LEARNING A TRANSFERABLE SCHEDULING POLICY FOR VARIOUS VEHICLE ROUTING PROBLEMS BASED ON GRAPH-CENTRIC REPRESENTATION LEARNING

Anonymous authors
Paper under double-blind review

ABSTRACT

Reinforcement learning has been used to learn to solve various routing problems. however, most of the algorithm is restricted to finding an optimal routing strategy for only a single vehicle. In addition, the trained policy under a specific target routing problem is not able to solve different types of routing problems with different objectives and constraints. This paper proposes an reinforcement learning approach to solve the min-max capacitated multi vehicle routing problem (mCVRP), the problem seeks to minimize the total completion time for multiple vehicles whose one-time traveling distance is constrained by their fuel levels to serve the geographically distributed customer nodes. The method represents the relationships among vehicles, customers, and fuel stations using relationship-specific graphs to consider their topological relationships and employ graph neural network (GNN) to extract the graph's embedding to be used to make a routing action. We train the proposed model using the random mCVRP instance with different numbers of vehicles, customers, and refueling stations. We then validate that the trained policy solve not only new mCVRP problems with different complexity (weak transferability but also different routing problems (CVRP, mTSP, TSP) with different objectives and constraints (storing transferability).

1 INTRODUCTION

The Vehicle Routing Problem (VRP), a well-known NP-hard problem, has been enormously studied since it appeared by Dantzig & Ramser (1959). There have been numerous attempts to compute the exact (optimal) or approximate solutions for various types of vehicle routing problems by using mixed integer linear programming (MILP), which uses mostly a branch-and-price algorithm appeared in Desrochers et al. (1992) or a column generation method (Chabrier, 2006), or heuristics ((Cordeau et al., 2002; Clarke & Wright, 1964; Gillett & Miller, 1974; Gendreau et al., 1994)). However, these approaches typically require huge computational time to find the near optimum solution. For more information for VRP, see good survey papers (Cordeau et al., 2002; Toth & Vigo, 2002).

There have been attempts to solve such vehicle routing problems using learning based approaches. These approaches can be categorized into supervised-learning based approaches and reinforcement-learning based approaches (Bengio et al., 2020); supervised learning approaches try to map a target VRP with a solution or try to solve sub-problems appears during optimization procedure, while reinforcement learning (RL) approaches seek to learn to solve routing problems without supervision (i.e, solution) but using only repeated trials and the associated reward signal. Furthermore, the RL approaches can be further categorized into improvement heuristics and construction heuristics (Mazyavkina et al., 2020); improvement heuristics learn to modify the current solution for a better solution, while construction heuristics learn to construct a solution in a sequential decision making framework. The current study focuses on the RL-based construction heuristic for solving various routing problems.

Various RL-based solution construction approaches have been employed to solve the traveling salesman problem (TSP) (Bello et al., 2016; Khalil et al., 2017; Nazari et al., 2018; Kool et al., 2018) or the capacitated vehicle routing problem (CVRP) (Nazari et al., 2018; Kool et al., 2018). (Bello et al., 2016; Nazari et al., 2018; Kool et al., 2018) has used the encoder-decoder structure to

sequentially generate routing schedules, and (Khalil et al., 2017) uses graph based embedding to determine the next assignment action. Although these approaches have shown the potential that the RL based approaches can learn to solve some types of routing problems, these approaches have the major two limitations: (1) only focus on routing a single vehicle over cities for minimizing the total traveling distance (i.e., min-sum problem) and (2) the trained policy for a specific routing problem cannot be used for solving other routing problems with different objective and constraints (they show that trained policy can be used to solve the same type of the routing problems with different problem sizes).

In this study, We proposed the Graph-centric RL-based Transferable Scheduler (GRLTS) for various vehicle routing problems. GRLTS is composed of graph-centric representation learning and RL-based scheduling policy learning. GRLTS is mainly designed to solve min-max capacititated multi vehicle routing problems (mCVRP); the problem seeks to minimize the total completion time for multiple vehicles whose one-time traveling distance is constrained by their fuel levels to serve the geographically distributed customer nodes. The method represents the relationships among vehicles, customers, and fuel stations using relationship-specific graphs to consider their topological relationships and employ graph neural network (GNN) to extract the graph's embedding to be used to make a routing action. To effectively train the policy for minimizing the total completion time while satisfying the fuel constraints, we use the specially designed reward signal in RL framework. The representation learning for graph and the decision making policy are trained in an end-to-end fashion in an MARL framework. In addition, to effectively explore the joint combinatorial action space, we employ curriculum learning while controlling the difficulty (complexity) of a target problem.

The proposed GRLTS resolves the two issues raised in other RL-based routing algorithms:

- GRLTS learns to coordinate multiple vehicles to minimize the total completion time (makespan). It can resolve the first issue of other RL-based routing algorithms and can be used to solve practical routing problems of scheduling multiple vehicles simultaneously. (Kang et al., 2019) also employed the graph based embedding (random graph embedding) to solve identical parallel machine scheduling problem, the problem seeking to minimize the makespan by scheduling multiple machines. However, our approach is more general in that it can consider capacity constraint and more fast and scalable node embedding strategies.
- GRLTS transfers the trained scheduling policy with random mCVRP instances to be used for solving not only new mCVRP problems with different complexity (weak transferability but also different routing problems (CVRP, mTSP, TSP) with different objectives and constraints (storing transferability).

2 FORMULATION

2.1 MIN-MAX SOLUTION FOR MCVRP

We define the set of vehicles $V_V = 1, ..., N_V$, the set of customers $V_C = 1, ..., N_C$, and the set of refueling stations $V_R = 1, ..., N_R$, where N_A, N_C, and N_R are the numbers of vehicles, customers, and refueling stations, respectively. The objective of min-max mCVRP is minimizing the makespan that is the longest distance among all vehicle's traveling distance, i.e., $\min \max_{i \in V_V} L_i$ with L_i being the traveling distance of vehicle i, while each vehicle's one-time traveling distance is constrained by its remaining fuel. The detailed mathematical formulation using mixed integer linear programming (MILP) is provided in Appendix. Figure (1) (left) shows a snapshot of a mCVRP state and Figure (1) (right) represents a feasible solution of the mCVRP.

2.2 DEC-MDP FORMULATION FOR MCVRP

We seek to sequentially construct an optimum solution. Thus, we frame the solution construction procedure as a decentralized Markov decision problem (Dec-MDP) as follows.

2.2.1 STATE

We define the vehicle state $s_t^v, \forall v \in V_V$, the customer state $s_t^c, \forall c \in V_C$, and the refueling station state $s_t^r, \forall r \in V_R$ as follows:

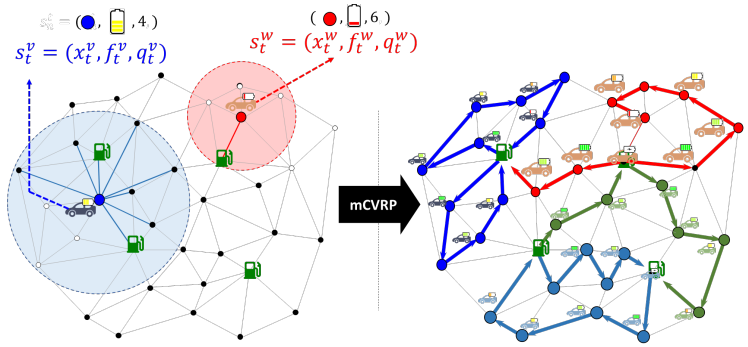

Figure 1: An example of mCVRP. (Left) a snapshot of environment state at time t. The circular range of each vehicle indicates the possible moving range with the current fuel level. (Right) a feasible solution of the environment example.

- State of vehicle v, $s_t^v = (x_t^v, f_t^v, q_t^v)$. x_t^v is the allocated node that vehicle v to visit; f_t^v is the current fuel level; and q_t^v is the number of customers served by the vehicle v so far.
- State of a customer c, $s_t^c = (x^c, \mathbb{v}^c)$. x^c is the location of customer node c (static). Visit indicator $\mathbb{v}^c \in \{0, 1\}$ becomes 1 if the customer c is visited and 0, otherwise.
- State of a refueling station r, $s_t^r = x^r$. x^r is the location of the refueling station r (static).

The global state $\mathbf{s}_t$ then becomes $\mathbf{s}_t = (\{s_t^v\}_{v=1}^{N_v}, \{s_t^c\}_{c=1}^{N_C}, \{s_t^r\}_{r=1}^{N_R})$.

2.2.2 Actions & State transition

Action a_t^v for vehicle v at time t is indicating a node to be visited by vehicle v at time $t+1$, that is, $a_t^v = x_{t+1}^v \in \{V_C \cup V_R\}$. Therefore, the next state of vehicle v becomes $s_{t+1}^v = (x_{t+1}^v, f_{t+1}^v, q_{t+1}^v)$ where f_{t+1}^v and q_{t+1}^v are determined deterministically by an action a_t^v as follows:

- Fuel capacity update: $f_{t+1}^v = \begin{cases} F^v, & \text{if } a_t^v \in V_R \\ f_{tv} - d(x_t^v, a_t^v), & \text{otherwise.} \end{cases}$
- Customer visit number update: $q_{t+1}^v = \begin{cases} q_t^v, & \text{if } a_t^v \in V_R \\ q_t^v + 1, & \text{otherwise.} \end{cases}$

2.2.3 Rewards

The goal of mCVRP is to force all agents to coordinate to finish the distributed tasks quickly while satisfying the fuel constraints. To achieve this global goal in a distributed manner, we use the specially designed independent reward for each agent as:

- **visiting reward**: To encourage vehicles to visit the customer nodes faster, in turn, minimizing makespan, we define customer visit reward $r_{visit}^v = q_t^v$. This reward is provided when an agent visits a customer; the more customer nodes a vehicle agent n visits, the greater reward it can earn.
- **Refueling reward**: To induce a strategic refueling, we introduce refuel reward $r_{refuel}^v = q^v \times \left((F^v - f_t^v)/(F^v - 1)\right)^\alpha$. We define the refuel reward as an opportunity cost. That is, vehicles with sufficient fuel are not necessary to refuel (small reward). In contrast, refueling vehicles with a lack of fuel is worth as much as visiting customers.

2.3 Relationships with other class of VRPs

mCVRP, the target problem of this study, has three key properties: 1) the problem seeks to minimize the total completion time of vehicles by forcing all vehicles to coordinate (in a distributed manner), 2) the problem employs fuel capacity constraints requiring the vehicles to visit the refueling stations strategically, 3) the problem considers multiple refueling depots (revisit allowed).

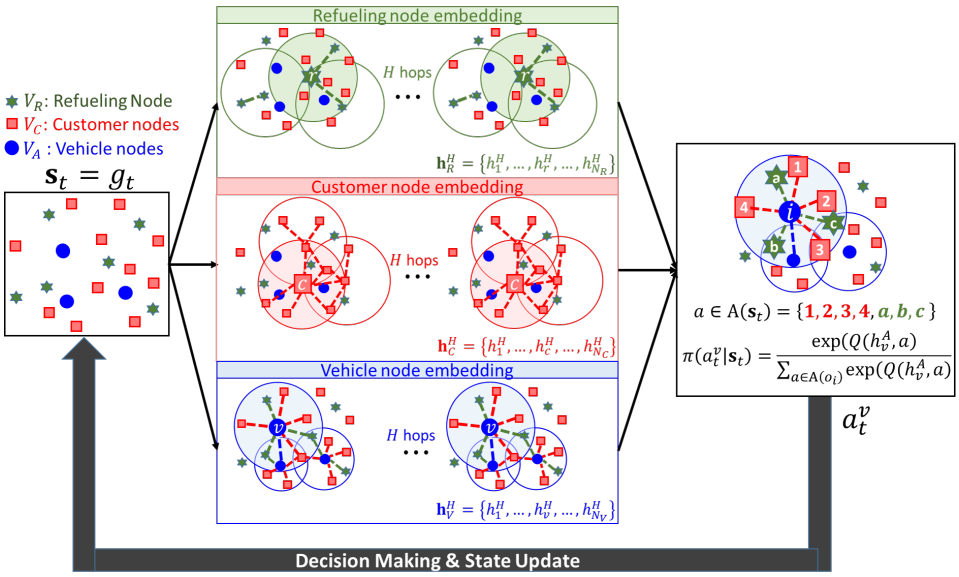

Figure 2: Sequential decision-making framework with trained GRLTS.

If some of these requirements are relaxed, min-max mCVRP can be degenerated into simpler conventional routing problems:

- TSP is the problem where a single vehicle is operated to serve every customer while minimizing the total traveling distance. The agent needs or needs not come back to the depot.
- CVRP (capacity-constrained TSP) is the problem where a single vehicle is operated to serve every customer while minimizing the total traveling distance and satisfying the fuel constraint. The vehicles need to comeback depot to charge.
- mTSP (multi-agent TSP) is the problem where multiple vehicles should serve all the customers as quickly as possible. This problem does not have capacity constraints.
- mCVRP (multi-agent, capacity-constrained TSP) is our target problem having the properties of both mTSP and CVRP. Additionally, we add more than one refueling depot.

The detailed mathematical formulations for these problems are provided in Appendix. We train the policy using random mCVRP instances with varying numbers of agents and customers and employ the trained policy without parameter changes to solve TSP, CVRP and mTSP to test its domain transferability.

3 METHOD

This section explains how the proposed model, given a state (a partial solution), assigns an idle vehicle to one of unvisited customer nodes under the sequential decision-making framework (see Figure 2). The method conducts the three steps: (1) representing an intermediate state of the problem as a graph, (2) extracting from the graph state the node embedding that is relevant to make the next sequential assignment, and (3) making decision about the next assignments.

3.1 STATE REPRESENTATION USING RELATIONSHIP-SPECIFIC MULTIPLE GRAPHS

The proposed model represents the global state $\mathbf{s}_t$ using as a weighted graph $\mathcal{G}_t = \mathcal{G}(V, E, w)$ where $V = \{V_V, V_C, V_R\}$, and E is the set of edges between node $i, j \in V$ and w is weight for edge (i, j) (here, distance d_{ij}). Each node corresponding to vehicle, customer, and refueling station will be initialized with its associated states defined earlier.

Although we can assume that all nodes are connected with each other regardless of types and distance, we restrict the edge connection to its neighboring nodes to reduce the computational cost. Specifically, each type of node can define its connectivity range and connect an edge if any node is located within its range as follows (see Figure 2):

$$e_{vj} = 1 \quad \forall v \in V_v, \quad d(v,j) \leq R_V = f_t^v \tag{1}$$

$$e_{cj} = 1 \quad \forall c \in V_c, \quad d(c,j) \leq R_C \tag{2}$$

$$e_{rj} = 1 \quad \forall r \in V_R, \quad d(r,j) \leq R_R = F^v \tag{3}$$

That is, the vehicle node v connects the edges with nodes that are located within its traveling distance (i.e., the current fuel level f_t^v). Note that the target node j that can be connected to each node can be any types of nodes.

3.2 Node embedding using GNN

The proposed model employs Graph neural network (GNN) (Scarselli et al., 2008) to compute the node embeddings for all nodes. The node embedding procedure starts with constructing the graph $\mathcal{G}_t$ out of the global state $\mathbf{s}_t = (\{s_t^v\}_{v=1}^{N_v}, \{s_t^c\}_{c=1}^{N_C}, \{s_t^r\}_{r=1}^{N_R})$. The method then compute the initial node embeddings h_i and edge embeddings h_{ij} for all the nodes and edges of $\mathcal{G}_t$ by employing encoder network. The sequel will explain how the GNN layer update these node and edge embeddings.

3.2.1 Edge update

The edge feature h_{ij}^τ at τ iteration is updated using the edge updater function ϕ_E as

$$h_{ij}^\tau = \phi_E(h_{ij}^{\tau-1}, h_i^{\tau-1}, h_j^{\tau-1}), \quad \forall i \in V, \forall j \in \mathcal{N}_i \tag{4}$$

where $h_i^{\tau-1}$ and $h_j^{\tau-1}$ are node embedding vectors of node i and node j at $\tau - 1$ iteration.

3.2.2 Edge feature aggregation

The updated edge feature h_{ij}^τ can be thought of as an message sent from node j to node i. Node i aggregates these messages from its neighboring nodes $j \in \{\mathcal{N}_V(i), \mathcal{N}_C(i), \mathcal{N}_R(i)\}$, where $\mathcal{N}_V(i)$ is the neighboring vehicle nodes of node i, as

$$(\bar{h}_{i,V}^\tau, \bar{h}_{i,C}^\tau, \bar{h}_{i,R}^\tau) = \left(\sum_{j \in \mathcal{N}_V(i)} \alpha_{ij} h_{ij}^\tau, \sum_{j \in \mathcal{N}_C(i)} \alpha_{ij} h_{ij}^\tau, \sum_{j \in \mathcal{N}_R(i)} \alpha_{ij} h_{ij}^\tau \right) \tag{5}$$

where the attention weight $\alpha_{ij} = \text{sofmax}_i(e_{ij})$, where $e_{ij} = f_\alpha(s_i, s_j; w_\alpha)$, scores the significance of node j to node i. Note that message aggregation is separately conducted for different types of noes and the aggregated messages per type are concatenated.

3.2.3 Node feature update

The aggregated edge node embeddings $(\bar{h}_{i,V}^\tau, \bar{h}_{i,C}^\tau, \bar{h}_{i,R}^\tau)$ per its neighborhood type are then used to update the node embedding vector h_i^τ using node update function ϕ_V as

$$h_i^\tau = \phi_V \left(h_i^{\tau-1}, (\bar{h}_{i,V}^\tau, \bar{h}_{i,C}^\tau, \bar{h}_{i,R}^\tau) \right) \tag{6}$$

The node embedding procedure is repeated H (hop) times for all nodes, and the final node embeddings $\mathbf{h}^H = \{h_i^H\}_{i=1}^N$ is used to determine the next assignment action of an idle vehicle.

3.3 Decision making using node embedding

When an vehicle node i reaches the assigned customer node, an event occurs and the vehicle node i computes its node embedding h_i^H and selects one of its feasible actions, choosing one of unvisited customer nodes or refueling nodes around vehicle node i. The probability for agent i to choose node j, $a_t^i = j$, is computed by the actor $\pi(a_t^i = j|\mathbf{s}^t; \phi)$ as:

$$\pi(a_t^i = j|\mathbf{s}^t; \phi) = \frac{\exp(Q(h_i^H, h_j^H; \phi)}{\sum_{k \in V_C \cup V_R} \exp(Q(h_i^H, h_k^H; \phi)} \tag{7}$$

where $Q(a_t^i, \mathbf{s}^t) \approx Q(h_i^H, h_j^G; \theta)$ is the critic parameterized with θ.

3.4 TRAINING GRLTS

We train the proposed model using random mCVRP instances with varying numbers of vehicles, customers. We employ the actor-critic algorithm to train the parameters for the GNN, critic, and actor (GRLTS). Furthermore, we use a curriculum learning method to control the difficulty of the target tasks during training to boost the learning. The problem formulation for RL training, network structures with hyperparameters, training data generation, and the pseudo codes are provided in Appendix.

4 EXPERIMENTS

The trained policy is used without changing the parameters to solve various types of vehicle routing problems, mCVRP, mTSP, CVRP, and TSP. For each type except mCVRP, we use two types of data sets: random instances with varying numbers of vehicles and customers whose locations are sampled randomly, and (2) benchmark problems obtained from the library (mTSP library, CVRP library, and TSP library).

To validate the proposed GRLTS, we develop mixed integer linear programming (MILP) formulations and compute the solutions using CPLEX 12.9 (all the optimization formulations are provided in the Appendix). We also use Google OR-Tools(Perron & Furnon) as the representative heuristic solvers. For the problems where other deep RL based approaches tried to solve (TSP and CVRP), we compare the performance of the proposed approach to those of deep RL baselines. All experiments are conducted on Windows 10, Intel(R) Core(TM) i9-9900K CPU 3.6 GHz, and 32GB RAM. GPU acceleration is not used in testing.

4.1 PERFORMANCE COMPARISON OF MCVRPS

Table 1: Performance comparison of mCVRPs on random instances

	$N_c = 25$				$N_c = 50$				$N_c = 100$		
N_v	CPX	ORT	RL	N_v	CPX	ORT	RL	N_v	CPX	ORT	RL
2	20 (∞)	**15** (3.9)	20 (0.96)	2	-	34 (38.3)	**30** (4.8)	5	-	**27** (510.3)	29(11.9)
3	14 (∞)	**10** (36.6)	16 (1.01)	3	-	**22** (2.2)	**22** (4.1)	10	-	-	**20** (12.1)
5	12 (∞)	-	**8** (0.98)	5	-	**14** (1.7)	**14** (3.8)				

Table 2: Scalability test of the trained GRLTS with large-scale mCVRPs

$N_c = 25, N_v = 5$	$N_c = 100, N_v = 10$	$N_c = 400, N_v = 20$	$N_c = 2,500, N_v = 50$
8 (0.98)	20 (12.1)	42 (420.2)	176 (3,861.2)

We apply the already trained model to solve mCVRP with different numbers of vehicles $N_v \in 2, 3, 5$ and the numbers of customers $N_c \in 25, 50, 100$. The number of refueling stations are set to be 4, 5 and 10 in case of $N_c = 25$, $N_c = 50$ and $N_c = 100$. For all cases, we compute the total completion time and the computational time (the number in the parenthesis) required to construct a scheduling. We set the limit of the computational time to be 18,000 (sec) for all cases. The symbol ∞ indicate the case where the computational time is reached and the solution at that moment is used in the table. The blanks with the hyphen (-) indicate the case where the algorithm could not find any feasible solution.

First, CPX (CPLEX) can only solve for mCVRPs with $N_c = 25$; for large scale problems, it cannot compute any feasible solution. ORT (Google OR-tool) produces the best results for the small sized problem with reasonable computational time; however, for the large scale problem it requires extensive amount of time or even fail to compute any feasible solution. Notably, the proposed GRLTS achieves the good performance with the least amount of computational time.

To validate the scalability of the proposed model, we further conduct test experiments with the two cases: $N_c = 400$ with $N_v = 20$ and $N_c = 2,500$. Table 2 shows how the makespan and the

computational time increases with the size of the problem. By comparing the ratio between the number of customers that each vehicle need to serve, (5: 10: 20: 50), and the makespan , (8: 20: 42: 176), we can roughly confirms that the trained model can perform reasonably well even in large sized problem that have never been experience during the training.

4.2 PERFORMANCE COMPARISON ON MTSP

We apply the trained network (without parameter changes) to solve mTSP, the problem that seeks to minimize the total completion time of multiple vehicles (minmax mTSP). This problem is a relaxed version of mCVRP in that it does not require the capacity constraints (we set the fuel level to be constant during execution). We solve the randomly generated mTSP with $N_c \in 49, 100, 400$ and $N_v \in 2, 4, 6, 8$. The trained model outperforms the others in the large size problems (see $N_c = 100$ and $N_c = 400$ case in Table 3) in terms of both makespan and the computational time. Note that CPLEX cannot even find any feasible solutions when $N_c = 400$. We mark an asterisk (*) on the optimal solution by CPLEX.

Table 3: Performance comparison on mTSP for random instances

	$N_c = 49$				$N_c = 100$				$N_c = 400$		
N_v	CPX	ORT	RL	N_v	CPX	ORT	RL	N_v	CPX	ORT	RL
2	**24*** (7.9)	26 (2.1)	32 (3.0)	2	51 (∞)	**57** (32.2)	**57** (15.9)	10	-	63 (1619.8)	**55** (361.2)
4	**13** (∞)	18 (1.1)	23 (3.2)	4	54 (∞)	31 (16.9)	**26** (10.7)	20	-	**41** (782.8)	43 (415.9)
6	13 (∞)	13 (1.1)	**12** (3.3)	6	46 (∞)	25 (15.4)	**24** (10.5)	30	-	44 (1192.1)	**28** (452.3)
8	**7*** (6467.3)	9 (1.6)	8 (3.4)	8	51 (∞)	19 (13.8)	**19** (10.8)	40	-	39 (627.0)	**24** (485.1)

We also employed the trained model to solve mTSP benchmark problems in MTSPLib [1] solving MinMax mTSP. The performance results on 124 instances (four different number of vehicles per 31 different maps) are provided in Table 7 in Appendix.

4.3 PERFORMANCE COMPARISON ON CVRP (PAYLOAD CAPACITY)

Table 4: Performance comparison on CVRP for random instances

Methods	CVRP20			CVRP50			CVRP100		
	Obj.	Gap (%)	Time	Obj.	Gap (%)	Time	Obj.	Gap (%)	Time Time
L2I (Lu et al., 2019)	6.12	-	12m	10.35	-	17m	15.57	-	24m
LKH3 (Helsgaun, 2017)	6.14	0.33	2h	10.38	0.29	7h	15.64	0.45	13h
OR-Tools	6.46	5.56	2m	11.27	8.89	13m	17.12	9.96	46m
AM (Kool et al., 2018) greedy	6.4	4.58	1s	10.61	2.51	3s	16.17	3.85	8s
AM Sampling	6.25	2.12	6m	10.59	2.32	28m	16.12	3.53	2h
Nazari et al. (2018)	6.4	4.58		11.15	7.73		16.96	8.93	
Chen & Tian (2019)	6.16	0.65		10.51	1.55		16.1	3.40	
Random Sweep	7.08	15.69		12.96	25.22		20.33	30.57	
Random CW	6.81	11.27		12.25	18.36		18.96	21.77	
Ours	6.97	13.89	2s	12.91	24.73	5s	19.74	26.78	15s

We employ the trained model to solve randomly generated 100 CVRP instances with $N_c = 20, 50, 100$ with a single capacitated vehicle. All nodes are randomly scattered in the unit square of $[0, 1] \times [0, 1]$. To make the trained policy compatible with the CVRP settings, we fix the number of vehicles to be one and use a single refueling node as if it were the depot in CVRP environment. As a result, the single vehicle needs to revisit the depot due to payload capacity (which is equivalent to fuel

[1] www.infoiasi.ro/ mtsplib

constraint). The payload capacity is set to be 30, 40 and 50 for CVRP20, CVRP50 and CVRP100, respectively. Demand are uniformly sampled from $\{1, \ldots, 9\}$. These test cases have been widely used by the studies seeking to develop RL-based solvers for CVRP problems.

Table 4 summaries results for the random CVRP environment. L2I and LKH3 are well-known best performing heuristic algorithms developed from the OR community, thus can serve as an oracle for comparing the performance. We also consider other RL-based approaches (Kool et al., 2018; Nazari et al., 2018; Chen & Tian, 2019). In general, our model is not outperforming other RL-based approaches and OR-tool. However, all RL-based approaches, except our model, are trained under the same environment of CVRP cases (the training and test instances are sampled from distribution). However, our model is trained with the complete different mCVRP environment and tested with the CVRP environment (strong transferability).

We also employed the trained policy to solve the CVRP benchmark problem instances from CVRPLib (Uchoa et al., 2017). Table 8 in Appendix provides all the performance results compared with other RL-baseline models. The results show that our model has better performance than one of the state-of-art RL-based approach (Kool et al., 2018).

4.4 Performance comparison of TSP

Table 5: Performance comparison on TSP for random instances

Methods	CVRP20			CVRP50			CVRP100		
	Obj.	Gap (%)	Time	Obj.	Gap (%)	Time	Obj.	Gap (%)	Time Time
Concorde	3.84	0.00	1m	5.70	0.00	2m	7.76	0	3m
LKH3	3.84	0.00	18s	5.70	0.00	5m	7.76	0	21m
OR-Tools	3.85	0.26	0s	5.80	1.75	1s	7.99	2.96	3s
AM (greedy)	3.85	0.26	0s	5.80	1.75	2s	8.12	4.64	6s
AM (sampling)	3.84	0.00	5m	5.73	0.53	24m	7.94	2.32	1h
Nearest Insertion	4.33	12.76	1s	6.78	18.95	2s	9.46	21.91	6s
Random Insertion	4.00	4.17	0s	6.13	7.54	1s	8.52	9.79	3s
Farthest Insertion	3.93	2.34	1s	6.01	5.44	2s	8.35	7.60	7s
Bello et al. (2016)	3.89	1.30		5.95	4.39		8.3	6.96	
Khalil et al. (2017)	3.89	1.30		5.99	5.09		8.31	7.09	
Ours	3.92	2.08	1s	6.32	10.88	3s	8.79	13.27	9s

Lastly, we employ the trained model to solve 100 randomly generated TSP with different number of customers $N_c = 20, 50, 100$ and a single vehicle. Table 5 shows summarized the results. Although GRLTS is not outperforming other RL-based scheduling methods, it shows the reasonably performance that is comparable to some of well known heuristic algorithms. Given that the GRLTS is trained with mCVRP environment and have never seen TSP instances, this result can validate that GRLTS can be transferred to TSP as well.

We also employed the trained policy to solve the TSP benchmark problem instances fromTSPLib (Reinhelt, 2014). Table 6 in Appendix provides all the performance results compared with other RL-baseline models. The results show that our model has comparable performance with the-state-of-art RL-based approaches (GPN (Ma et al., 2019) and S2V-DQN (Khalil et al., 2017)) and heuristic algorithms, especially for large-scale TSP problems.

5 Conclusion

We proposed the Graph-centric RL-based Transferable Scheduler for various vehicle routing problems using graph-centric state presentation (GRLTS) that can solve any types of vehicle routing problems such as mCVRP, mTSP, CVRP, and TSP. The transferability is achieved by graph-centric representation learning that can generalize well over various relationships among vehicles, customers, and refuel stations (depot). GRLTR is computationally efficient for solving very large-scale vehicle routing problems with complex constraints, which provides potential that such RL-based scheduler can be used for large-scale realistic applications in logistics, transportation, and manufacturing.

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A APPENDIX

A.1 MILP FORMULATIONS FOR ROUTING PROBLEMS

The MILP formulations for mCVRP, mTSP, and TSP are provided in this section. All problems are formulated to find the optimum routes of vehicles to minimize the makespan. For TSP formulation, the makespan minimization is equal to total distance minimization due to single-vehicle operation. Although mTSP here is same as multi-VRP by its definition, we denote the problem as mTSP because we focus on the makespan minimization problem with multi-vehicle operation.

A.1.1 TSP

A fundamental formulation in the context of routing problems is based on TSP formulation by Miller-Tucker-Zemlin (MTZ) formulation (Miller et al., 1960):

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i \in V} \sum_{j \in V, i \neq j} d_{ij} x_{ij} & & \\
\text{subject to.} \quad & \sum_{i \in V} x_{ij} = 1, & \forall j \in V : i \neq j, & \quad (1) \\
& \sum_{j \in V} x_{ij} = 1, & \forall i \in V : i \neq j, & \quad (2) \\
& u_i - u_j + |V| x_{ij} \leq |V| - 1, & \forall i, j \in V \setminus vStart, & \quad (3) \\
& 0 \leq u_i \leq |V| - 1, & \forall i \in V \setminus vStart, & \quad (4) \\
& x_{ij} \in \{0, 1\}, & \forall i, j \in V, & \quad (5) \\
& u_i \in \mathbb{Z}, & \forall i \in V \setminus vStart. & \quad (6)
\end{aligned}
$$

The routing problem is defined in a graph $G(V, E, w)$ where V, E and w are nodes, edges, and weight (distance d or cost c). x_{ij} is 1 if the edge between node i and node j exists, otherwise, 0. Here, corresponding distance of the edge is d_{ij}. Constraint (3,4) with dummy variable u_i is for subtour elimination. As setting $d_{ij} = 0, \forall j \in vStart$, the problem can be arbitrary end assumption.

A.1.2 MTSP

The goal of mTSP is to find the sub-routes of multiple vehicles to minimize the makespan. Thus, decision variables are expanded to multi-vehicle settings, and the objective function is modified from A.1.1 to consider the makespan minimization setting (MinMax problem).

$$
\begin{aligned}
\text{minimize} \quad & Q & & \\
\text{subject to.} \quad & \sum_{i \in V} \sum_{j \in V} d_{ij} x_{aij} \leq Q, & \forall a \in A : i \neq j, & \quad (1) \\
& \sum_{j \in V} x_{aij} = 1, & \forall a \in A, i \in vStart_a : i \neq j, & \quad (2) \\
& \sum_{a \in A} \sum_{i \in G} \sum_{j \in G} x_{aij} = 1, & \forall j \in vTask : i \neq j, & \quad (3) \\
& \sum_{i \in G} x_{aij} - \sum_{i \in G} x_{aji} = 0, & \forall a \in A, j \in V : i \neq j, & \quad (4) \\
& u_{ai} - u_{aj} + |V| x_{aij} \leq |V| - 1, & \forall a \in A, j \in V \setminus vStart_a : i \neq j, & \quad (5) \\
& 0 \leq u_{ai} \leq |V| - 1, & \forall a \in A, i \in V \setminus vStart_a, & \quad (6) \\
& x_{aij} \in \{0, 1\}, & \forall a \in A, \forall i, j \in V, & \quad (7) \\
& u_{ai} \in \mathbb{Z}, & \forall a \in A, i \in V \setminus vStart_a. & \quad (8)
\end{aligned}
$$

Here, we denoted Q as the makespan, which is the longest traveling distance among multiple vehicles. By minimizing this maximum traveling distance, the above formulation can minimize the makespan. Therefore, the above formulation is to minimize the makespan. We also allow a vehicle to start a tour at any staring node $vStart_a$ (constraint (2)).

A.1.3 MCVRP

We extend the above mTSP formulation in A.1.2 to include the fuel constraint (i.e., allowable traveling distance per sub-tour). In this formulation, vehicles can start their tour at an arbitrary location.

$$
\begin{aligned}
\text{minimize} \quad & Q \\
\text{subject to.} \quad & \sum_{i \in V} \sum_{j \in V} d_{ij} x_{aij} \leq Q, && \forall a \in A : i \neq j, && (1) \\
& \sum_{j \in V} x_{aij} = 1, && \forall a \in A, i \in vStart_a : i \neq j, && (2) \\
& \sum_{a \in A} \sum_{i \in G} \sum_{j \in G} x_{aij} = 1, && \forall j \in vTask : i \neq j, && (3) \\
& \sum_{i \in G} x_{aij} - \sum_{i \in G} x_{aji} = 0, && \forall a \in A, j \in V : i \neq j, && (4) \\
& u_{ai} - u_{aj} + |V| x_{aij} \leq |V| - 1, && \forall a \in A, j \in V \setminus vStart_a : i \neq j, && (5) \\
& 0 \leq u_{ai} \leq |V| - 1, && \forall a \in A, i \in V \setminus vStart_a, && (6) \\
& x_{aij} \in \{0, 1\}, && \forall a \in A, \forall i, j \in V, && (7) \\
& u_{ai} \in \mathbb{Z}, && \forall a \in A, i \in V \setminus vStart_a && (8) \\
& f_{ai} = F_a, && \forall a \in A, i \in vRefuel, && (9) \\
& f_{ai} - d_{ij} x_{aij} \geq 0, && \forall a \in A, \forall i, j \in vRefuel, && (10) \\
& f_{aj} - f_{ai} + d_{ij} x_{aij} \geq F_a (1 - x_{aij}), && \forall a \in A, \forall i, j \in vTask. && (11)
\end{aligned}
$$

Constraint (9) indicates that a vehicle can charge fuel as much as the vehicle's maximum fuel capacity F_a. In addition, constraint (10) requires that a vehicle must have enough remaining fuel to visit a refueling node. Constraint (11) indicates that a vehicle cannot consume (travel) more than its fuel capacity. We allow vehicles to visit refueling nodes as many times as possible by introducing a sufficient number of dummy variables.

A.1.4 CVRP

Similar to the above mCVRP with fuel constraint, we also solve CVRP constrained by payload capacity (not fuel constraint) and with single vehicle ($m = 1$ so that $|A| = 1$). We consider a single depot to start a mission and unload some burdens. Therefore, $vRefuel$ in the MILP of mCVRP becomes same as $vStart_a$. Fuel consumption as much as distance d_{ij} between node i and node j is equivalent to payload capacity consumption as much as demand d_j at node j. As a result, constraint (9-11) of the mCVRP model becomes:

$$
\begin{aligned}
& c_{ai} = C_a, && \forall a \in A, i \in vStart, && (9) \\
& c_{aj} - c_{ai} + d_j x_{aij} \geq C_a (1 - x_{aij}), && \forall a \in A, \forall i, j \in vTask. && (10)
\end{aligned}
$$

The constraint (10) in the mCVRP model is dropped because payload capacity is independent in returning to the starting depot.

A.2 Experiment Results on Benchmark Problems

We apply the trained network to solve benchmark problems for TSP, mTSP and CVRP.

A.2.1 TSP

Table 6 shows results of TSPLib (Reinhelt, 2014) with some RL-based approaches (GPN (Ma et al., 2019) and S2V-DQN (Khalil et al., 2017) and heuristics. The average performance of the proposed model is not outperforming the state of the art RL approaches. However, we observed that the proposed method is scalable in that the performance on the large-scale problem does not degrade compared to other approaches. We compare the performance by problem instance size (eil51 $\sim$ tsp225 Vs. pr226 $\sim$ pch442). Our model showed 1.1% reduction in the optimality gap (15.4% $\rightarrow$

14.3%). However, the other RL-based approaches show much higher increases in the optimality gap (Drori et al. (2020): 3.1% → 10.0%, GPN: 14.1% → 33.1%, Drori et al. (2020): 4.7% → 10.6%).

Table 6: Performance comparison on TSP library

		RL				Approx.		
City	Opt.	Ours	Drori et al. (2020)	GPN	S2V-DQN	Farthest	2-opt	Nearest
eil51	426	471	439	485	439	448	452	514
berlin52	7542	8269	7681	8795	7734	8121	7778	8981
st70	675	770	684	701	685	729	701	806
eil76	538	648	555	591	558	583	597	712
pr76	108159	135145	112699	118032	111141	119649	125276	153462
rat99	1211	1368	1268	1472	1250	1319	1351	1565
kroA100	21282	25282	21452	24806	22335	23374	23306	26856
kroB100	22141	25939	22488	24369	22548	24035	23129	29155
kroC100	20749	23644	21427	24780	21468	21818	22313	26327
kroD100	21294	23397	21555	23494	21886	22361	22754	26950
kroE100	22068	26326	22267	23467	22820	23604	25325	27587
rd100	7910	9237	8243	8844	8305	8652	8832	9941
eil101	629	753	650	704	667	687	694	825
lin105	14379	17173	14571	15795	14895	15196	16184	20363
pr124	59030	66554	59729	67901	61101	61645	61595	69299
bier127	118282	125233	120672	134089	123371	127795	136058	129346
ch130	6110	6987	6208	6457	6361	6655	6667	7575
pr136	96772	111563	98957	110790	100185	104687	103731	120778
pr144	58537	59197	60492	67211	59836	62059	62385	61651
ch150	6528	7264	6729	7074	6913	6866	7439	8195
kroA150	26528	32492	27419	30260	28076	28789	28313	33610
kroB150	26130	32438	27165	29141	26963	28156	28603	32825
u159	42080	51542	43687	52642	45620	46842	42976	53637
rat195	2323	2774	2384	2686	2567	2620	2569	2762
d198	15780	16217	17754	19249	16855	16161	16705	18830
kroA200	29368	34160	30553	34315	30732	31450	32378	35798
kroB200	29437	35848	30381	33854	31910	31656	32853	36982
ts225	126643	141754	130493	147092	140088	140625	143197	152494
tsp225	3916	4536	4091	4988	4219	4233	4046	4746
pr226	80369	83861	86438	85186	82869	84133	85306	94390
gil262	2378	2889	2523	5554	2539	2638	2630	3218
pr264	49135	51836	52838	67588	53790	54954	58115	58634
a280	2579	3059	2742	3019	3007	3011	2775	3311
pr299	48191	59257	53371	68011	55413	52110	52058	61252
lin318	42029	49167	45115	47854	45420	45930	45945	54034
rd400	15281	16631	16730	17564	16850	16864	16685	19168
fl417	11861	12768	13300	14684	12535	12589	12879	15288
pr439	107217	124333	126849	137341	122468	122899	111819	131258
pch442	50778	61757	55750	58352	59241	57149	57684	60242
Gap	0	15.1	4.9	19.0	6.2	8.4	9.1	24.7

A.2.2 MTSP

Table 7 shows results from mTSPLib [2] solving MinMax TSP. Our model shows 3% longer results than ORT. However, computational time is, on average, about 30 seconds faster, which is about 45%. In addition, we calculate difference of the optimality gap by $N_v = 2, 3, 5, 7$. Our model shows better performance as N_v increases ($N_v = 2$ case: 9.8%, $N_v = 3$ case: 8.9%, $N_v = 5$ case: 2.8%, $N_v = 7$ case: -7.5%)

[2] www.infoiasi.ro/ mtsplib

Table 7: Performance comparison on mTSP library

		ORT		Ours					ORT		Ours		
Maps	N_v	MS	Time	MS	Time	gap	Maps	N_v	MS	Time	MS	Time	gap
eil51	2	223	1.14	281	2.95	26.2	kroB100	2	12214	15.06	13721	9.41	12.3
	3	159	0.67	244	2.92	53.7		3	8957	14.81	10197	9.52	13.8
	5	120	1.48	165	2.93	37.5		5	7108	10.31	6271	9.19	-13.3
	7	109	1.03	86	2.67	-26.5		7	7108	10.24	6119	9.49	-16.2
berlin52	2	4634	1.27	4247	3.01	-9.1	kroC100	2	11440	8.90	12453	9.39	8.9
	3	3195	1.35	3452	3.03	8.0		3	8725	12.58	7985	9.48	-9.3
	5	2606	1.18	2179	3.03	-19.6		5	6616	15.66	6291	9.50	-5.2
	7	2440	1.42	2182	2.87	-11.8		7	6154	12.83	5740	9.30	-7.2
eil76	2	288	4.46	361	5.81	25.4	kroD100	2	13130	11.27	12815	9.32	-2.5
	3	212	3.26	227	5.75	7.2		3	8889	9.36	9359	9.40	5.3
	5	179	4.79	170	5.61	-5.1		5	6976	9.67	6660	9.34	-4.7
	7	179	4.78	121	5.41	-47.4		7	6485	12.00	7246	9.25	11.7
st70	2	388	3.77	451	4.87	16.2	kroE100	2	13424	9.02	13611	9.53	1.4
	3	285	2.04	282	4.98	-1.2		3	9334	14.61	8929	10.76	-4.5
	5	251	2.26	249	4.69	0.8		5	7599	20.27	6992	10.05	-8.6
	7	251	2.22	253	4.78	0.9		7	8727	17.14	5868	9.42	-18.7
pr76	2	60679	4.99	80789	5.69	33.1	eil101	2	345	11.23	386	9.70	11.8
	3	51074	6.25	65567	5.67	28.4		3	237	10.32	264	9.64	11.3
	5	39167	3.55	48143	5.48	22.9		5	163	11.15	167	9.67	2.3
	7	38116	5.85	33082	5.50	-15.2		7	123	15.29	141	9.63	14.6
rat99	2	709	16.47	756	9.11	6.6	lin105	2	8846	18.50	10220	10.20	15.5
	3	549	13.86	565	9.28	2.9		3	7112	16.20	8168	10.30	14.8
	5	463	11.12	373	9.14	-24.0		5	7060	18.79	5487	10.34	-28.7
	7	440	12.01	367	9.24	-19.8		7	6444	18.84	4802	10.35	-34.2
rd100	2	4406	17.94	4520	9.54	2.6	pr107	2	31692	14.29	26369	10.96	20.2
	3	3224	20.31	3136	9.61	-2.8		3	24904	12.82	23283	10.90	-7.0
	5	2435	20.78	2629	9.32	8.0		5	21455	16.70	20985	10.80	-2.2
	7	2803	17.38	2375	9.38	-18.0		7	21365	17.45	20620	10.81	-3.6
kroA100	2	12647	12.19	13560	9.49	7.2	pr124	2	35094	18.82	36472	15.02	3.9
	3	9116	14.37	9635	9.49	5.7		3	27257	16.70	27598	14.75	1.3
	5	8229	13.11	7462	9.42	-10.0		5	23231	21.44	23651	14.75	1.8
	7	8229	13.31	7147	9.24	-15.1		7	22725	25.29	21526	14.75	-5.6
bier127	2	62458	22.72	67706	15.83	8.4	u159	2	26171	47.80	27864	26.35	6.5
	3	47347	22.61	52749	15.68	11.4		3	19014	56.26	19519	25.87	2.7
	5	29560	38.21	30946	15.76	4.7		5	14308	72.44	16719	26.34	16.9
	7	24891	47.49	25074	15.48	0.7		7	14776	67.59	16077	26.03	8.8
ch130	2	3393	31.44	3755	16.85	10.7	rat195	2	1285	205.86	1524	43.66	18.6
	3	2550	23.90	3425	16.65	34.3		3	975	101.40	937	43.82	-4.0
	5	1575	39.03	2307	16.69	46.5		5	708	152.43	638	43.57	-10.9
	7	1338	32.88	2077	16.30	55.2		7	631	130.31	495	43.85	-27.5
pr136	2	59999	34.54	59113	18.18	-1.5	d198	2	10416	170.17	9226	45.20	-12.9
	3	40643	14.45	45017	18.25	10.8		3	9663	71.48	7813	45.36	-23.7
	5	29958	29.59	32858	18.14	9.7		5	8577	113.45	6086	45.21	-40.9
	7	30138	20.66	30222	17.98	0.2		7	8577	115.69	5892	45.64	-45.6
pr144	2	38104	23.11	52344	20.72	37.4	kroA200	2	17148	154.68	20696	46.46	20.7
	3	31305	36.20	42179	20.72	34.7		3	12253	161.74	13778	46.44	12.4
	5	25262	46.69	28153	20.55	1.4		5	8185	190.01	10770	46.33	31.6
	7	24361	42.25	27168	20.52	11.5		7	6675	186.54	7142	46.36	7.0
ch150	2	3914	43.40	3637	22.75	-7.6	kroB200	2	16127	92.48	18758	47.00	16.3
	3	2663	42.67	2870	22.78	7.7		3	11814	103.17	13199	46.13	11.7
	5	1915	75.00	1994	22.66	4.1		5	8525	126.04	10618	46.83	24.6
	7	1661	66.54	1603	23.10	-3.6		7	7720	120.19	8678	46.41	12.4
kroA150	2	15326	40.34	16578	22.77	8.2	ts225	2	70660	230.60	71594	64.01	1.3
	3	10810	47.01	12600	22.83	16.6		3	55237	245.30	52330	63.12	-5.6
	5	7323	54.11	8567	23.05	17.0		5	38748	189.31	31318	64.13	-23.7
	7	6290	42.09	7901	22.66	25.6		7	35460	213.09	27921	63.53	-27.0
kroB150	2	14633	54.68	18342	22.84	25.3	tsp225	2	2145	277.23	2481	62.39	15.7
	3	10565	36.80	11512	22.91	8.9		3	1612	145.43	1743	63.56	8.1
	5	7257	35.87	7539	22.82	3.9		5	1183	184.62	1308	63.83	10.6
	7	8221	31.13	6057	22.92	-35.7		7	1044	209.54	1016	63.89	-2.7
pr152	2	43922	52.08	51982	23.71	18.3							
	3	38752	50.79	48238	23.35	24.5							
	5	33006	91.95	43940	23.27	33.1							
	7	32245	102.58	41624	23.57	29.1							

A.2.3 CVRP

Table 8 shows results from CVRPLib (Uchoa et al., 2017). Those values are the total traveling distance. Although our model is trained to minimize makespan (MinMax), our model shows better performance than AM in the total distance (MinSum).

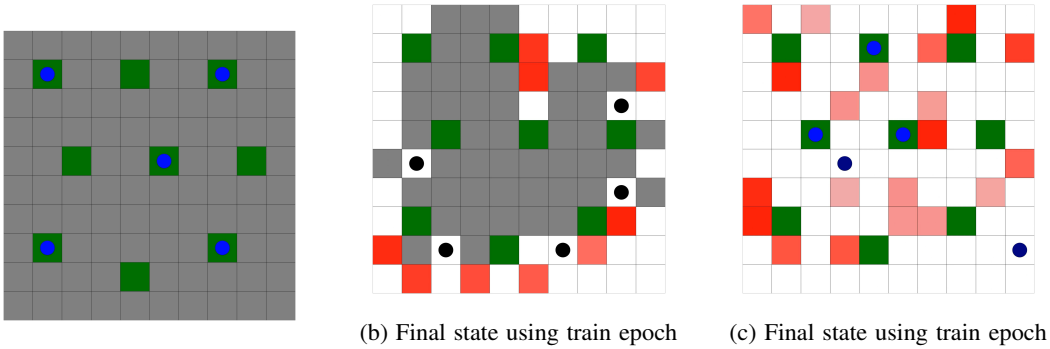
Table 8: Performance comparison on CVRP library

Instance	Opt	OR-Tools	AM (N=1280)	AM (N=5000)	Ours
X-n101-k25	27591	29405	39437	37702	39133
X-n106-k14	26362	27343	28320	28473	27940
X-n110-k13	14971	16149	15627	15443	17388
X-n115-k10	12747	13320	13917	13745	19292
X-n120-k6	13332	14242	14056	13937	16589
X-n125-k30	55539	58665	75681	75067	69919
X-n129-k18	28940	31361	30399	30176	36649
X-n134-k13	10916	13275	13795	13619	14800
X-n139-k10	13590	15223	14293	14215	16368
X-n143-k7	15700	17470	17414	17397	23548
X-n148-k46	43448	46836	79611	79514	62240
X-n153-k22	21220	22919	38423	37938	33079
X-n157-k13	16876	17309	21702	21330	19702
X-n162-k11	14138	15030	15108	15085	19491
X-n167-k10	20557	22477	22365	22285	25676
X-n172-k51	45607	50505	86186	87809	63191
X-n176-k26	47812	52111	58107	58178	66997
X-n181-k23	25569	26321	27828	27520	27220
X-n186-k15	24145	26017	25917	25757	30086
X-n190-k8	16980	18088	37820	36383	18651
X-n195-k51	44225	50311	79594	79276	66957
X-n200-k36	58578	61009	78679	76477	68502
Gap	-	8.06	32.97	31.62	30.22

A.3 GRAPHICAL SOLUTIONS EXAMPLE

This section provides the visual results for several vehicle routing problems solved by the trained policy.

A.3.1 TRAINING RESULTS ON MCVRP



(a) Initial state

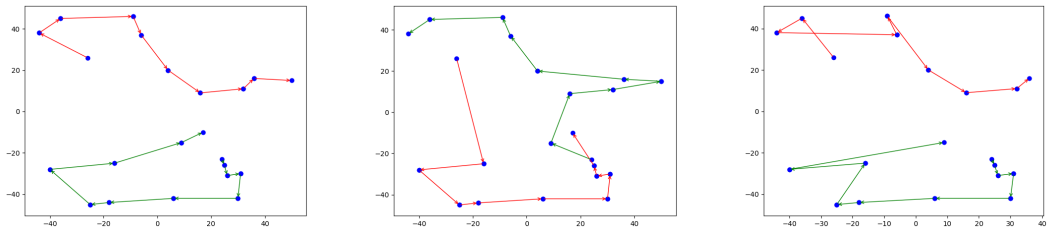
(b) Final state using train epoch #2 model

(c) Final state using train epoch #420 model

Figure 3: $N_c = 100, N_v = 5$ case of mCVRP

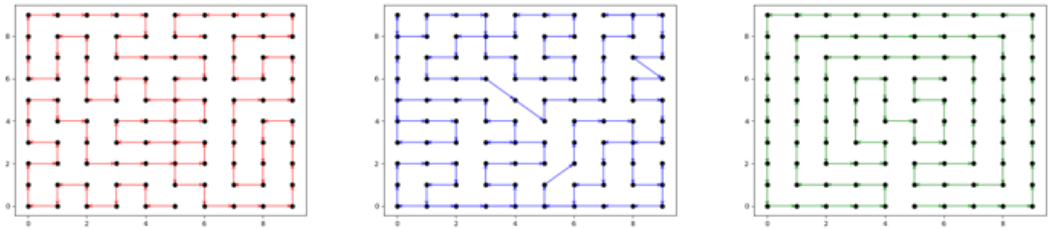
Figure 3 depicts the evaluation case ($N_c = 100, N_v = 5$ case of mCVRP) during training process. This test environment is designed to mimic the search and rescue problems, operating multiple drones to search victims distributed over a map.

Green grids are for refueling nodes, blue circles are for vehicles (color becomes black as fuel capacity diminishes), and grey grids are for unvisited customers. If a customer is visited by a vehicle, the color becomes red or white; the red girds indicate the victims, while the white indicates no victim in that cell. These red grids do not have any effect on performance measure, because visiting all customers (grey grids) is top-priority.



(a) CPLEX (b) OR-Tools (c) Ours (trained on mCVRP)

Figure 4: $N_c = 20, N_v = 2$ case of mTSP



(a) CPLEX (b) OR-Tools (c) Ours (trained on mCVRP)

Figure 5: TSP with a single vehicle

At the very beginning of the training process (figure 3 (b)), the trained model cannot have vehicles search over the cells and make them die due to lack of fuel. At the end of the training process (figure 3 (c)), however, all customers are visited by the vehicles.

A.3.2 MVRP (MTSP)

Figure 4 depicts an example solutions of mTSP using CPLEX, OR-Tools, and RL. The RL-based solver is trained under mCVRP environment but we solve mTSP with the trained model.

A.3.3 TSP

Figure 5 depicts an example solutions of TSP using CPLEX, OR-Tools, and RL. The RL-based solver is trained under mCVRP environment but we solve TSP with the trained model.

A.4 PARAMETER UPDATE

The proposed framework is trained following algorithm (1). We use the decentralized actors so that PPO method is also paralleled. Note that the batch size T_i depends on the length of current episode and the number of decision making epoch for each agent because we assume Monte-Carlo simulation. Algorithm (1) focus on sequence of the parameter optimization problem. Detailed algorithm for interaction between simulation, GNN, and multi-agent will be provided in the later section.

Algorithm 1 Parameter update in decentralized actors with PPO and Monte-Carlo simulation

for Agent $i = 1, 2, \ldots, N$ **do**
  for $Update\ iteration = 1, 2, \ldots, K$ **do**
    Evaluate policy $\pi_{\theta_{i,old}}$ in environment for an experienced episode
    Compute $r_t(\theta_i)$ in Equation (10)
    Compute advantage estimates $\hat{A}_1, \ldots, \hat{A}_T$ in Equation (12)
    Optimize surrogate objective in Equation (11) with batch size T_i
    $\theta_{i,old} \leftarrow \theta_i$, $\phi_{i,old} \leftarrow \phi_i$
  end for
end for

A.5 NETWORK ARCHITECTURE AND HYPERPARAMETERS

Table 9 is summary of hyperparameters used in training process.

Table 9: Hyperparameters

Parameters (for PPO update)	Value	Parameters (for GRLTS)	Value
Optimizer	Adam	MLP units	$(32, 32)$
betas (β_1, β_2)	$(0.9, 0.999)$	Neuron initialization	Kaiming normal
Learning rate (η_{policy})	2×10^{-4}	Activation fn.	ReLU
Learning rate $(\eta_{Q-network})$	2×10^{-4}	Node umbedding epoch (N^{hop})	5
gamma (γ)	0.99	Node feature dim.	5
clip ratio (ϵ)	0.2	Edge feature dim.	10
value function coeffi.	0.5	Max. decision epoch	$3 \times N_c$
entropy coeffi.	0.01		
PPO update epoch (K)	10		
Training epoch	1		

A.6 FORMULATION OF LEARNING PROBLEM

As a policy optimization method, we implement a PPO (Schulman et al., 2017) in decentralized actors to update the policy parameter $\boldsymbol{\theta}$. PPO is similar to the true region policy optimization (TRPO) (Schulman et al., 2015) in that both use the trust region method, but PPO is a computational-friendly form of TPRO.

Trust region methods commonly try to solve an surrogate objective function maximization problem. However, there is a penalty term to avoid the destructively large policy update. Therefore, the optimization problem is

$$\underset{\theta}{\text{maximize}}\ \hat{\mathbb{E}}_t \left[\frac{\pi_\theta\left(a_t|s_t\right)}{\pi_{\theta_{\text{old}}}\left(a_t|s_t\right)} \hat{A}_t - \beta\, \mathrm{KL}\left[\pi_{\theta_{\text{old}}}\left(\cdot|s_t\right), \pi_\theta\left(\cdot|s_t\right)\right] \right] \tag{8}$$

where π_{old} is the policy paramter before an update and KL indicates the Kullback-Leibler divergence. Trust region methods penalize any urgent updates, nominated as the destructively large policy updates, with the penalty coefficient β and the KL divergence of both policies while maximizing the surrogate objective function. However, it is hard to choose a value of the penalty coefficient β. Therefore, the penalty term becomes a contraint for hard bound without the coefficent β in TRPO.

To approximate the action-value function Q_i^π as a function of crtic paramter ϕ_i, the critic is trained to minimize the loss $\mathcal{L}$:

$$\begin{aligned} \mathcal{L}(\phi_i) &= \mathbb{E}_{\mathbf{o},\mathbf{a},r_i,\mathbf{o}'\sim\mathcal{D}}[(Q^{\pi_i}(\mathbf{o},\mathbf{a};\phi_i) - y_i)^2] \\ &= \mathbb{E}_{\mathbf{o},\mathbf{a},r_i,\mathbf{o}'\sim\mathcal{D}}[(Q^{\pi_i}(\mathbf{h}^{N_{hop}};\phi_i) - y_i)^2] \end{aligned} \tag{9}$$

where $y_i = r_i + \gamma Q^{\pi_i'}(\mathbf{h'}^{N_{hop}};\phi_i')$, $(\cdot)'$ refers target networks of critic and actor and $\mathcal{D}$ is a state transition memory.

To train the actor network, we use PPO method to maximize $J(\boldsymbol{\theta})$. PPO, as one of the trust region methods, aims to solve similar problem in equation (8) with clipped surrogate objective function for agent i as follow:

$$J^{\text{CLIP}}(\theta_i) = \mathbb{E}_t[\min(r_t(\theta_i)\hat{A}_i^t, \text{clip}(r_t(\theta_i), 1-\epsilon, 1+\epsilon)\hat{A}_i^t)] \tag{10}$$

where $r_t(\theta_i) = \frac{\pi_{\theta_i}(a_i^t|o_i^t)}{\pi_{\theta_{i,\text{old}}}(a_i^t|o_i^t)}$. In addition, equation (10) is added by a value function error and an entropy bonus for sufficient exploration because critic and actor share parameters as follow:

$$J_t^{\text{CLIP}'}(\theta_i) = \mathbb{E}_t[J_t^{\text{CLIP}}(\theta_i) - c_1(V_{\theta_i}(o_i^t) - V_{\text{target}})^2 + c_2 H(o_i^t, \pi_{\theta_i}(\cdot))] \tag{11}$$

where $V_{\theta_i}(o_i^t) \approx V(h_i^{N_{hop},t}; \phi_i)$ from critic, H denotes an entropy bonus and c_1, c_2 are hyperparameters. We compute the advantage estimator $\hat{A}_i^t$ by running the policy for T timesteps. The advantage estimator for agent i, $\hat{A}_i^t$, becomes

$$\begin{aligned} \hat{A}_i^t &= \delta_i^t + \gamma\delta_i^{t+1} + \cdots + \cdots + \gamma^{T-t+1}\delta_i^{T-1} \\ \text{where } \delta_i^t &= r_i^t + \gamma V(h_i^{N_{hop},t+1}; \phi_i) - V(h_i^{N_{hop},t}; \phi_i) \end{aligned} \tag{12}$$

A.7 Training performance

Figure (6) shows how the trained policy perform on the thee set of validation problems while the policy is being trained by the random mCVRP inc stances. The three validation problems are 1) 100 customers covered by 5 vehicles, 2) 100 customers by 10 vehicles and 3) 400 customers covered by 20 vehicles. The first row show the performance on the random training instances while the second, third, and fourth row show the performances on the three validation problems.

As training processes progress, the trained model gradually becomes more efficient, that is, a fleet of vehicles visits more customers faster. After about 200 training epochs, the trained model converges, but there are some randomness due to the nature of policy gradient structure. Although the makespan curves in the second column of training case seems relatively constant (the first row row), the level of difficulty is increasing as curriculum becomes harder (The number of refueling is also increasing as curriculum becomes harder).

Comparison of case #1 and case #2 give us two interpretations: 1) case #1 shows the model is trained to visit all customers and 2) in case #2, the model achieves consistent performance as training iterations progress in spite of same visit ratio as 1 around 100 epoch and 350 epoch. Besides, the makespans of both training epochs are very close. Case #2 is relatively easier environment than case #1 in that more vehicles are deployed to serve same number of customers.

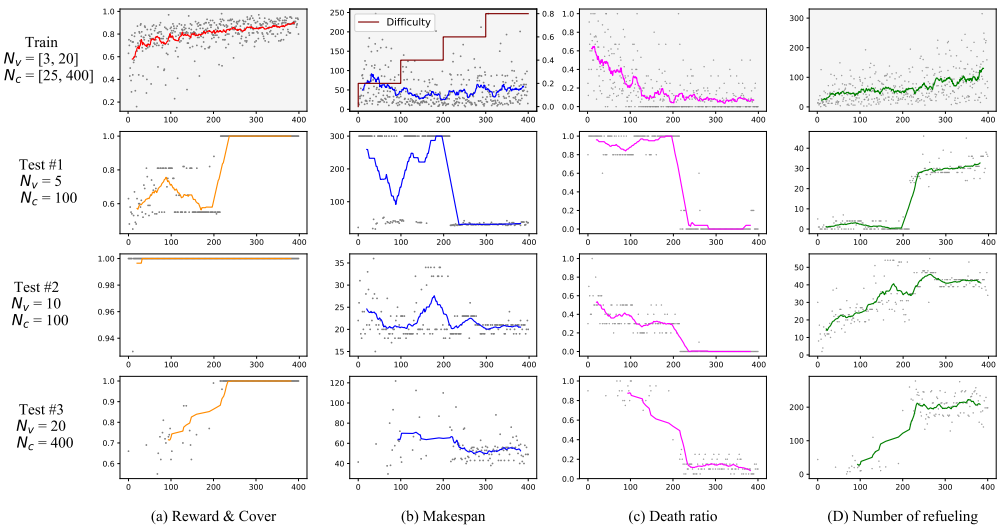


Figure 6: Performance curves of training (1st row) and testing (2nd, 3rd and 4th rows). Note that we plot a reward curve and cover ratio curves in 1st column for training and tesing, respectively.

A.8 Pseudo Code

Simulation: Problem Instance generation We generate random mCVRP instances and use these training instances to train the proposed model. For training, we randomly select the number of

vehicles, customers, and refueling stations. We randomly assign these entities over the grid world with the distance between each grid cell being 1. When transforming the snapshot of the mCVRP to a graph, we use the Manhattan distance between the two cells as the edge weight. In addition, the speed of each vehicle is set to be 1; thus, each vehicle moves over one cell during a single time-tic. Although the world is represented as discrete gird, the mCVRP problem's state transition is event-based. Whenever the agent reaches the assigned customer, the event occurs, and all the edge distance is updated.

Initially, this grid world is developed to use the trained policy for search and rescue problems, seeking distributed victims over the grid-world. Because each vehicle (drone) can search a specific zone within a certain amount of time, using a cell-based grid world is a reasonable choice.

Once the policy is trained, it can be used for both a discrete world or a continuous world because these environments can be transformed into a graph without any difference. We validated the proposed method using CVRP and TSP instances defined over the continuous state to compare the performance of the proposed model to other deep baseline line models. Other deep baseline line models all use the continuous state.

To boost training, we vary the difficulty level of random mCVRP instances during training process (see Algorithm (2)). For the curriculum instance generation in line 9, we compute N_{curr}, the number of customers which are assumed to be *visited* already, as $N_{curr} = N_c \times (1 - 2 \times currLevel/10)$. Then, we choose N_{curr} tasks randomly and mark as $visited$ and distribute N_{curr} for all agents' visit number q^t.

Algorithm 2 Training instance generation

1: Generate random problem instances N_v, N_c, N_r, x_v^0, x_r^0,
2: $currLevel \leftarrow 0$
3: **for** $i_{training} = 1, \ldots, N_{training}$ **do**
4: **for** each 100 training iterations **do**
5: $currLevel \leftarrow currLevel + 1$
6: **end for**
7: Generate a random number $p_{curr} \sim \mathrm{U}[0, 1]$
8: **if** $p_{curr} \geq 0.5$ **then**
9: Curriculum instance generation
10: **else**
11: Non-curriculum instance generation (followed by line 1)
12: **end if**
13: **end for**

Simulation: Episode generation As a medium for the interaction of the two components such as environment and the GRLTS, simulation conducts the agents' actions and stores the state transitions which are used to update the GRLTS. Starting from the problem instance generation (Algorithm (2)), simulation assigns the agents' assignment as a_{agt}^t using the action probability $\pi(\cdot|o_{agt}^t)$ computed from the proposed GRLTS. Simulation computes the agent agt's transition time Δ_{agt} as follow:

$$\Delta_{agt} = {}^{Dist(x_{agt}^t, a_{agt}^t)}/_{v_{agt}} = Dist(x_{agt}^t, a_{agt}^t) \tag{13}$$

where velocity of agents are constant as 1 and distance between i and j $Dist(i, j)$ can be the Manhattan distance or Euclidean distance.

Algorithm 3 Episode generation

1: Initialize problem instances (based on Algorithm 2)
2: $termination, done \leftarrow False, t \leftarrow 0, memory \leftarrow \emptyset$
3: $idleAgts \leftarrow \{1, 2, \ldots, N_v\}, actingAgts \leftarrow \emptyset$
4: **while** $termination$ **do**
5: **for** $agt \in idleAgts$ **do**
6: Send an obervation $\Pi_{agt} o^t_{agt}$ to a GRLTS
7: Choose an action $a^t_{agt} \sim \pi(\cdot|o^t_{agt})$ where an action probability is from GRLTS
8: Assign the action a^t_{agt} to the agent
9: **end for**
10: Compute and update transition time Δ_{agt} for $agt \in idleAgts$ (Equation (13))
11: $actingAgts \leftarrow \{agt|\Delta_{agt} = \min_{agt} \Delta_{agt}\}$ for all agents
12: $t \leftarrow t + \min_{agt} \Delta_{agt}$
13: **for** $agt \in actingAgts$ **do**
14: Perform the recently assigned action $a^{t'}_{agt}$ assigned by $\pi(\cdot|o^{t'}_{agt})$
15: Observe the observation o^t_{agt}, reward r^t_{agt} and $done$
16: Append a state transition tuple $< t', o^{t'}_{agt}, a^{t'}_{agt}, r^t_{agt}, o^t_{agt} >$ to $memory$
17: **end for**
18: **if** $done$ **then**
19: $termination \leftarrow True$
20: **else**
21: $idleAgts \leftarrow actingAgts$
22: **end if**
23: **end while**

RL networks: GRLTS Most of algorithms in the RL model is explained the paper so that we focus on the interaction between simulation and GRLTS networks and internal message exchanges.

Algorithm 4 GRLTS

1: Receive an observation $\Pi_{agt} o^t_{agt}$ at time t from simulation (line 6 of Algorithm 3)
2: Generate $\mathcal{G}^{t,(0)}$ with an input of observations $\Pi_i o^t_i$
3: **for** node embedding iteration $\tau = 1, \ldots, N_{hop}$ **do**
4: Compute connectivity C_{ij} for all nodes i, j:

For agent nodes $n, C_{nj} = \begin{cases} f^n, & \forall j \in V_T \cup V_R \\ \infty, & \forall j \in V_A. \end{cases}$

For refueling nodes r, $C_{rj} = F, \forall j \in V$
For task nodes t, $C_{tj} = 5, \forall j \in V$

5: Store the node embedding vector h^A_i at $memeory_i$
6: Update the connected edges following Equation (3)
7: Aggregate the incoming edge featured at each nodes following Equation (4)
8: Update the node feature with the node updator function ϕ_v
9: **end for**
10: Compute $Q(h^A_i, a)$ for agent i where $a \in A(o_i) = \{v \in \{V_R \cup V_T\}|C_{iv} = 1\}$
11: Compute $\pi(a|o_i) = \frac{\exp\left(Q\left(h^A_i, a\right)\right)}{\sum_{a \in \mathrm{A}(o_i)} \exp\left(Q\left(h^A_i, a\right)\right)}$
12: Send the action probability $\pi(a|o_i)$ to simulation

RL networks: PPO-based update in GRLTS We provide an extended pseudo-code in Algorithm (1) with interaction between the simulation and the GRLTS.

Algorithm 5 Parameter update in decentralized actors with PPO and Monte-Carlo simulation

(Input) $memory$ from GRLTS in Algorithm (4), old policy parameters $\boldsymbol{\theta}_0 = \{\theta_0^1, \ldots, \theta_0^{N_v}\}$, old GNN parameters $\boldsymbol{\phi_0}$
for $i \in Agents$ **do**
    Compute discounted reward r_i^t for $t \in memory_i$
    for Update iteration $k = 1, 2, \ldots, K$ **do**
        Compute the entropy term $H(\theta_k^i)$ with $memory_i$ in Equation (11)
        Compute the advantage estimates $\hat{A}_t$ in Equation (12) with discounted reward r_i^t
        Update the policy parameter $\boldsymbol{\theta}$ and the GNN parameter $\boldsymbol{\phi}$:
$$\theta_{k+1}^i, \phi_{k+1} \leftarrow \arg\max_{\theta^i, \phi_k o^i, a^i \sim \pi_{\theta_k^i}} \mathbb{E} \left[J_t^{\text{CLIP}}(\theta_i) - c_1 (V_{\phi_k}(o_i^t) - V_{\text{target}})^2 + c_2 H(o_i^t, \pi_{\theta_i}(\cdot)) \right]$$
    end for
end for