robert L Goldstone. Perceptual learning. *Annual review of psychology*, 49(1):585–612, 1998.

Siavash Golkar, Michael Kagan, and Kyunghyun Cho. Continual learning via neural pruning. *arXiv preprint arXiv:1903.04476*, 2019.

Ian J Goodfellow, Mehdi Mirza, Da Xiao, Aaron Courville, and Yoshua Bengio. An empirical investigation of catastrophic forgetting in gradient-based neural networks. *arXiv preprint arXiv:1312.6211*, 2013.

Tyler L Hayes and Christopher Kanan. Lifelong machine learning with deep streaming linear discriminant analysis. *arXiv preprint arXiv:1909.01520*, 2019.

Tyler L Hayes, Nathan D Cahill, and Christopher Kanan. Memory efficient experience replay for streaming learning. In *2019 International Conference on Robotics and Automation (ICRA)*, pp. 9769–9776. IEEE, 2019.

Alexander Hinneburg, Charu C. Aggarwal, and Daniel A. Keim. What is the nearest neighbor in high dimensional spaces? In *Proceedings of the 26th International Conference on Very Large Data Bases*, VLDB '00, pp. 506–515, San Francisco, CA, USA, 2000. Morgan Kaufmann Publishers Inc. ISBN 1-55860-715-3.

Devon Hjelm, Alex Fedorov, Samuel Lavoie-Marchildon, Karan Grewal, Phil Bachman, Adam Trischler, and Yoshua Bengio. Learning deep representations by mutual information estimation and maximization. In *ICLR 2019*. ICLR, April 2019.

Yen-Chang Hsu, Yen-Cheng Liu, Anita Ramasamy, and Zsolt Kira. Re-evaluating continual learning scenarios: A categorization and case for strong baselines. In *NeurIPS Continual learning Workshop*, 2018.

Gabriel Huang, Hugo Larochelle, and Simon Lacoste-Julien. Centroid networks for few-shot clustering and unsupervised few-shot classification. *arXiv preprint arXiv:1902.08605*, 2019.

X. Ji, J. Henriques, and A. Vedaldi. Invariant infromation clustering for unsupervised image classification and segmentation. In *Proceedings of the International Conference on Computer Vision (ICCV)*, 2019.

Zhuxi Jiang, Yin Zheng, Huachun Tan, Bangsheng Tang, and Hanning Zhou. Variational deep embedding: An unsupervised and generative approach to clustering. In *Proceedings of the 26th International Joint Conference on Artificial Intelligence*, IJCAI'17, pp. 1965–1972. AAAI Press, 2017a. ISBN 9780999241103.

Zhuxi Jiang, Yin Zheng, Huachun Tan, Bangsheng Tang, and Hanning Zhou. Variational deep embedding: An unsupervised and generative approach to clustering. In *Proceedings of the 26th International Joint Conference on Artificial Intelligence*, IJCAI'17, pp. 1965–1972. AAAI Press, 2017b. ISBN 978-0-9992411-0-3.

Ronald Kemker and Christopher Kanan. Fearnet: Brain-inspired model for incremental learning. *International Conference on Learning Representations (ICLR)*, 2018.

Ronald Kemker, Marc McClure, Angelina Abitino, Tyler Hayes, and Christopher Kanan. Measuring catastrophic forgetting in neural networks. *AAAI Conference on Artificial Intelligence*, 2018.

Diederik P Kingma and Max Welling. Auto-encoding variational bayes. *arXiv preprint arXiv:1312.6114*, 2013.

Diederik P. Kingma, Danilo J. Rezende, Shakir Mohamed, and Max Welling. Semi-supervised learning with deep generative models. In *Proceedings of the 27th International Conference on Neural Information Processing Systems - Volume 2*, NIPS'14, pp. 3581–3589, Cambridge, MA, USA, 2014. MIT Press.

James Kirkpatrick, Razvan Pascanu, Neil Rabinowitz, Joel Veness, Guillaume Desjardins, Andrei A Rusu, Kieran Milan, John Quan, Tiago Ramalho, Agnieszka Grabska-Barwinska, et al. Overcoming catastrophic forgetting in neural networks. *Proceedings of the national academy of sciences*, 2017.

Adam Kosiorek, Hyunjik Kim, Yee Whye Teh, and Ingmar Posner. Sequential attend, infer, repeat: Generative modelling of moving objects. In S. Bengio, H. Wallach, H. Larochelle, K. Grauman, N. Cesa-Bianchi, and R. Garnett (eds.), *Advances in Neural Information Processing Systems 31*, pp. 8606–8616. Curran Associates, Inc., 2018.

Adam Kosiorek, Sara Sabour, Yee Whye Teh, and Geoffrey E Hinton. Stacked capsule autoencoders. In *Advances in Neural Information Processing Systems*, pp. 15486–15496, 2019.

Alex Krizhevsky, Vinod Nair, and Geoffrey Hinton. The cifar-10 dataset. *online: http://www. cs. toronto. edu/kriz/cifar. html*, 55, 2014.

Dharshan Kumaran, Demis Hassabis, and James L McClelland. What learning systems do intelligent agents need? complementary learning systems theory updated. *Trends in cognitive sciences*, 20(7): 512–534, 2016.

Chia-Wen Kuo, Chih-Yao Ma, Jia-Bin Huang, and Zsolt Kira. Manifold graph with learned prototypes for semi-supervised image classification. *arXiv preprint arXiv:1906.05202*, 2019.

Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner. Gradient-based learning applied to document recognition. *Proceedings of the IEEE*, 86(11):2278–2324, Nov 1998. ISSN 0018-9219. doi: 10.1109/5.726791.

Dong-Hyun Lee. Pseudo-label : The simple and efficient semi-supervised learning method for deep neural networks. *ICML 2013 Workshop : Challenges in Representation Learning (WREPL)*, 07 2013.

Kibok Lee, Kimin Lee, Jinwoo Shin, and Honglak Lee. Overcoming catastrophic forgetting with unlabeled data in the wild. In *Proceedings of the IEEE International Conference on Computer Vision*, pp. 312–321, 2019.

Soochan Lee, Junsoo Ha, Dongsu Zhang, and Gunhee Kim. A neural dirichlet process mixture model for task-free continual learning. *arXiv preprint arXiv:2001.00689*, 2020.

Yanchao Li, Yongli Wang, Qi Liu, Cheng Bi, Xiaohui Jiang, and Shurong Sun. Incremental semi-supervised learning on streaming data. *Pattern Recognition*, 88, 11 2018. doi: 10.1016/j.patcog. 2018.11.006.

Zhizhong Li and Derek Hoiem. Learning without forgetting. *IEEE transactions on pattern analysis and machine intelligence*, 40(12):2935–2947, 2017.

Xialei Liu, Chenshen Wu, Mikel Menta, Luis Herranz, Bogdan Raducanu, Andrew D Bagdanov, Shangling Jui, and Joost van de Weijer. Generative feature replay for class-incremental learning. *arXiv preprint arXiv:2004.09199*, 2020.

Vincenzo Lomonaco and Davide Maltoni. Core50: a new dataset and benchmark for continuous object recognition. *arXiv preprint arXiv:1705.03550*, 2017.

H. R. Loo and M. N. Marsono. Online data stream classification with incremental semi-supervised learning. In *Proceedings of the Second ACM IKDD Conference on Data Sciences*, CoDS '15, pp. 132–133, New York, NY, USA, 2015. Association for Computing Machinery. ISBN 9781450334365. doi: 10.1145/2732587.2732614.

David Lopez-Paz and Marc'Aurelio Ranzato. Gradient episodic memory for continual learning. In *Proceedings of the 31st International Conference on Neural Information Processing Systems*, NIPS'17, pp. 6470–6479, USA, 2017. Curran Associates Inc. ISBN 978-1-5108-6096-4.

Davide Maltoni and Vincenzo Lomonaco. Continuous learning in single-incremental-task scenarios. *Neural Networks*, 116:56–73, 2019.

Takeru Miyato, Shin-ichi Maeda, Shin Ishii, and Masanori Koyama. Virtual adversarial training: a regularization method for supervised and semi-supervised learning. *IEEE transactions on pattern analysis and machine intelligence*, 2018.

Yuval Netzer, Tao Wang, Adam Coates, Alessandro Bissacco, Bo Wu, and Andrew Y. Ng. Reading digits in natural images with unsupervised feature learning. In *NIPS Workshop on Deep Learning and Unsupervised Feature Learning 2011*, 2011.

Avital Oliver, Augustus Odena, Colin A Raffel, Ekin Dogus Cubuk, and Ian Goodfellow. Realistic evaluation of deep semi-supervised learning algorithms. In *Advances in Neural Information Processing Systems*, pp. 3235–3246, 2018.

Aaron van den Oord, Yazhe Li, and Oriol Vinyals. Representation learning with contrastive predictive coding. *arXiv preprint arXiv:1807.03748*, 2018.

S.J. Pan and Q. Yang. A Survey on Transfer Learning. *IEEE Transactions on Knowledge and Data Engineering*, 22(10):1345–1359, 2010.

German I Parisi, Jun Tani, Cornelius Weber, and Stefan Wermter. Lifelong learning of spatiotemporal representations with dual-memory recurrent self-organization. *Frontiers in neurorobotics*, 12:78, 2018.

German I. Parisi, Ronald Kemker, Jose L. Part, Christopher Kanan, and Stefan Wermter. Continual lifelong learning with neural networks: A review. *Neural Networks*, 113:54 – 71, 2019. ISSN 0893-6080. doi: https://doi.org/10.1016/j.neunet.2019.01.012.

Cengiz Pehlevan, Alexander Genkin, and Dmitri B Chklovskii. A clustering neural network model of insect olfaction. In *2017 51st Asilomar Conference on Signals, Systems, and Computers*, pp. 593–600. IEEE, 2017.

Dushyant Rao, Francesco Visin, Andrei Rusu, Razvan Pascanu, Yee Whye Teh, and Raia Hadsell. Continual unsupervised representation learning. In *Advances in Neural Information Processing Systems 32*, pp. 7645–7655. Curran Associates, Inc., 2019.

Antti Rasmus, Mathias Berglund, Mikko Honkala, Harri Valpola, and Tapani Raiko. Semi-supervised learning with ladder networks. In C. Cortes, N. D. Lawrence, D. D. Lee, M. Sugiyama, and R. Garnett (eds.), *Advances in Neural Information Processing Systems 28*, pp. 3546–3554. Curran Associates, Inc., 2015.

Sylvestre-Alvise Rebuffi, Alexander Kolesnikov, Georg Sperl, and Christoph H. Lampert. icarl: Incremental classifier and representation learning. In *2017 IEEE Conference on Computer Vision and Pattern Recognition*, CVPR'17, pp. 5533–5542, 2017.

Mengye Ren, Eleni Triantafillou, Sachin Ravi, Jake Snell, Kevin Swersky, Joshua B. Tenenbaum, Hugo Larochelle, and Richard S. Zemel. Meta-learning for semi-supervised few-shot classification. In *Proceedings of 6th International Conference on Learning Representations ICLR*, 2018.

Sebastian Ruder. An overview of multi-task learning in deep neural networks. *arXiv preprint arXiv:1706.05098*, 2017.

Andrei A Rusu, Neil C Rabinowitz, Guillaume Desjardins, Hubert Soyer, James Kirkpatrick, Koray Kavukcuoglu, Razvan Pascanu, and Raia Hadsell. Progressive neural networks. *arXiv preprint arXiv:1606.04671*, 2016.

Hanul Shin, Jung Kwon Lee, Jaehong Kim, and Jiwon Kim. Continual learning with deep generative replay. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett (eds.), *Advances in Neural Information Processing Systems 30*, pp. 2990–2999. Curran Associates, Inc., 2017.

Jake Snell, Kevin Swersky, and Richard Zemel. Prototypical networks for few-shot learning. In *Advances in neural information processing systems*, pp. 4077–4087, 2017.

Kihyuk Sohn, David Berthelot, Chun-Liang Li, Zizhao Zhang, Nicholas Carlini, Ekin D Cubuk, Alex Kurakin, Han Zhang, and Colin Raffel. Fixmatch: Simplifying semi-supervised learning with consistency and confidence. *arXiv preprint arXiv:2001.07685*, 2020.

Jost Tobias Springenberg. Unsupervised and semi-supervised learning with categorical generative adversarial networks. *arXiv preprint arXiv:1511.06390*, 2015.

Antti Tarvainen and Harri Valpola. Mean teachers are better role models: Weight-averaged consistency targets improve semi-supervised deep learning results. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett (eds.), *Advances in Neural Information Processing Systems 30*, pp. 1195–1204. Curran Associates, Inc., 2017.

Michael Tschannen, Olivier Bachem, and Mario Lucic. Recent advances in autoencoder-based representation learning. *arXiv preprint arXiv:1812.05069*, 2018.

Gido M van de Ven and Andreas S Tolias. Three scenarios for continual learning. *arXiv preprint arXiv:1904.07734*, 2019.

Rajasekar Venkatesan and Meng Joo Er. A novel progressive learning technique for multi-class classification. *Neurocomput.*, 207(C):310–321, September 2016. ISSN 0925-2312. doi: 10.1016/j. neucom.2016.05.006.

Junyuan Xie, Ross Girshick, and Ali Farhadi. Unsupervised deep embedding for clustering analysis. In Maria Florina Balcan and Kilian Q. Weinberger (eds.), *Proceedings of The 33rd International Conference on Machine Learning*, volume 48 of *Proceedings of Machine Learning Research*, pp. 478–487, New York, New York, USA, 20–22 Jun 2016. PMLR.

Jianwei Yang, Devi Parikh, and Dhruv Batra. Joint unsupervised learning of deep representations and image clusters. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pp. 5147–5156, 2016.

Suet-Peng Yong, Jeremiah D. Deng, and Martin K. Purvis. Novelty detection in wildlife scenes through semantic context modelling. *Pattern Recogn.*, 45(9):3439–3450, September 2012. ISSN 0031-3203. doi: 10.1016/j.patcog.2012.02.036.

Jaehong Yoon, Eunho Yang, Jeongtae Lee, and Sung Ju Hwang. Lifelong learning with dynamically expandable networks. In *International Conference on Learning Representations*, 2018.

Jason Yosinski, Jeff Clune, Yoshua Bengio, and Hod Lipson. How transferable are features in deep neural networks? In Z. Ghahramani, M. Welling, C. Cortes, N. D. Lawrence, and K. Q. Weinberger (eds.), *Advances in Neural Information Processing Systems 27*, pp. 3320–3328. Curran Associates, Inc., 2014.

Friedemann Zenke, Ben Poole, and Surya Ganguli. Continual learning through synaptic intelligence. In *International Conference on Machine Learning*, 2017.

Chen Zeno, Itay Golan, Elad Hoffer, and Daniel Soudry. Task agnostic continual learning using online variational bayes. *arXiv preprint arXiv:1803.10123*, 2018.

Guanyu Zhou, Kihyuk Sohn, and Honglak Lee. Online incremental feature learning with denoising autoencoders. In Neil D. Lawrence and Mark Girolami (eds.), *Proceedings of the Fifteenth International Conference on Artificial Intelligence and Statistics*, volume 22 of *Proceedings of Machine Learning Research*, pp. 1453–1461, La Palma, Canary Islands, 21–23 Apr 2012. PMLR.