Differentially Private CutMix for Split Learning with Vision Transformer

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Abstract

Recently, vision transformer (ViT) has started to outpace the conventional CNN in computer vision tasks. Considering privacy-preserving distributed learning with ViT, federated learning (FL) communicates models, which becomes ill-suited due to ViT's large model size and computing costs. Split learning (SL) detours this by communicating smashed data at a cut-layer, yet suffers from data privacy leakage and large communication costs caused by high similarity between ViT's smashed data and input data. Motivated by this problem, we propose *DP-CutMixSL*, a differentially private (DP) SL framework by developing *DP patch-level randomized CutMix (DP-CutMix)*, a novel privacy-preserving inter-client interpolation scheme that removes randomly selected patches in smashed data. By experiment, we show that DP-CutMixSL not only boosts privacy guarantees and communication efficiency, but also achieves higher accuracy than its Vanilla SL counterpart. Theoretically, we analyze that DP-CutMix amplifies Rényi DP (RDP), which is upper-bounded by its Vanilla Mixup counterpart.

1 Introduction

Motivation: Privacy-Preserving Distributed ML for ViT Edge devices such as phones, cameras, and e-health wearables generate the sheer amount of fresh data [1]. To exploit these user data for machine learning (ML) without violating data privacy, federated learning (FL) is gaining increasing attention, which keeps raw data locally stored while only exchanging and averaging model parameters across devices [2, 3]. In particular, FL has been notably successful in computer vision tasks with the *de facto* standard convolutional neural network (CNN) architectures [4, 5]. However, recently vision transformer (ViT) has been aggressively taking over the throne of CNN [6], questioning the effectiveness of FL. In fact, ViT is often larger than CNN, and this bodes ill for FL by imposing excessive energy and communication burdens on devices [7, 8]. Alternatively, split learning (SL) can cope with large models via model partitioning [9, 10]. In SL, each device locally stores only a tiny fraction of the entire model, and offloads the rest to a parameter server, between which devices exchange their cut-layer forward activations with the server, referred to as *smashed data*. Notwithstanding, ViT commonly lacks pooling and convolutional layers [6, 11, 12], making smashed data similar to their raw data as visualized in Fig. 4 of Appendix A. This may entail huge costs and privacy leakage as opposed to its counterpart SL with CNN [13, 14, 15].

Contributions: DP-CutMixSL To address the aforementioned issues, inspired from the patchfied smashed data in ViT [6] and the CutMix technique [16], we propose *DP-CutMixSL* [1], a differentially private (DP) SL framework with ViT via patch-level randomized CutMix. As Fig. 1 demonstrates, following the Gaussian DP mechanism [17, 18, 19], each device in DP-CutMixSL first injects random Gaussian noise into smashed data, followed by punching randomly selected patches, yielding *Cutout*

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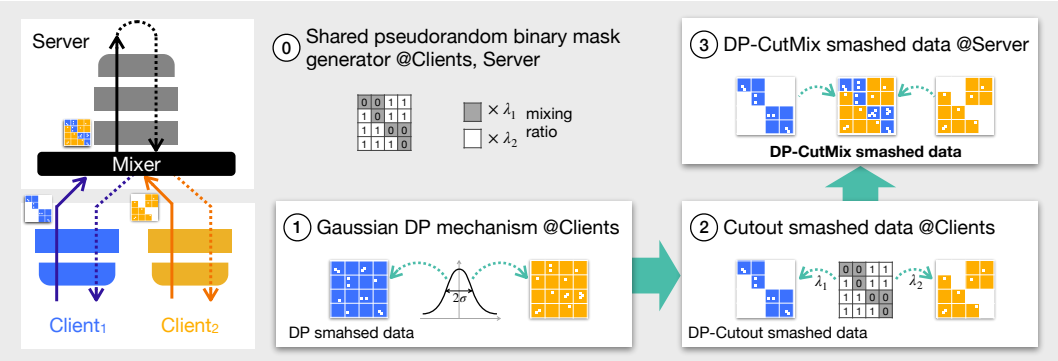


Figure 1: A schematic illustration of DP-CutMixSL operations.

smashed data as analogous to those of Cutout [20]. These Cutout smashed data are uploaded to and put together by the server, resulting in *DP-CutMix smashed data* that continue feed-forward propagation. Compared to SL with the Gaussian DP mechanism (*DP-SL*), we theoretically prove that the proposed randomized CutMix operation in DP-CutMixSL amplifies the DP guarantee of smashed data, by up to its upper-bound baseline *DP-MixSL* obtained by replacing CutMix with Mixup [21] that simply superimposes the entire patches from each of different smashed data. By experiment, we show that DP-CutMixSL achieves the highest accuracy, followed by DP-MixSL and DP-SL. It is worth noting that while most of the existing works apply Cutout and CutMix at pixel levels for intra-dataset interpolations [22, 23], we utilize them at patch levels for privacy-preserving inter-dataset interpolations across different devices, i.e., privacy-preserving distributed ML.

2 DP-CutMixSL: Patch-Level Randomized CutMix Operations for ViT

The major difference between ViT and CNN can be summarized as follows: i) As shown in Fig. 4 of Appendix A, ViT has less feature distortion for the input data of the hidden representation (i.e. smashed data) due to the absence of a pooling layer, ii) Due to its own self-attention mechanism driven by embedding process, ViT captures global spatial information whereas CNN focuses on local spatial information, iii) The above operations of ViT run at patch-level.

At first, i) implies that regularization of the hidden representation in ViT is as efficient as in the input data. Conversely, the mutual information about the input data of the hidden representation is high, leading to data privacy leakage. Next, due to the property of ViT to learn global spatial information mentioned in ii), ViT has more robustness to large-scale noise applied to the fraction of the image [25], thereby it is suitable for Cutout [20] or CutMix augmentation. Finally, iii) suggests the possibility of a patch-scale regularizer. Integrating the above yields a common solution, patch-level randomized CutMix of hidden representations, short for patch CutMix.

Let i and $\mathbf{C}$ be a subscript for a client and a set of clients, respectively. As observed in Fig. 1, a mixer, which may be a third-party entity, first generates random sequences M_i with the mixing ratio $\lambda_i \in [0, 1]\ \forall i \in \mathbf{C}$ following a Dirichlet multinomial distribution [26], where $\sum_i \lambda_i = 1$. For instance, if a smashed data s_i consists of N patches, M_i is a random binary sequence of length N to control the on-off of each patch, and its non-zero element is $\lfloor \lambda_i \cdot N \rfloor$. Then, the i-th client acquires the Cutout smashed data $\bar{s}_i$ by masking the smashed data ($\bar{s}_i = M_i \odot s_i$), obtained by passing the input data through the local model, via the random sequence downloaded from the mixer. The mixer aggregates Cutout smashed data from all clients and generates *DP-CutMix smashed data* in the following way:

$$\tilde{s}_{i,j} = \bar{s}_i + \bar{s}_j = M_i \odot s_i + M_j \odot s_j, \tag{1}$$

$$\tilde{y}_{i,j} = \lambda_i \cdot y_i + \lambda_j \cdot y_j, \qquad \text{for } j \neq i. \tag{2}$$

When the DP-CutMix smashed data and its label are uploaded to the server, we assume that a gaussian mechanism is applied to them, generating the following DP-CutMix smashed data and label

[1]An early version of this work was presented at FL-IJCAI 2022 [24]. Compared to [24] proposing CutMixSL and focusing its communication efficiency, this work proposes DP-CutMixSL while studying its DP analysis and the privacy-accuracy trade-off.

containing white gaussian noise of N_s and N_y, respectively:

$$\tilde{s}'_{i,j} = \tilde{s}_{i,j} + N_s, \quad \tilde{y}'_{i,j} = \tilde{y}_{i,j} + N_y. \tag{3}$$

Next, the rest of DP-CutMixSL's operation, equal to that of SL, performing FP & BP on the server-side model follows. The said operation of CutMixSL is detailed by the pseudo code of Appendix B.

As a result, DP-CutMixSL can benefit both in terms of privacy leakage and communication cost, in a way that only fraction of the smashed data is shared to the server, even ejected with gaussian noise. Note that random sequences used for smashed data masking are mutually exclusive and collectively exhaustive at the patch-level, so that there are no blank patches in DP-CutMix smashed data.

3 DP-CutMixSL: Differential Privacy Analysis

Let $\mathcal{D} = \{(s_1, y_1), .., (s_n, y_n)\}$ be a set consisting of n clients' pairs of smashed data $s_i \in \mathbb{R}^{N \times P^2 \times C}$ and the corresponding label $y \in \mathbb{R}^L$ is a one-hot encoding vector, where P denotes the size of patch, respectively. We assume that the each element of the smashed data and ground-truth label is upper bounded as follows: $s_i \in [0, \Delta]^{D_s}$ and $y_i \in [0, 1]^{D_y}$, where $D_s = NP^2C$ and $D_y = L$. In addition, λ_i is the mixing ratio of the i-th client, and N_s and N_y are white gaussian noise with dimensions D_s and D_y, respectively, i.e., $N_s \sim N(0, \sigma_s^2 I_{D_s})$ and $N_y \sim N(0, \sigma_y^2 I_{D_y})$ for some (σ_s, σ_y). Then, we derive the *Rényi differential privacy (RDP)* [19] of the proposed DP-CutMixSL, which is compared with those of DP-SL and DP-MixSL as follows.

Theorem 1. For a given order α, the RDP privacy budgets $\epsilon_o(\alpha)$, $\epsilon_{Mix}(\alpha)$, and $\epsilon_{CutMix}(\alpha)$ of DP-SL, DP-MixSL and DP-CutMixSL satisfy $\epsilon_{Mix}(\alpha) \leq \epsilon_{CutMix}(\alpha) \leq \epsilon_{SL}(\alpha)$ where:

$$\epsilon_o(\alpha) = \frac{\alpha}{2}\left(\frac{\Delta^2 D_s}{\sigma_s^2} + \frac{D_y}{\sigma_y^2}\right), \tag{4}$$

$$\epsilon_{Mix}(\alpha) = \epsilon_o(\alpha) \cdot \left(\max_{i \in \mathbf{C}} \lambda_i\right)^2, \tag{5}$$

$$\epsilon_{CutMix}(\alpha) = \epsilon_o(\alpha) \cdot \max_{i \in \mathbf{C}} \lambda_i. \tag{6}$$

Sketch of Proof. For each technique, we derive its output representation, followed by calculating the RDP bound using the output via the Rényi divergence formula for a multivariate Gaussian distribution [18]. Applying this to both the smashed data and the label and combining them via the sequential composition rule completes the proof. The details are deferred to Appendix C. ■

Since $\lambda_i \in [0, 1]$, DP-MixSL achieves the highest RDP guarantee (i.e., tightest RDP bound) compared to DP-CutMixSL and DP-SL, with with the help of the inherent distortion property of interpolations [27, 28, 29]. It is worth noting that the case only when $\max_{i \in \mathbf{C}} \lambda_i = 1$, i.e., a single client scenario with $|\mathbf{C}| = 1$, the equality conditions $\epsilon_{Mix}(\alpha) = \epsilon_{CutMix}(\alpha) = \epsilon_{SL}(\alpha)$ hold. In other words, none of equality does not hold for multi clients. Note here that we focus only on the RDP guarantees of smashed data. Smashed data are vulnerable to reconstruction attacks [30], threatening the privacy of raw data, which is discussed in Appendix D.

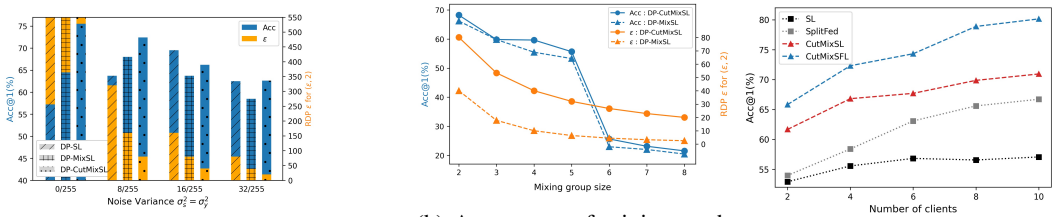
4 Numerical Evaluation

In this section, we measure the accuracy, RDP bound (ϵ), and scalability of those of DP-CutMixSL compared to SplitFed [14], DP-SL, DP-MixSL, and etc. In Table 1, both the CIFAR-10 data set [31] and the Fashion-MNIST data set [32] are utilized under three types of models: ViT-tiny, PiT-tiny [33], and VGG-16 [34]. Here, PiT is a transformer structure equipped with a pooling layer and is a model between ViT and CNN. For all SL algorithms, we assume that the cut-layer is located after embedding process. Other parameters especially for RDP calculation are as follows: patch size $N = 64$, $D_s = 20$, $D_y = 10$, $\Delta = 0.05$, $\lambda_i = 1/n \; \forall i$ (uniform), and RDP parameter $\alpha = 2$.

Table 1 shows the top-1 accuracy for several SL methods including the proposed CutMixSL in a noiseless environment. As seen at Table 1, except for one case, where SL w. Mixup is used with CIFAR-10 data set and VGG-16 model, the CutMixSL outperforms other state-of-the-art SL algorithms in terms of top-1 accuracy. This is rooted in the difference between ViT and CNN

Table 1: Top-1 accuracy of SL-based techniques w.r.t various model types and data sets.

Method ($\lvert \mathbf{C} = 10 \rvert$)	Models w/ CIFAR-10			Models w/ Fashion-MNIST		
	ViT-Tiny	PiT-Tiny	VGG-16	ViT-Tiny	PiT-Tiny	VGG-16
Standalone	48.84	47.77	54.97	77.65	78.21	80.12
SL [10]	57.21	52.28	62.62	85.68	82.35	84.39
SplitFed [14]	67.88	55.63	63.98	89.17	84.27	87.34
Standalone w. Cutout [20]	53.86	50.28	56.65	88.46	86.48	88.17
SL w. Mixup	69.23	64.89	**68.20**	88.21	87.62	88.53
SL w. Vanilla CutMix	71.78	58.21	33.50	87.86	86.31	89.01
CutMixSL (proposed)	**73.77**	**71.26**	67.53	**89.75**	**89.25**	**89.45**



(a) Accuracy-ϵ per noise variance.

(b) Accuracy-ϵ of mixing methods.

(c) Scalability.

Figure 2: Accuracy and ϵ under the CIFAR-10 data set: (a) accuracy and ϵ of DP-SL, DP-MixSL, and DP-CutMixSL w.r.t noise variance; (b) accuracy and ϵ of DP-MixSL and DP-CutMixSL w.r.t the mixing group size; (c) accuracy of various SL-based techniques according to number of clients.

mentioned in Sec. 2. When learning spatial information, ViT focuses on globality due to the self-attention mechanism, whereas CNN focuses on locality. Hence, a patch CutMix in which certain patches are replaced by patches of other smashed data may cause significant information loss in CNN. On the other hand, interpolation such as mixup is less likely to yield large information loss, thereby CNN and ViT are suitable for mixup and patch CutMix, respectively. Comparing CutMixSL and SL w. vanilla CutMix in Table 1 proves that patch-level random punching is more efficient than bounding box-based process. This is because the random punching method is a regularizer well suited to ViT, where all operations operate at patch-level, unlike vanilla CutMix's bounding box where the size is regardless of patch size. From a dropout [35] perspective, the random punching method of CutMixSL is more similar to dropout than mixup or vanilla CutMix, leading to accuracy gains.

Fig. 2a shows the effect of noise variance on accuracy and ϵ. In terms of accuracy, DP-CutMixSL is the best, except when the noise variance is $16/255$. Compared to DP-MixSL, DP-CutMixSL has superior performance in all cases. Looking at ϵ, however, DP-CutMixSL has a tighter RDP bound compared to DP-SL, but has a larger ϵ in comparison with DP-MixSL, showing the *accuracy-privacy trade-off*. In Fig. 2b, the size of a mixing group, a set of clients taking a mixup or CutMix, varies both in DP-CutMixSL and DP-MixSL. In both DP-CutMixSL and DP-MixSL, the accuracy and ϵ decrease as the mixing group size increases, also resulting in the accuracy-privacy trade-off. The decrease in ϵ according to the mixing group size can be explained by the "Hiding in the crowd" effect [36], and the decrease in accuracy is also explained in connection with the information loss mentioned above. Fig. 2c shows the curve of scalability, in terms of accuracy increase according to the number of clients. In Fig. 2c, all SL techniques including CutMixSL guarantee scalability when the client increases from 2 to 10, and the accuracy of CutMixSFL, which introduced SplitFed's weight averaging of local model to CutMixSL, is further improved.

5 Concluding Remarks

In this work, we proposed DP-CutMixSL for privacy-preserving distributed ML for ViT by exploiting CutMix for intra-dataset interpolations. We theoretically analyzed its DP guarantee, and numerically showed its achieving the highest accuracy compared to two baelines, DP-SL and DP-MixSL. While we focus only on generating a single CutMix output for two or more inputs, it is possible to generate multiple outputs by changing the mixing ratio for interpolations with finer resolution as done in [37], which could be an interesting topic for future study. Furthermore, based on the simulation results showing the effectiveness of the proposed method even in the presence of several pooling and convolutional layers, it is worth investigating other patchified architectures in different domains for future research.

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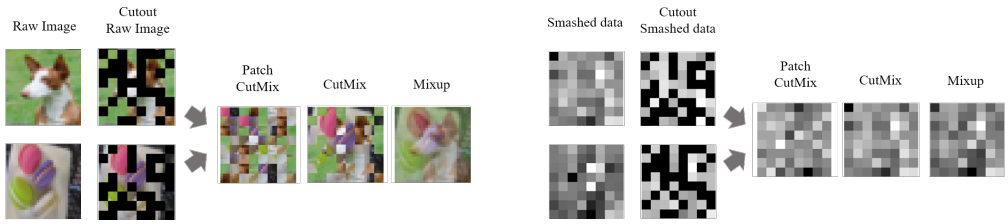
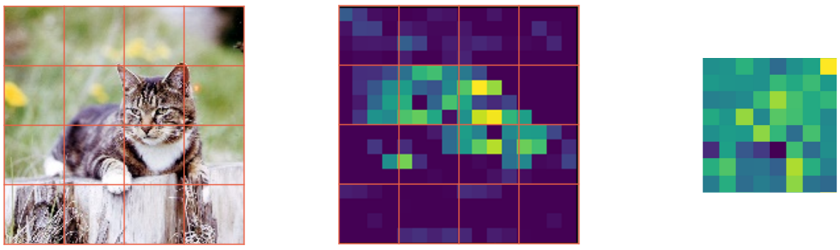
A Visualization of various types of images



(a) Raw images. (b) Smashed data.

Figure 3: Examples of data obtained by performing various operations on (a) raw image and (b) smashed data.



(a) Input raw image. (b) Smashed data of ViT. (c) Smashed data of CNN.

Figure 4: Comparison between raw image and smashed data of ViT and CNN.

B Pseudo code of DP-CutMixSL

Algorithm 1 DP-CutMixSL

requirements: $w = [w_{c,i}, w_s]^T$ ($w_{c,i}$: lower model segment, w_s: upper model segment)
η: learning rate
while w not converged **do**
Mixer:
  samples $\{a_1, .., a_n\} \sim \text{Dir}(\alpha)$
  generates pseudo random sequences M_i for all i ▷ *Pseudo random sequence generation*
  unicasts M_i to i-th client for all i
Client $i \in \mathbf{C}$:
  generates smashed data s_i by passing input data x_i through $w_{c,i}$
  produces Cutout smashed data $\bar{s}_i$ by masking s_i via M_i ▷ *Cutout smashed data*
  uploads $\bar{s}_i$ to the mixer
Mixer:
  produces DP-CutMix smashed data $\tilde{s}_i$ via $\bar{s}_i$ aggregation for all i ▷ *DP-CutMix smashed data*
  uploads $\tilde{s}_i$ to the server ▷ *Gaussian mechanism applied*
Server:
  generates loss $\sum_i L_i$ by passing $\tilde{s}'_i$ through w_s in parallel
  updates upper model segment via $w_s \leftarrow w_s - \eta \cdot \nabla_{w_s}(\sum_i L_i)$ ▷ *Weight update*
  unicasts i-th cut-layer gradient to i-th client for all i
Client $i \in \mathbf{C}$:
  updates lower model segment via $w_{c,i} \leftarrow w_{c,i} - \eta \cdot \nabla_{w_{c,i}}(\sum_i L_i)$ ▷ *Weight update*
end while

C RDP Analysis

Before going further, the definition of RDP is as follows:

Definition C.1 ((α, ϵ)-RDP [19])**.** A randomized mechanism $f : \mathcal{D} \to \mathcal{R}$ is said to have ϵ-Rényi differential privacy of order α, or (α, ϵ)-RDP for short, if for any adjacent $D, D' \in \mathcal{D}$ it holds that

$$D_\alpha(f(D)||f(D')) \leq \epsilon \tag{7}$$

A strong privacy guarantee implies that one cannot distinguish whether D or D' was used to produce an outcome of mechanism.

C.1 DP-SL

Here, our baseline DP mechanism is Gaussian mechanism. Then, the output of DP-SL, which are the noisy smashed data and its corresponding label, is as follows:

$$s'_i = s_i + N_s, \tag{8}$$

$$y'_i = y_i + N_y, \tag{9}$$

where $N_s \sim N(0, \sigma_s^2 I_{D_s})$ and $N_y \sim N(0, \sigma_y^2 I_{D_y})$ for some (σ_s, σ_y).

By using the definition C.1, the RDP bound of gaussian mechanism $\mathcal{M}$ for DP-SL, $\epsilon_{SL}(\alpha)$ can be expressed as:

$$\epsilon_{SL}(\alpha) = \sup_{\mathcal{D},\mathcal{D}'} D_\alpha(\mathcal{M}(\mathcal{D})||\mathcal{M}(\mathcal{D}')) = \sup_{\mathcal{D},\mathcal{D}'} \frac{\alpha}{2\sigma^2} ||\mu_X^{\mathcal{D}} - \mu_X^{\mathcal{D}'}||^2, \tag{10}$$

where $\mathcal{M}(\mathcal{D})$ and $\mathcal{M}(\mathcal{D}')$ follow gaussian distribution $\mathcal{M}(\mathcal{D}) \sim N(\mu_X^{\mathcal{D}}, \sigma_X^2)$, and $\mathcal{M}(\mathcal{D}') \sim N(\mu_X^{\mathcal{D}'}, \sigma_X^2)$. Let $s_{(i,k)}$ denote the k-th element of smashed data s_i, where smashed data $s_i = [s_{(i,1)}, ..., s_{(i,D_s)}]$. Then, ${s'}_{(i,k)} \sim N(s_{(i,k)}, \sigma_s^2)$ for $k \in [D_s]$. We assume that $s'^{\mathcal{D}}$ and $s'^{\mathcal{D}'}$ denote noisy smashed data with a given dataset $\mathcal{D}$ and $\mathcal{D}'$, respectively. Then, $s'^{\mathcal{D}} \sim N(s^{\mathcal{D}}, \sigma_s^2)$, and $s'^{\mathcal{D}'} \sim N(s^{\mathcal{D}'}, \sigma_s^2)$.

When the two data sets $\mathcal{D}$ and $\mathcal{D}'$ are assumed to be adjacent except for the i'-th element, from (10), we have

$$\|\mu_X^{\mathcal{D}} - \mu_X^{\mathcal{D}'}\|^2 = \|s^{\mathcal{D}} - s^{\mathcal{D}'}\|^2 = \sum_{k=1}^{D_s} \left(s_{(i',k)}^{\mathcal{D}} - s_{(i',k)}^{\mathcal{D}'}\right)^2. \tag{11}$$

Considering the element-wise upper bound of DP-SL's smashed data yields the following formula:

$$\sum_{k=1}^{D_s} \left(s_{(i',k)}^{\mathcal{D}} - s_{(i',k)}^{\mathcal{D}'}\right)^2 \leq \Delta^2 \cdot D_s. \tag{12}$$

Therefore, gaussian mechanism for smashed data in DP-SL is (α, ϵ_{SL})-RDP

$$\epsilon_{SL}(\alpha) = \alpha \frac{\Delta^2 \cdot D_s}{2\sigma_s^2}. \tag{13}$$

Likewise, RDP bound of groud-truth label in DP-SL can be calculated, yielding $\epsilon_{SL}(\alpha)$ as below:

$$\epsilon_{SL}(\alpha) = \frac{\alpha}{2}\left(\frac{\Delta^2 D_s}{\sigma_s^2} + \frac{D_y}{\sigma_y^2}\right). \tag{14}$$

C.2 DP-MixSL

The output of DP-MixSL can be expressed as $\hat{s}_i = \sum_{i=1}^{n} \lambda_i s_i$ for $i \in [n]$ where $\{\lambda_1, ..., \lambda_n\}$ is a set of mixing ratio of each client's smashed data. Also, DP-MixSL executes mixup operation similar to its label. Then, the output of DP mechanism for DP-MixSL follows $N(\hat{s}', \sigma_s^2 I_{D_s})$ when the base mechanism is gaussian mechanism.

From (11), for adjacent data sets $\mathcal{D}$ and $\mathcal{D}'$,

$$\|\mu_X^{\mathcal{D}} - \mu_X^{\mathcal{D}'}\|^2 = \sum_{k=1}^{D_s} \lambda_{i'}^2 (s_{(i',k)}^{\mathcal{D}} - s_{(i',k)}^{\mathcal{D}'})^2. \tag{15}$$

This bound can be maximized when the i'-th mixing ratio is the largest value of λ_i for all $i \in \mathbf{C}$. In this case, from 15, we have

$$\sum_{k=1}^{D_s} \lambda_{i'}^2 (s_{(i',k)}^{\mathcal{D}} - s_{(i',k)}^{\mathcal{D}'})^2 \leq (\max_{i \in \mathbf{C}} \lambda_i)^2 \Delta^2 D_s. \tag{16}$$

By applying the same process above to the label, we have

$$\epsilon_{Mix}(\alpha) = \epsilon_o(\alpha) \cdot \left(\max_{i \in \mathbf{C}} \lambda_i\right)^2. \tag{17}$$

where

$$\epsilon_o(\alpha) := \frac{\alpha}{2}\left(\frac{\Delta^2 D_s}{\sigma_s^2} + \frac{D_y}{\sigma_y^2}\right). \tag{18}$$

C.3 DP-CutMixSL

When a set of mixing ratio $\{\lambda_1, ..., \lambda_n\}$ is given, where $\sum_{i=1}^{n} \lambda_i = 1$, and $\lambda_i = \frac{|M_i|}{N}$. Here, $M_i = \{b_i^1, ..., b_i^{|M_i|}\}$ is a set of positions of i-th smashed data for mix operation. DP-CutMixSL mechanism fills the blank set of patches with client's patches given a pseudo random sequence. The assumption for two adjacent sets changes the only patches from i'-th client in the DP-CutMix smashed data.

That is, from (11),

$$(s_{(i)}^{\mathcal{D}} - s_{(i)}^{\mathcal{D}'})^2 = 0, \tag{19}$$

where $i \in \{[n] - M_{i'}\}$.

For $i \in M_{i'}$,

$$(s_{(i)}^{\mathcal{D}} - s_{(i)}^{\mathcal{D}'})^2 = \sum_{k=1}^{D_s} (s_{(i,k)}^{\mathcal{D}} - s_{(i,k)}^{\mathcal{D}'})^2 \leq \Delta^2 D_s. \tag{20}$$

Combining (19) and (20) yields the following formula:

$$\sum_{i=1}^{n} (s_{(i)}^{\mathcal{D}} - s_{(i)}^{\mathcal{D}'})^2 = \Delta^2 D_s. \tag{21}$$

Since data sample with highest mixing ratio has largest range of impact on the output, the (11) is upper bounded as below.

$$\sum_{i=1}^{n} (s_{(i)}^{\mathcal{D}} - s_{(i)}^{\mathcal{D}'})^2 \leq \max_{i \in \mathbf{C}} |M_i| \Delta^2 D_s = (\max_{i \in \mathbf{C}} \lambda_i) \Delta^2 D_s. \tag{22}$$

The corresponding mixed label follows mixup operation like [29]. Here, we have mixing ratio followed by dirichlet distribution unlike to [29] where even mixing ratio are used. Let $y_i = \{y_{(i,1)}, ..., y_{(i,D_y)}\}$ denote one hot encoding where $y_{(i,k)} \in \{0,1\}$ for $k \in [D_y]$. The corresponding mixed label can be represented as $y = \sum_{i=1}^{n} \lambda_i y_i$.

$$\|\mu_Y^{\mathcal{D}} - \mu_Y^{\mathcal{D}'}\|^2 = \sum_{i=1}^{n} \sum_{k=1}^{D_y} \left(\lambda_i y_{(i,k)}^{\mathcal{D}} - \lambda_j y_{i,k}^{\mathcal{D}'}\right)^2 \leq (\max_{i \in \mathbf{C}} \lambda_i)^2 D_y \tag{23}$$

Combining (10), (21), and (23), we have $\epsilon_{CutMix}(\alpha)$ for DP-CutMixSL mechanism as follows:

$$\epsilon_{CutMix}(\alpha) = \frac{\alpha \cdot \max_{i \in \mathbf{C}} \lambda_i}{2} \left(\frac{\Delta^2 D_s}{\sigma_s^2} + \frac{(\max_{i \in \mathbf{C}} \lambda_i) D_y}{\sigma_y^2} \right). \tag{24}$$

Integrating (14), (17), and (24) only in terms of ϵ for smashed data, not including those of label, the equation holds as presented in Theorem 1.

D Robustness to Reconstruction Attack

Table 2: Privacy leakage measured by the reconstruction loss (MSE).

Type	Train Dataset(10%)	Train Dataset(100%)
Smashed data	0.0091	0.0056
Cutout Smashed data	**0.0920**	**0.0829**
Mixup	0.0402	0.0351
Patch CutMix	0.0458	0.0434

Table 2 shows loss between raw data and reconstructed data generated from different types of mixing methods or the type of data. To restore its original data, we use a decoder model, comprised of two convolutional layers and additional interpolation methods to adjust the dimensions to the aimed data size. The decoder is trained with two different amount of training data, which is in comparison. Obviously, throughout the all types, when training with larger amount of data, it better reconstructs to the origin data, thus resulting smaller loss. Mixup shows the worst privacy leakage after smashed data, certainly because the smashed data has entire information of both mixed data. Our proposed, patch CutMix lowers privacy leakage then mixup, by throwing some part of information, but still performs lower than Cutout smashed data, since patch CutMix combines two of those data while Cutout discards it.