Curiosity-Driven Exploration via Temporal Contrastive Learning

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Abstract

Effective exploration in reinforcement learning requires not only tracking where an agent has been, but also understanding how the agent perceives and represents the world. To learn powerful representations, an agent should actively explore states that facilitate a richer understanding of its environment. Temporal representations can capture the information necessary to solve a wide range of potential tasks while avoiding the computational cost associated with full-state reconstruction. In this paper, we propose an exploration method that leverages temporal contrastive representations to guide exploration, aiming to maximize state coverage as perceived through the lens of these learned representations. Temporal contrastive representations have shown promising results in goal-conditioned reinforcement learning, unsupervised skill discovery, and empowerment-based learning. We demonstrate that such representations can enable the learning of complex exploratory behaviors in locomotion, manipulation, and embodied-AI tasks, revealing previously inaccessible capabilities and behaviors that traditionally required extrinsic rewards.

1 Introduction

Exploration remains a key challenge in reinforcement learning (RL), especially in tasks that demand reasoning over increasingly long horizons (Thrun, 1992) or with seemingly high-dimensional observations (Stadie et al., 2015; Burda et al., 2019b; Pathak et al., 2017). Perhaps the defining feature of RL, relative to other areas of ML, is the ability to find *new* strategies. Realizing this benefit would unlock important capabilities in robotics, LLM agents, and myriad other application domains. To unlock these capabilities, we need methods that can reduce the effective state space, but preserve the structure required for most tasks.

A common approach to exploration in reinforcement learning (RL) is to estimate the density of visited states. This estimated density is then used to construct an intrinsic reward function that encourages agents to visit novel states, thereby promoting state coverage. To enhance exploration in high-dimensional environments, prior work has proposed representation-based methods that learn compact representations, those sufficient to predict actions (Pathak et al., 2017), entirely random encodings (Burda et al., 2019b), or reconstructions of original observations (Stadie et al., 2015). These methods guide exploration along the *manifold* of meaningful states by discarding irrelevant information, as determined by the learned representations. However, identifying which aspects of the observation space are truly relevant remains a fundamental challenge. As a result, recent research has focused on learning representations that are aligned with the underlying task structure (Gelada et al., 2019; Zhang et al., 2021). In this paper, we ask: *How can we learn representations that facilitate exploration and are provably linked to the RL objective, retaining task-relevant features while excluding irrelevant ones?*

*Equal Advising.

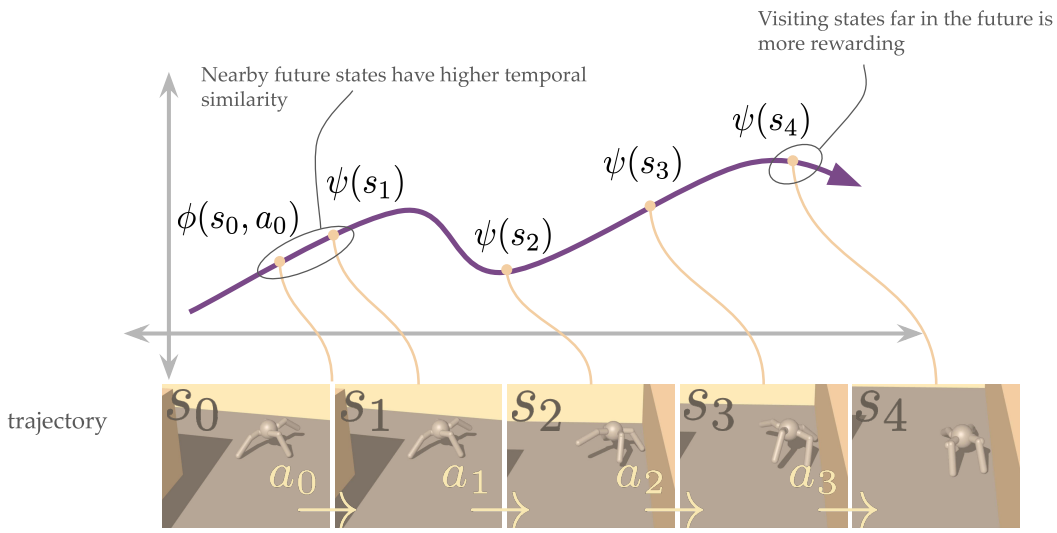

Figure 1: **Curiosity-Driven Exploration via Temporal Contrastive Learning** The agent's starting state is (s_0). We train a contrastive model such that the temporal similarity between the representation of $((s_0, a_0))$ and $(s_{2,3,4,\ldots})$ is high, and we reward the agent for visiting states that are further in the future. For example, the reward for visiting (s_4) from (s_0) should be larger than the reward for visiting (s_3) from the same (s_0).

For a representation to be suitable for exploration, it should satisfy several key properties. Most importantly: (1) it should be invariant to uncontrollable aspects of the environment, (2) it should be predictive of future states under the agent's policy, (3) it should scale with the dimensionality of the observation space, and (4) it should remain up to date with the agent's ongoing experience.

One of the main challenges with previous works is the complexity of the representation learning process Pathak et al. (2017); Burda et al. (2019b), which limits the agent's ability to keep the representation up-to-date with the agent's current distribution (Castanyer et al., 2024). Our proposed method aims to overcome these issues by building on temporal contrastive representations (Sermanet et al., 2018; Qian et al., 2021; Eysenbach et al., 2022; Dave et al., 2022), which are closely related to the successor representation (Dayan, 1993) and have been shown to be provably sufficient to represent Q-values for any reward function (Mazoure et al., 2023). While prior work has primarily leveraged these representations for encoding high-dimensional observations (Laskin et al., 2020) and for learning goal-directed skills (Eysenbach et al., 2022), we take a different direction: we use them to guide exploration. Specifically, we propose to reward the agent for visiting states that lie farther along its future trajectory, thereby encouraging it to expand the boundaries of its current state visitation and to explore less-visited regions of the environment.

This work makes two primary contributions. First, we introduce a novel exploration algorithm—Curiosity-Driven Exploration via Temporal Contrastive Learning (C-TeC)—which achieves state-of-the-art state coverage across navigation, manipulation, and open-world environments. Second, we establish a connection between the contrastive learning-based objective and information-theoretic control objectives. Specifically, we show that the reward signal from C-TeC encourages mode-seeking behavior, prompting the RL agent to prioritize visiting novel future states and thereby achieving broad state coverage relative to its future state marginal.

2 Related Work

Unsupervised RL. Prior work on unsupervised RL (Laskin et al., 2021) has proposed various task-agnostic methods for learning behaviors. A key direction in this area is intrinsic motivation, which encourages novelty-seeking behavior by maximizing state coverage or surprise. In low-dimensional

and/or discrete environments, count-based exploration methods (Gardeux et al., 2016; Bellemare et al., 2016; Tang et al., 2017; Ostrovski et al., 2017; Martin et al., 2017; Xu et al., 2017; Machado et al., 2020) have demonstrated effective exploration in Atari games. However, these methods often struggle in high-dimensional or continuous state spaces. In such settings, prediction-error-based exploration approaches (Pathak et al., 2017; Burda et al., 2019a;b; Lee et al., 2019) have been more effective, both in video game environments and in continuous control tasks. Another line of research focuses on representation-based novelty, where a representation learning component is used to extract compact features from raw state inputs. An entropy estimator is then applied to these learned representations to assess state novelty Liu & Abbeel (2021); Laskin et al. (2022).

A different approach to unsupervised RL involves training the agent to control the environment by either maximizing mutual information between states and actions (empowerment)(kly, 2005; Klyubin et al., 2005) or minimizing surprise(Friston, 2010; Berseth et al., 2021; Rhinehart et al., 2021). Empowerment-based methods (Biehl et al., 2015; Zhao et al., 2021; Mohamed & Jimenez Rezende, 2015; Karl et al., 2019; Hayashi & Takahashi, 2025; Levy et al., 2024; Jung et al., 2011; Du et al., 2020; Myers et al., 2024) encourage the agent to take actions that exert significant influence over future states, although solving the full problem remains intractable. In contrast, surprise minimization drives the agent to regulate the environment and maintain an orderly niche, giving rise to complex behaviors in both fully observed (Berseth et al., 2021; Hugessen et al., 2024) and partially observed settings (Rhinehart et al., 2021).

Representation learning for RL. Prior work on representation learning for RL focuses on self-supervised methods to improve the data efficiency of RL agents. A notable approach in this category involves the use of unsupervised auxiliary tasks, where a pseudo-reward is added to the task reward to shape the learned representations and provide an additional training signal. Examples of this approach include (Jaderberg et al., 2017; Farebrother et al., 2023; Oord et al., 2018; Laskin et al., 2020; Schwarzer et al., 2021). Another line of work focuses on forward-backward representations (Touati & Ollivier, 2021; Touati et al., 2023), which aim to capture the dynamics under all optimal policies and have been shown to exhibit zero-shot generalization capabilities. Moreover, contrastive learning has been applied in various exploration settings, including goal-conditioned learning (Eysenbach et al., 2022; Liu et al., 2025), skill discovery (Laskin et al., 2022; Yang et al., 2023; Zheng et al., 2025), and state coverage or curiosity (Liu & Abbeel, 2021; Du et al., 2021; Yarats et al., 2021). In the context of curiosity-driven exploration, Du et al. (2021); Yarats et al. (2021) employ contrastive learning to learn visual representations in image-based environments, where the RL agent is trained to maximize the error of the representation learner (similar in spirit to prediction-error approaches). We consider C-TeC to fall under the state coverage category, while learning contrastive representations that facilitate density estimation and compress the agent experience into a low-dimensional space.

3 Background

We consider a controlled Markov process (i.e., an MDP without a reward function), defined by time-indexed states s_t and actions a_t. The initial state is sampled from $p_0(s_0)$, and subsequent states are sampled from the Markovian dynamics $p(s_{t+1} \mid s_t, a_t)$. Actions are selected by a stochastic, parameterized policy $\pi(a_t \mid s_t)$. Without loss of generality, we assume that episodes have an infinite horizon; the finite-horizon problem can be incorporated by augmenting the dynamics with an absorbing state. The key to C-TeC is to use a self-supervised, or intrinsic reward, built on temporal contrastive representations. We detail the necessary preliminaries below.

Discounted state occupancy measure Formally, we define the γ-discounted state occupancy measure conditioned on a state and an action (Ho & Ermon, 2016; Eysenbach et al., 2021; 2022) as

$$p(s_f \mid s, a) \triangleq (1-\gamma) \sum_{t=0}^{\infty} \gamma^t p(s_t = s_f \mid s, a), \tag{1}$$

where $p(s_t = s_f \mid s, a)$ is the probability of being at future state s_f at time step t conditioned on s, a. In continuous settings, the future state distribution $p(s_t = s_f \mid s, a)$ is a probability *density*.

Traditionally, the discounted state occupancy measure is defined with respect to a policy as $p_\pi(s_f \mid s, a)$. However, in this work, the intrinsic reward r_{intr} is defined using a discounted state occupancy measure over the trajectory buffer $\mathcal{T}$, which contains trajectories collected from a history of policies:

$$p_{\mathcal{T}}(s_f \mid s, a) \triangleq (1-\gamma) \sum_{0}^{\infty} \gamma^t p_{\mathcal{T}}(s_t = s_f \mid s, a).$$

To sample from the *trajectory buffer* distribution $p_{\mathcal{T}}(s_t = s_f \mid s, a)$, we first sample an offset $\Delta \sim \textsc{Geom}(1-\gamma)$, then set the future state $s_f = s_{t+\Delta}$. Here, future state $s_f = s_{t+\Delta}$ is the state reached from (s, a) after executing Δ-number of actions within a sampled stored trajectory.

Contrastive learning Contrastive representation learning methods (cho, 2005; Oord et al., 2018; Chen et al., 2020) train a critic function C_θ that takes as input pairs of positive and negative examples, and learn representations so that positive pairs have similar representations and negative pairs have dissimilar representations. To estimate the discounted state occupancy, positive examples are sampled from a joint distribution $p_{\mathcal{T}}((s, a), s_f) = p_{\mathcal{T}}(s, a) p_{\mathcal{T}}(s_f \mid s_t, a_t)$, while the negative examples are sampled from the product of marginal distributions $p_{\mathcal{T}}(s, a) p_{\mathcal{T}}(s_f)$. Here, $p_{\mathcal{T}}(s_f)$ is the marginal discounted state occupancy:

$$p_\tau(s_f) = \int p_\tau(s_f \mid s, a) p_{\mathcal{T}}(s, a) \; ds\, da.$$

We use the InfoNCE loss to train the contrastive learning model Oord et al. (2018). Let $\mathcal{B} = \{(s_t^{(i)}, a_t^{(i)}, s_f^{(i)})\}_{i=1}^K$ be the sampled batch, where $s_f^{(1)}$ is the positive example sampled from conditional distribution $p_{\mathcal{T}}(s_f \mid s_i, a_i)$ and $\{s_f^{(2:K)}\}$ are the $K-1$ negatives sampled from the marginal distribution $p_{\mathcal{T}}(s_f)$ (independently from (s_i, a_i)). In addition to the standard InfoNCE objective, prior work has shown that a LogSumExp regularizer is necessary to learn a critic function for control Eysenbach et al. (2021). The full contrastive reinforcement learning (CRL) loss is as follows:

$$\mathcal{L}_{\text{CRL}}(\theta) = -\mathbb{E}_{\substack{(s,a)\sim p_{\mathcal{T}}(s,a) \\ s_f^{(1)} \sim p_{\mathcal{T}}(s_f \mid s,a) \\ s_f^{(2:K)} \sim p_{\mathcal{T}}(s_f)}} \left[\log \left(\frac{e^{C_\theta((s,a), s_f^{(1)})/\tau}}{\sum_{j=1}^{K} e^{C_\theta((s,a), s_f^{(j)})/\tau}} \right) - 0.01 \cdot \log \left(\sum_{j=1}^{K} e^{C_\theta((s,a_)), s_f^{(j)})/\tau} \right)^2 \right]. \tag{2}$$

Where τ is a temperature parameter. The optimal critic $C^*((s_t, a_t), s_f)$ corresponds to the following log probability ratio (Ma & Collins, 2018)

$$C^*((s_t, a_t), s_f) \approx \log \frac{p_{\mathcal{T}}(s_f \mid s_t, a_t)}{p_{\mathcal{T}}(s_f)},$$

where we use the following two parametrizations of the critic:

$$C_\theta((s_t, a_t), s_f)_{L2} = -||\phi_\theta(s_t, a_t) - \psi_\theta(s_f)||_2 \tag{3}$$
$$C_\theta((s_t, a_t), s_f)_{L1} = -||\phi_\theta(s_t, a_t) - \psi_\theta(s_f)||_1. \tag{4}$$

Conceptually, the critic C_θ gives a temporal similarity score between state-action pairs (s, a) and future states s_f via learned representation ϕ_θ and ψ_θ. A visual overview of the method is shown in Figure 1. These representations are powerful yet simple tools that capture complex temporal correlations between states-actions and future states. In our method C-TeC, we leverage these learned temporal contrastive representations to do exploration.

4 Curiosity-Driven Exploration via Temporal Contrastive Learning

To improve exploration, we learn representations that encode the agent's future state occupancy using temporal contrastive learning. We begin by describing how contrastive representation learning can be used to estimate state occupancy by learning a similarity function that assigns high scores to frequently visited future states and low scores to rarely visited ones (Eysenbach et al., 2022; Oord et al., 2018). We then explain how this similarity score can be leveraged to derive an intrinsic reward signal for exploration.

4.1 Training the contrastive model

As detailed in Section 3, the contrastive model $C_\theta(s_t, a_t, s_f)$ is trained on batches $\mathcal{B}$ of (s_t, a_t, s_f) tuples, where each s_f is sampled from the discounted future state distribution. Specifically, a geometric offset $\Delta \sim \textsc{Geom}(1-\gamma)$ is sampled, and the future state is set to $s_f = s_{t+\Delta}$.

We use two parameterized encoders to define the contrastive model: $\phi_\theta(s_t, a_t)$ for state-action pairs and $\psi_\theta(s_f)$ for future states. A batch of state-action pairs $\{(s_t^{(i)}, a_t^{(i)})\}_{i=1}^K$ is passed through ϕ_θ, while the corresponding batch of future states $\{s_f^{(i)}\}_{i=1}^K$ is passed through ψ_θ. The resulting representations are then normalized to have unit norm.

To compute the similarity between representations in practice, we found that using either the negative L_1 distance or the negative L_2 distance was effective, depending on the environment. The contrastive encoder is trained to minimize the InfoNCE loss (Equation (15)) (Oord et al., 2018) . For each batch sample, the positive examples of other samples are treated as negatives, following common practice (Chen et al., 2020). The temperature parameter τ is learned during training as a learnable parameter. The details of the implementation are provided in Appendix A.

4.2 Extracting an exploration signal from the contrastive model

Given the contrastive model, a useful intrinsic reward can be constructed. We begin with intuition for a proper intrinsic reward to encourage exploration behavior. In this method, our aim is to reach unexpected but *meaningful* states. This is in contrast to surprise maximization or similar objectives which may prioritize unexpected but meaningless (i.e. random) states as in the noisy TV problem Gruaz et al. (2024).

Recall from the background section that training the critic function with the InfoNCE loss (Equation (15)) produces a similarity score between a state-action pair (s_t, a_t) and a future state s_f. This similarity score is proportional to the probability of reaching s_f from (s_t, a_t) based on the trajectory buffer. *Negating* this similarity score results in our exploration signal r_{intr}: this signal encourages the agent to visit states that, in expectation, *have previously had unpredictable futures*. Because the intrinsic reward has stochasticity from the future state sampling procedure (see Section 4.1), we write the expression for the expectation of r_{intr}:

$$\mathbb{E}[r_{\text{intr}}(s_t, a_t)] = \mathbb{E}_{p_\mathcal{T}(s_f|s_t,a_t)}\left[-C_\theta((s_t, a_t), s_f)\right] = \mathbb{E}_{p_\mathcal{T}(s_f|s_t,a_t)}\left[||\phi_\theta(s_t, a_t) - \psi_\theta(s_f)||\right]. \quad (5)$$

The norm can be taken to be the ℓ^1 norm or ℓ^2 norm (See Section 6). This reward may seem counterintuitive – we should, perhaps, prioritize reaching states with high empowerment or surprise minimization. However, we claim that Equation (5) rewards meaningful exploration: the reward can be interpreted as identifying possible inconsistencies in the contrastive model, where the contrastive model overestimates the distance between state-actions and their futures. If the policy *were* to easily reach future states that the internal model wrongly believes are distant, this would lead to a stronger update of the internal model. We expand on information-theoretic interpretations of r_{intr} in Section 5.

Algorithm 1 Curiosity-Driven Exploration via Temporal Contrastive Learning

Initialize: $\pi, \phi_\theta, \psi_\theta$,trajectory buffer $\mathcal{T}$
for each iteration **do**
  for each environment step $1 \leq t \leq T$ **do**
    $a_t \sim \pi(a_t \mid s_t)$
    $s_{t+1} \sim p(s_{t+1} \mid s_t, a_t)$
    $\tau_j \leftarrow \tau_j \cup \{s_t, a_t, s_{t+1}\}$
  $\mathcal{T} \leftarrow \mathcal{T} \bigcup \tau_j$
  Sample $\{(s_t^i, a_t^i)\}_{i=1}^{|\mathcal{B}|} \sim \mathcal{T}$ ▷ Sample a batch of state,action pairs
  Sample $\Delta_i \sim \textsc{Geom}(1-\gamma)\ \forall i \in \{1, 2, \ldots, |\mathcal{B}|\}$ ▷ Sample a geometric offsets
  Set $s_f^i = s_{t+\Delta_i}^i \forall i \in \{1, 2, \ldots, |\mathcal{B}|\}$ ▷ Set the future state s_{f_i} according to Δ_i
  Compute intrinsic rewards: $\mathbf{r}_i = -C_\theta((s_t^i, a_t^i), s_f^i)$ ▷ Equation (5)
  Update representations: $\theta \leftarrow \theta - \eta \nabla_\theta \mathcal{L}_{\text{InfoNCE}}(\mathcal{B} = \{(s_t^i, a_t^i, s_f^i)\}_{i=1}^{|\mathcal{B}|}; \theta)$ ▷ Equation (15)
  RL update using $\{(s_t^i, a_t^i, \mathbf{r}_t^i)\}_{i=1}^{|\mathcal{B}|}$ ▷ Update the policy using PPO/SAC

After learning the contrastive representations that define the r_{intr}, we train the parameterized policy $\pi(a_t \mid s_t)$ to maximize the discounted sum of rewards:

$$J(\pi) = \mathbb{E}_{\pi(a_t|s_t)}\left[\sum_{t=0}^{\infty} \gamma^t r_{\text{intr}}(s_t, a_t)\right]. \tag{6}$$

The experiments use PPO (Schulman et al., 2017) and SAC (Haarnoja et al., 2018b;a;c) for policy training (pseudocode in Algorithm 1). In practice, we found that using a single sample future state to approximate the expectation in Equation (5) works well, except in Craftax-Classic, where we used a Monte Carlo estimate. Additional details are provided in Section 5 and Appendix A.4.1.

4.3 Future states sampling

An important design choice is how to sample the future state when computing the intrinsic reward. For example, consistent sampling from states far in the future could lead to a high variance reward signal that might hinder the agent's learning, and sampling from the nearby states can be inefficient as the agent might already have a strong model over these states. One natural strategy is to sample according to the discounted occupancy measure, as described in Equation (1). Alternatively, we can sample uniformly from the future states (conditioned on the current state and action). We observe that sampling from the according to the discounted occupancy measure yields good performance across environments and we stick to this strategy in our experiments. We also show the performance differences between these sampling strategies in the experiments Section 6.3.

5 Interpretation of C-TeC

In addition to quantifying temporal similarity, the converged InfoNCE loss $\mathcal{L}^*_{\text{CRL}}$ provides a lower bound on the mutual information (MI) Oord et al. (2018); Eysenbach et al. (2021):

$$I(S_f; S_t, A_t) \geq \log K - \mathcal{L}^*_{\text{CRL}}(\mathcal{B}; \theta).$$

Thus, we can think of contrastive learning as finding representations that maximize a lower bound on the MI between *current* states and actions and *future* state distributions. Critically, these representations do not have full expressivity: the representation-induced critic function is a low-dimensional (in latent space) approximation of the point-wise MI, where the rank of the point-wise MI matrix is upper bounded by the representation dimension. In the below sections, we provide intuition for how the information theoretic interpretation of the intrinsic reward and parameterization via representations may explain effective exploration behavior.

5.1 Information-Theoretic Expression of C-TeC

The intrinsic reward has a corresponding information-theoretic interpretation. We begin by considering the limit that the representations perfectly capture the underlying point-wise MI. In this regime, the intrinsic reward evaluates to the negative of the KL-divergence between the conditional future-state distribution $p_{\mathcal{T}}(s_f \mid s_t, a_t)$ and the marginal future-state distribution $p_{\mathcal{T}}(s_f)$:

$$\begin{aligned}\mathbb{E}[r_{\text{intr}}(s_t, a_t)] &= -\mathbb{E}_{p_{\mathcal{T}}(s_f|s_t,a_t)}\Big[\log \frac{p_{\mathcal{T}}(s_f \mid s_t, a_t)}{p_{\mathcal{T}}(s_f)}\Big] \\ &= -D_{\text{KL}}[p_{\mathcal{T}}(s_f \mid s_t, a_t) \,||\, p_{\mathcal{T}}(s_f)] \leq 0. \qquad (D_{\text{KL}} \text{ is always non-negative.})\end{aligned} \tag{7}$$

This intrinsic reward leads to mode-seeking behavior: notably, the probability distribution $p_{\mathcal{T}}(s_f \mid s_t, a_t)$ only has support where $p_{\mathcal{T}}(s_f)$ has support, promoting a mode-seeking coverage of the conditional over the broader marginal state distribution. Since the sampled future states $s_f \sim p_{\mathcal{T}}(s_f \mid s_t, a_t)$ are temporally close to the state s_t, a_t and are not arbitrarily fit distributions, this optimization is distinct from minimizing the *forward* KL-divergence:

$$D_{\text{KL}}[p_{\mathcal{T}}(s_f) \,||\, p_{\mathcal{T}}(s_f \mid s_t, a_t)] \geq 0. \tag{8}$$

which instead prioritizes mean-seeking behavior over regions of the state space where the marginal $p_{\mathcal{T}}(s_f)$ may *not* have support.

This mode-seeking behavior can be interpreted as prioritizing new trajectories via (s, a) to previously encountered states (s_f). New conditioned trajectories are as spread out as possible within the marginal state distribution:

$$\begin{aligned}\mathbb{E}[r_{\text{intr}}(s_t, a_t)] &= -D_{\text{KL}}[p_{\mathcal{T}}(s_f \mid s_t, a_t) \,||\, p_{\mathcal{T}}(s_f)] \\ &= \underbrace{H[S_f \mid s_t, a_t]}_{\text{surprise}} + \underbrace{\mathbb{E}_{p_{\mathcal{T}}(s_f|s_t,a_t)}[\log p_{\mathcal{T}}(s_f)]}_{\text{“familiarity” term}},\end{aligned}$$

where S_f denotes the future state *random variable* and $s_f \sim p_{\mathcal{T}}(s_f \mid s_t, a_t)$. The first term is surprise, traditionally defined as $H[S' \mid s_t, a_t]$, and rewards a spread out future state distribution. The latter cross-entropy term encourages familiarity, only rewarding exploration over states more commonly seen during training. Thus, the intrinsic reward captures a form of surprise/state-coverage *regularized* with the marginal state distribution $p_{\mathcal{T}}(s_f)$ – the method prioritizes more unlikely trajectories over states that *have been seen before*, building a stronger model of states, actions, and transition dynamics. New states are then incidentally encountered during roll-out and added to the marginal, and the process repeats.

Taking an additional expectation of the rollout state-occupancy reveals that the PPO/SAC objective is a minimization of MI

$$J^{\pi} = \mathbb{E}_{p_{\pi}(s,a), p_{\mathcal{T}}(s_f|s,a)}[r_{\text{intr}}(s_t, a_t)] \approx -I[S_f; S_{\pi}, A_{\pi}]. \tag{9}$$

where p_{π} is the policy induced discounted state-occupancy measure (see 1.

To test the hypothesis that mode-seeking behavior is important to the success of C-TeC, we ran experiments where the intrinsic reward is the forward KL-divergence Equation (8). The results in Appendix C.5 show that the objective succeeds not because it is minimizing a divergence that promotes coverage of the marginal, but because it is minimizing this mode-seeking reverse KL-divergence.

Finally, successfully reaching these hard-to-compress trajectories should increase their presence in the marginal, leading to the prioritization of *other* trajectories in the next iteration of C-TeC. A linear stability analysis on the fixed points of C-TeC for a simplified setting is in Appendix E; there are generally no practical, stable fixed points, leading to sustained exploration behavior.

5.2 Representations are Necessary for C-TeC to Succeed

Finally, we note that all of the above analysis assumes a fully-expressive critic that captures the point-wise MI (e.g. a perfect table of relative probabilities). However, in practice, we are actually using low-dimensional representations to parameterize the intrinsic reward. We find that these representations are crucial for effective exploration. Experimental results in Appendix C.4 show that the method is *not* robust to the usage of a monolithic critic $f(s, a, g)$. This result suggests that temporal contrastive representations are crucial to providing useful exploration signal.

Concretely, the expected intrinsic reward evaluates to the following for compressed representations:

$$\mathbb{E}[r_{\text{intr},\phi,\psi}(s_t, a_t)] = -\mathbb{E}_{p_{\mathcal{T}}(s_f|s_t,a_t)}\Big[\log \frac{p_{\phi,\psi}(s_f \mid s_t, a_t)}{p_{\phi,\psi}(s_f)}\Big]$$

where $p_{\phi,\psi}$ describe the (relative) probability distributions under the learned contrastive representations ϕ and ψ. This object rewards (s, a) pairs that result in the largest additional number of bits needed to encode the representation induced $p_{\phi,\psi}(s_f \mid s_t, a_t)$ with a code optimized for the marginal $p_{\phi,\psi}(s_f)$. The policy reaches state-action pairs that have had hard-to-compress future trajectories relative to an encoding scheme optimized for the (representation compressed) marginal.

Finally, representation learning helps enable the agent to ignore spurious noise like Noisy TV (see Fig. 15). The learned contrastive representations keep track of bits that distinguish future from random states. Noise randomly sampled from the same distribution every timestep does not lead to stronger classifier performance/lower contrastive loss in expectation. Thus, the distance between representations should be roughly independent of the observed noise. The resulting intrinsic motivation and learned policy should be independent of this noise.

5.3 Comparison with Previous Methods

At a high level, C-TeC is related to other intrinsic exploration objectives that reward uncertainty. Objectives such as RND Burda et al. (2019b) and Disagreement Pathak et al. (2019) explore unfamiliar states, presumably leading to these states becoming more familiar in future rounds. Such methods avoid trivial fixed points found in count-based (lose reward signal after reaching a uniform distribution, not easily scalable) Ostrovski et al. (2017), curiosity (lose reward signal after perfectly capturing local transitions and struggle with Noisy TV) Gruaz et al. (2024), or empowerment objectives (focus in controllable, but not novel, areas) kly (2005).

However, the key difference between prior methods and C-TeC lies in the prioritization of states that have previously led to surprising future states relative to the marginal distribution in a compressed contrastive model. Our method drives the agent to explore areas where future outcomes have been seen but remain unpredictable: expanding state coverage while prioritizing familiarity of the state. This objective is markedly different from next-step prediction error Pathak et al. (2017); Burda et al. (2019a): C-TeC does not use a decoder in latent or real space, relies on learned temporal correlations via a lightweight contrastive objective, and is designed to leverage short and long-horizon relations between states beyond next-step transitions. Importantly, these temporal representations not only encourage the model to reach states that naturally "compress" these distant futures to appear closer/more represented in the marginal. Taken together, our analysis and results show that temporal contrastive representations are simple yet powerful frameworks for intrinsic motivation.

6 Experiments

Our experiments show that contrastive representations can be used to reward the agent for visiting less-occupied or distant future states. We then use the C-TeC reward function for exploration in robotic environments and Craftax-Classic. We mainly study the following questions: (**Q1**) How well does C-TeC reward capture the agent's future state distribution? (**Q2**) How effectively does C-TeC explore in locomotion, manipulation, and Craftax environments compared to prior work? (**Q3**) How sensitive is C-TeC to the future state sampling strategy?

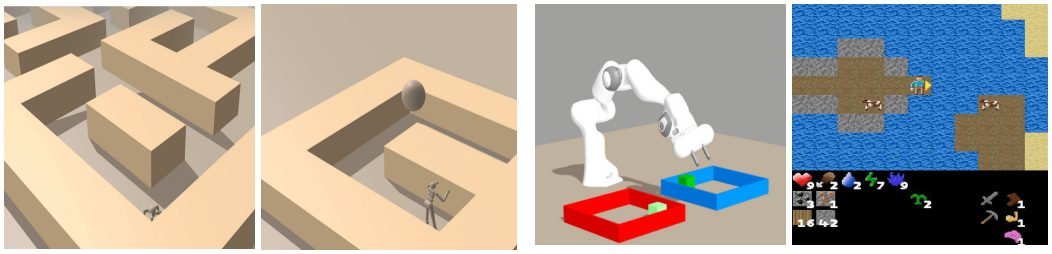

Figure 2: **Environments**. Maze coverage, robotic manipulation, and the survival game Craftax.

Environments We use environments from the JaxGCRL codebase (Bortkiewicz et al., 2025). Specifically, we evaluate C-TeC on the `ant_large_maze`, `humanoid_u_maze`, and `arm_binpick_hard` environments, which require solving long directed plans to reach goal states. In the maze-based environments, the agent's objective is to reach a designated goal specified at the start of each episode. Exploration in these settings corresponds to maze coverage: an agent that visits more unique positions in the maze demonstrates better exploration capabilities. In the `arm_binpick_hard` environment, which differs from the more navigation-themed tasks used in prior work, the agent must pick up a cube from a blue bin and place it at a specified target location in a red bin. This represents a challenging exploration task, as the agent must locate the cube, grasp it, and successfully place it at the correct target location.

Our experiments with the `ant` and `humanoid` agents assess the method's ability to achieve broad state coverage using two complex embodiments. Meanwhile, the `arm_binpick_hard` task evaluates the method's effectiveness at exploration in an object manipulation setting. We also run C-TeC on Craftax-Classic (Matthews et al., 2024), a challenging open-world survival game resembling a 2D Minecraft. The agent's goal is to survive by crafting tools, maintaining food and shelter, and defeating enemies

In the locomotion and manipulation environments, we compare C-TeC to common prior methods for exploration: **Random Network Distillation (RND)** (Burda et al., 2019b) and **Intrinsic Curiosity Module (ICM)** (Pathak et al., 2017) which are both popular intrinsic motivation methods for exploration. **Active Pre-training (APT)** (Liu & Abbeel, 2021): APT learns observation representations using contrastive learning, where positives are augmentations of the same observation and negatives are different observations. It uses the KNN distance between state representations as an exploration signal, which correlates with state entropy. Unlike C-TeC, APT does not learn representations predictive of the future. In Craftax, we compare against RND, ICM, and **exploration via elliptical episodic bonuses (E3B)** (Henaff et al., 2022), a count-based exploration method. We found that using the negative L_1 distance (Equation (4)) as the critic function works best in the robotics environments, while the negative L_2 distance (Equation (3)) performs best in Craftax. A comparison of different critic functions can be found in the appendix.

6.1 Capturing the future state distribution (Q1)

The goal of this experiment is to demonstrate that the C-TeC reward captures the future state distribution. As a result, it can be used to incentivize the agent to visit less-occupied and more distant future states. We visualize the C-TeC reward at different stages of training in the `ant_hardest_maze` environment. The contrastive critic is defined as the negative L_1 distance (Equation (4)), and the policy is trained to maximize the intrinsic reward defined in Equation (5). Figure 3 shows the reward values in a section of the maze, with the black circle in the lower-left corner indicating the starting state. In the early stages of training (3M steps), the reward is highest for nearby states. As training progresses, the agent explores farther, and the reward increases for more distant regions (e.g., at 400M and 500M steps). Over time, the reward becomes increasingly aligned with the maze's geometry.

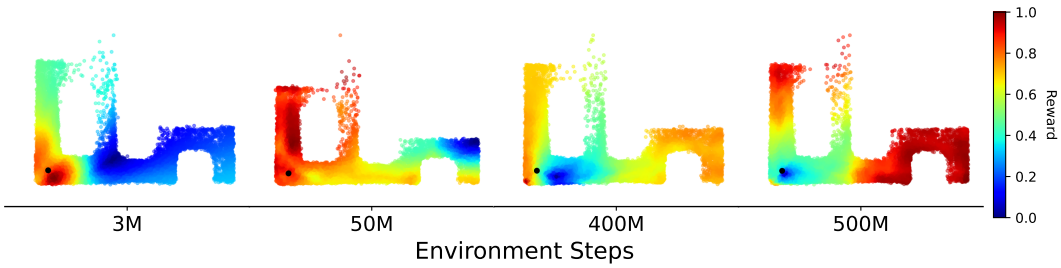

Figure 3: **Evolution of the C-TeC reward during training**. This figure shows how the intrinsic reward changes over the course of training based on future state visitation. The black circle in the lower-left corner represents the starting state. Early in training (3M steps), higher rewards are assigned to nearby states. As training progresses, the agent explores farther, and the reward increases for more distant regions. All reward values are normalized for visualization.

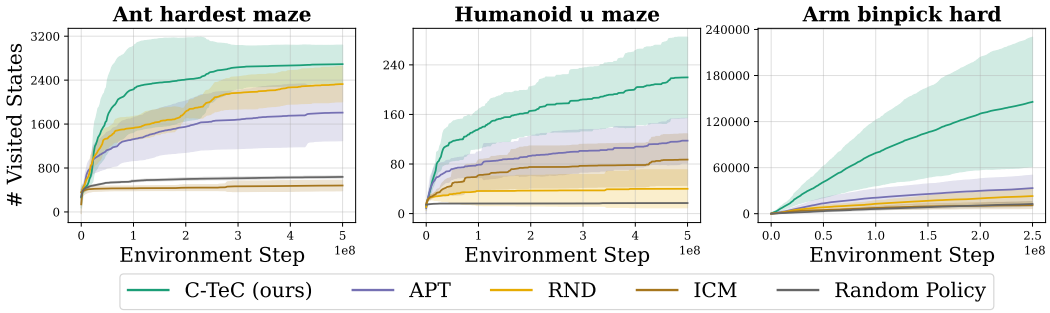

Figure 4: **C-TeC explores more states than prior methods.** We compare the state coverage of C-TeC to APT (Liu & Abbeel, 2021), RND (Burda et al., 2019b) and ICM (Pathak et al., 2017). We include a uniform random policy as well.

6.2 Exploration results (Q2)

In this experiment, we evaluate C-TeC in the `ant_large_maze`, `humanoid_u_maze`, and `arm_binpick_hard` environments. We run two variants of the experiment: (1) using the complete state vector as the future state, which is common in exploration tasks where the agent is encouraged to explore the entire state space; and (2) incorporating prior knowledge by narrowing the future state to specific components of the state vector. The latter allows us to assess whether C-TeC can flexibly explore subspaces of the state space, which is often useful in practice. In `ant_large_maze`, we define the future state as the future (x, y) position of the ant's torso. In `humanoid_u_maze`, we use the future (x, y, z) position of the humanoid's torso. Finally, in `arm_binpick_hard`, we define the future state as the future position of the cube.

As an evaluation metric, we count the number of unique discretized states covered by each agent. In `ant_large_maze`, we count the number of unique (x, y) positions in the maze visited by each agent. Similarly, in `humanoid_u_maze`, we count the number of visited (x, y, z) positions, and in `arm_binpick_hard`, we count the number of unique cube positions. We compare C-TeC to RND, ICM, APT, and a uniformly random policy. Figure 4 shows the learning curve when using the complete future state vector while Figure 5 shows the performance when we incorporate prior knowledge by restricting the future state to specific components of the state vector. . Each agent is run with 5 random seeds, and we plot the mean and standard deviation (Patterson et al., 2024).

Our agent outperforms the baselines in both variants of the experiment and learns interesting behaviors in the challenging `humanoid_u_maze` environment. Figure 6 shows screenshots of C-TeC behavior and more visuals are provided in Appendix F. This improvement can be the result of C-TeC's consistent reward properties. Methods like RND, ICM will eventually tend to a reward of 0 as the state distribution is covered. A nice property of C-TeC is that it does not have zero reward in the limit.[1]

[1]Videos are in the project website: `https://temp-contrastive-explr.github.io/`

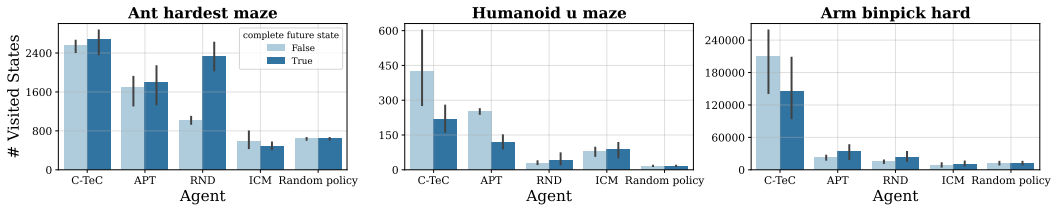

Figure 5: **State coverage when leveraging prior knowledge** C-TeC outperforms prior methods (Liu & Abbeel, 2021; Burda et al., 2018; Pathak et al., 2019) and can explore effectively when leveraging prior knowledge. This shows the flexibility of C-TeC in incorporating prior knowledge by narrowing the exploration space. Prior work does not offer this flexibility, as shown in the results.

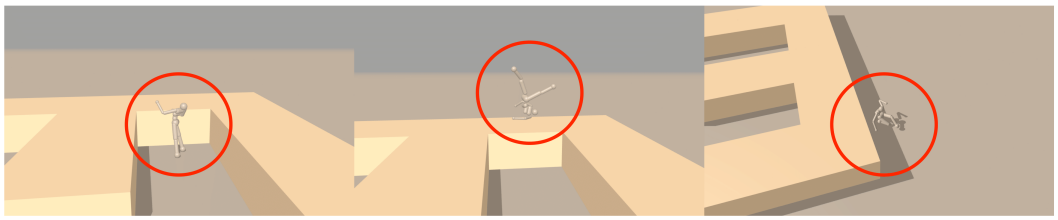

Figure 6: **C-TeC behavior in humanoid-u-maze**.C-TeC agent learns to escape the u-maze by jumping over the wall. None of the baseline intrinsic motivation methods discovered this kind of unexpected novel behavior.

6.2.1 Learning complex behavior in Craftax-Classic

Can an RL policy learn complex behavior in Craftax-Classic without any task reward? To answer this question, we run C-TeC on Craftax-Classic (Matthews et al., 2024), a complex survival game where the agent's goal is to survive by crafting tools, maintaining food and shelter, and defeating enemies.

In this experiment, we use the same PPO implementation as used in the baselines in the Craftax paper (Matthews et al., 2024), adding the C-TeC reward on top of it. We compare against RND, ICM, E3B, and a uniform random policy. We found that using PPO with memory (PPO-RNN) yields the best performance.

Figure 7 shows the results. The Y-axis represents the sum of the achievements success rate, which measures how many capabilities and useful objects the agent has discovered. C-TeC outperforms the baselines and unlocks more achievements. Figure 18 visualizes some of the achievements of the C-TeC agent during an evaluation episode.

6.3 Sensitivity to future state sampling strategy (Q3)

In this experiment, we investigate the sensitivity of C-TeC to the future state sampling strategy. Specifically, we consider two variants in addition to the geometric sampling: (1) sampling uniformly from the future. Unlike geometric sampling, uniform sampling does not prefer states that are sooner in the future over later ones. (2) geometric sampling with an increasing γ value. The intuition behind this strategy is that exploring nearby states is easier for the agent at the start of training, and as the agent becomes better at exploring them, it can progressively explore farther states in the future. We refer to this strategy as the γ-schedule, and we experiment with two different starting values of γ: one ranging from $\gamma = 0.9$ to $\gamma = 0.99$, and another from $\gamma = 0.1$ to $\gamma = 0.99$.

The results are shown in Figure 8. First, we note that regardless of the future state sampling strategy, the contrastive method explores better than the baselines in all three environments. However, the best-performing strategy is environment-specific. For example, in `ant_hardest_maze`, sampling according to the γ-schedule performs best, while in `arm_binpick_hard`, geometric sampling tends to perform slightly better. Overall, the method appears robust to future state sampling strategies.

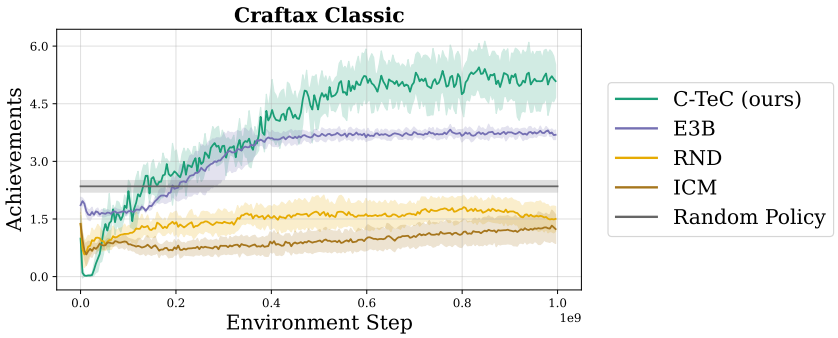

Figure 7: **Exploration in Craftax.** C-TeC outperforms the baselines in discovering more achievements in Craftax-Classic, E3B (Henaff et al., 2022) is the most competitive baseline.

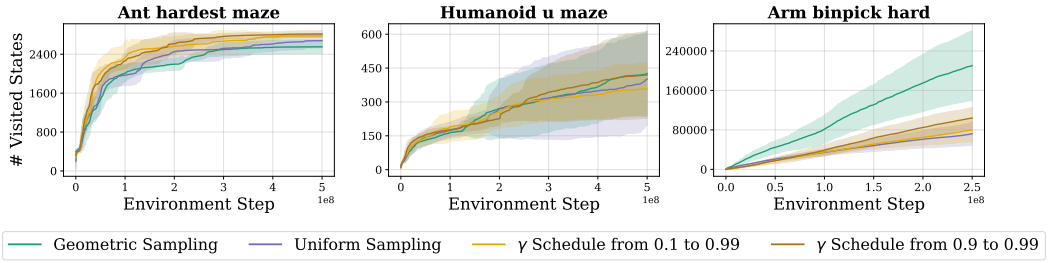

Figure 8: **Sensitivity to future state sampling strategy.** We compare variants of C-TeC with different future state sampling strategy, the method is robust to the choice of the sampling strategy and all the variants outperform the baselines.

7 Conclusion

This work has shown how to learn contrastive representations for intrinsic exploration. These representations over state action pairs and another model for future states can be combined to estimate the distribution over state visitation. It is shown that constructing a reward function that seeks out states with unexpected future states results in a significant performance gain over prior intrinsic objectives that also aim to estimate the state visitation. This is also shown via visualization of the state visitation distribution. These results are also demonstrated with different RL algorithms, and across environments, illustrating that the benefits are robust. In the future, this representation learning algorithm could be applied to image-based environments to better understand its capabilities on partially observed environments.

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Supplementary Materials

The following content was not necessarily subject to peer review.

Broader Impact Statement

This work proposes an exploration method for deep RL agents that facilitates finding better solutions across a broad range of sequential decision-making problems. Depending on the intended task and the reward function, the resulting policy may lead to either positive or negative consequences.

A Training Details and Ablations

We summarize the hyperparameters and model architectures for all experiments. In Appendix A.1, we provide the training details for the locomotion and manipulation experiments. In Appendix A.2, we provide the details of the Craftax experiments. In Appendix A.3, we provide the details of all the environments. In Appendix A.5, we include the codebase.

Finally, in Appendix C, we include the ablation experiments.

A.1 Robotics Environments

In the robotics environments, we used SAC as the RL algorithm. Table 1 shows the hyperparameters that are shared across all methods.Table 2 and Table 3 show the algorithm-specific hyperparameters for C-TeC and the baselines, respectively.

Table 1: Hyperparameters for all methods in robotics environments

Hyperparameter	Value
num_timesteps	500,000,000
max_replay_size	10,000
min_replay_size	1,000
episode_length	1,000
discounting	0.99
num_envs	1024 (256 for humanoid_u_maze)
batch_size	1024 (256 for humanoid_u_maze)
multiplier_num_sgd_steps	1
action_repeat	1
unroll_length	62
policy_lr	3e-4
critic_lr	3e-4
hidden layers (for both actor and critic)	[256,256]

Table 2: Hyperparameters for C-TeC in robotics environments

Hyperparameter	Value
contrastive_lr	3e-4
contrastive_loss_function	InfoNCE
similarity_function	L1
logsumexp_penalty	0.1
hidden layers (for both encoders)	[1024,1024]
representation dimension	64

Table 3: Hyperparameters for baselines in robotics environments

Hyperparameter	Value
rnd encoder lr	3e-4
rnd embedding dim	512
rnd encoder hidden layers	[256, 256]
icm encoder lr (forward and inverse models)	3e-4
icm embeddings_dim	512
icm encoders hidden layers	[1024, 1024]
icm weight on forward loss	0.2
apt contrastive lr	3e-4
apt similarity function	L1
apt contrastive hidden layers	[1024, 1024]
apt representation dimension	64
Augmentation type	$\mathcal{N}(0, 0.5)$

A.2 Craftax

In Craftax, we used PPO as the RL algorithm[2]. Table 4 shows the hyperparameters shared across all methods. Table 5 and Table 6 show the algorithm-specific hyperparameters for C-TeC and the baselines, respectively.

Table 4: Hyperparameters for all methods in robotics environments

Hyperparameter	Value
num_timesteps	1,000,000,000
num_steps	64
learning_rate	2e-4
anneal_learning_rate	True
update_epochs	4
discounting	0.99
gae_lambda	0.8
clip_epsilon	0.2
ent_coef	0.01
max_grad_norm	1.0
activation	tanh
action_repeat	1
RNN_layers (GRU)	[512 (embedding dim),512 (hidden dim)]
hidden layers (both actor and value)	[512, 512]

[2] https://github.com/MichaelTMatthews/Craftax_Baselines

Table 5: Hyperparameters for C-TeC in Craftax

Hyperparameter	Value
contrastive_lr	3e-4
contrastive_loss_function	InfoNCE
similarity_function	L2
logsumexp_penalty	0.0
hidden layers (for both encoders)	[1024,1024,1024]
representation dimension	64

Table 6: Hyperparameters for baselines in Craftax

Hyperparameter	Value
rnd encoder lr	3e-4
rnd embedding dim	512
rnd encoder hidden layers	[256, 256]
icm encoder lr (forward and inverse models)	3e-4
icm embeddings_dim	512
icm encoders hidden layers	[256, 256]
icm weight on forward loss	1.0
e3b (icm) lambda	0.1

A.3 Environment Details

- **Ant-hardest-maze** The observation space of this environment has 29 dimensions, consisting of joint angles, angular velocities, and the x,y position of the ant's torso. The action space is 7-dimensional, representing the torque applied to each joint.
- **Humanoid-u-maze** The observation space of this environment has 268 dimensions, consisting of joint angles, angular velocities, and the x,y position of the humanoid's torso. The action space is 17-dimensional, representing the torque applied to each joint.
- **Arm-binpick-hard** The observation space of this environment has 18 dimensions, consisting of joint angles, angular velocities, the cube position, and the end-effector position and offset. The action space is 5-dimensional, representing the displacement of the end-effector.
- **Craftax-Classic** The observation space is a one-hot encoding of size 1345, capturing player information (inventory, health, hunger, attributes, etc.) as well as the types of blocks and creatures within the player's visual field. The action space is discrete and consists of 17 actions.

A.4 Details on C-TeC reward

One important detail is that the policy's objective is slightly different from the (negative) representation objective (Equation (15)) because it omits the log-sum-exp term. This can be seen by rewriting the reward function as follows:

$$r_{\text{intr}}(s,a) = \mathbb{E}_{p_{\mathcal{T}}(s_f|s,a)}[\underbrace{\|\phi(s,a) - \psi(s_f)\| + \log \sum_{s'_f} e^{-\|\phi(s,a)-\psi(s'_f)\|}}_{\text{(neg) contrastive loss}} - \log \sum_{s'_f} e^{-\|\phi(s,a)-\psi(s'_f)\|}]. \tag{10}$$

To further gain intuition for what this is doing, we note that (in practice) the $\phi(s,a)$ representations are quite similar to the $\psi(s)$ representation evaluated at the same state. Thus, we can approximate

this second term as

$$\log \sum_{s'_f} e^{-\|\psi(s)-\psi(s'_f)\|} \approx \log \hat{p}(s), \tag{11}$$

which we identify as a kernel density estimate of the marginal likelihood of state s under the replay buffer distribution $p_{\mathcal{T}}(s)$. This observation helps explain why including the log-sum-exp term in the reward would degrade performance – it effectively corresponds to *minimizing* state entropy, which can often hinder exploration, especially in environments without much noise (Zheng et al., 2025). One additional consideration here is that, because the likelihood is measured using learned representations, it is sensitive to the policy's understanding of the environment. While ordinarily maximizing state entropy can lead to degenerate solutions (like the noisy TV), our approach mitigates this problem because the contrastive representations will only learn features that are predictive of future states (hence, they would ignore a noisy TV).

A.4.1 Variance reduction in the reward estimate

We can decrease the variance in our estimate of the expectation in Equation (5) by looking at all future states $s_f = s_{t+1}, s_{t+2}, \cdots$ and weighting each summand by γ^i:

$$\begin{aligned} r_t &= \mathbb{E}_{p(s_f|s_t,a_t)}[r_{\text{int}}(s_t, a_t, s_f)] && (12)\\ &= \mathbb{E}_{p(s_f|s_t,a_t)}[||\phi(s,a) - \psi(s_{t'})||_2] && (13)\\ &\approx \frac{1-\gamma^{H-t}}{1-\gamma} \sum_{t'=t}^{H} \gamma^{t'-t} ||\phi(s,a) - \psi(s_{t'})||_2 && (14) \end{aligned}$$

The (unbiased) approximation comes because we only look at future states that occur in one trajectory, and other trajectories might visit different future states. The ugly fraction is the normalizing constant for a truncated geometric series. In the last line, note that the summation $\sum_{t'=t}^{H} \gamma^{t'-t}\psi(s_{t'})$ can be quickly computed for every r_t by starting at $T = H$ and decrementing t, updating $\psi_{\text{sum}} = \psi(s_t) + \gamma\psi_{\text{sum}}$. This is the same trick that's usually used for computing the empirical future returns in REINFORCE, and decreases compute from $\mathcal{O}(H^2)$ to $\mathcal{O}(H)$. We use this estimator in Craftax-Classic but we found that omitting the normalization term results in much better performance.

A.5 Codebase

Our codebase for the robotics experiments and Craftax is provided below:

- **Robotics Environments** https://github.com/FaisalAhmed0/c-tec
- **Craftax** https://github.com/FaisalAhmed0/c-tec/tree/craftax

B Compute Resources

In all experiments, we use 2 CPUs, a single GPU, and 8 GB of RAM. The specific GPU type varies depending on the job scheduling system, but most experiments run on NVIDIA RTX 8000 or V100 GPUs. Training in the robotics environments takes approximately 24 hours on average, while Craftax experiments require around 30 hours.

C Ablation Study

To understand the contribution of each component to the overall performance of C-TeC, we conduct an ablation study on several key elements of the algorithm, illustraed in the following section.

C.1 Representation Normalization

Is it important to normalize the contrastive representations when computing the intrinsic reward? To answer this question, we compare the exploration performance of C-TeC across all environments, keeping all hyperparameters fixed except for the normalization of the representations.

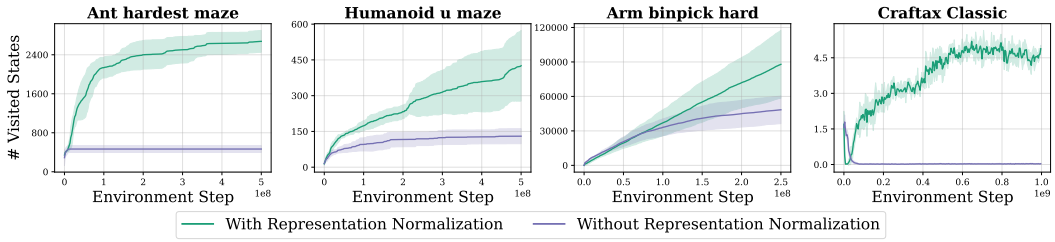

Figure 9: **Normalizing the contrastive representations.** Normalizing the representations is crucial for effective exploration—using unnormalized representations significantly degrades exploration performance.

C.2 Contrastive Losses

We compare the performance of C-TeC using different contrastive loss functions. Specifically, we evaluate InfoNCE, symmetric InfoNCE, NCE (Hjelm et al.), FlatNCE (Chen et al., 2021), and a Monte-Carlo version of the forward-backward (FB) Touati & Ollivier (2021) loss, as defined in [Equation (15)–Equation (19)]. Figure 10 presents the results. Overall, NCE leads to poorer exploration, particularly in Craftax. InfoNCE and symmetric InfoNCE exhibit similar performance across all environments. In general, the method is reasonably robust to the choice of contrastive loss.

$$\mathcal{L}_{\text{InfoNCE}}(\theta) = -\sum_{i=1}^{K} \log \left(\frac{e^{C_\theta((s_i,a_i),s_f^{(i)})}}{\sum_{j=1}^{K} e^{C_\theta((s_i,a_i),s_f^{(j)})}} \right) \tag{15}$$

$$\mathcal{L}_{\text{symmetric_InfoNCE}}(\theta) = -\left[\sum_{i=1}^{K} \log \left(\frac{e^{C_\theta((s_i,a_i),s_f^{(i)})}}{\sum_{j=1}^{K} e^{C_\theta((s_i,a_i),s_f^{(j)})}} \right) + \log \left(\frac{e^{C_\theta((s_i,a_i),s_f^{(i)})}}{\sum_{j=1}^{K} e^{C_\theta((s_j,a_j),s_f^{(i)})}} \right) \right] \tag{16}$$

$$\mathcal{L}_{\text{Binary(NCE)}}(\theta) = -\left[\sum_{i=1}^{K} \log \left(\sigma \left(C_\theta((s_i,a_i),s_f^{(i)}) \right) \right) - \sum_{j=2}^{K} \log \left(1 - \sigma \left(C_\theta((s_i,a_i),s_f^{(j)}) \right) \right) \right] \tag{17}$$

$$\mathcal{L}_{\text{FlatNCE}}(\theta) = -\sum_{i=1}^{K} \log \left(\frac{\sum_{j=1}^{K} e^{C_\theta(s_i,a_i,s_f^{(j)}) - C_\theta(s_i,a_i,s_f^{(i)})}}{\text{detach}\left[\sum_{j=1}^{K} e^{C_\theta(s_i,a_i,s_f^{(j)}) - C_\theta(s_i,a_i,s_f^{(i)})} \right]} \right) \tag{18}$$

$$\mathcal{L}_{\text{FB}}(\theta) = -\sum_{i=1}^{K} \left(e^{C_\theta(s_i,a_i,s_f^{(i)})} \right) + \frac{1}{2(K-1)} \sum_{i=1}^{K} \sum_{\substack{j=1 \\ j \neq i}}^{K} \left(e^{C_\theta(s_i,a_i,s_f^{(j)})} \right)^2 \tag{19}$$

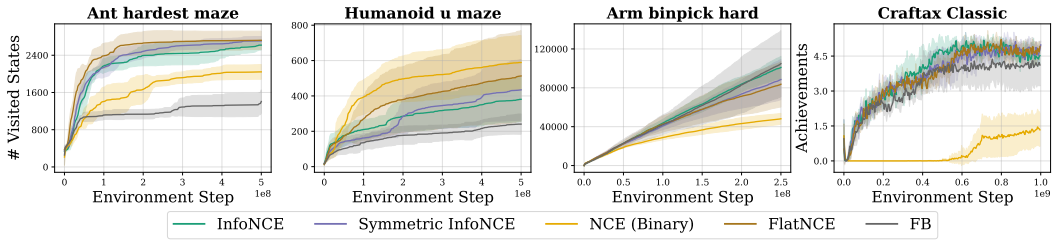

Figure 10: **Comparison of Different Contrastive Losses.** Overall, C-TeC is robust to the choice of contrastive loss. A notable exception is the Binary NCE loss in Craftax, where it performs relatively poorly.

C.3 Contrastive Critic Functions

We compare four critic similarity functions shown below:

$$C_\theta((s_t, a_t), s_f)_{L1} = -||\phi_\theta(s_t, a_t) - \psi_\theta(s_f)||_1. \tag{20}$$

$$C_\theta((s_t, a_t), s_f)_{L2} = -||\phi_\theta(s_t, a_t) - \psi_\theta(s_f)||_2 \tag{21}$$

$$C_\theta((s_t, a_t), s_f)_{L2-w/o-sqrt} = -||\phi_\theta(s_t, a_t) - \psi_\theta(s_f)||_2^2 \tag{22}$$

$$C_\theta((s_t, a_t), s_f)_{dot} = -\phi_\theta(s_t, a_t)^\top \psi_\theta(s_f) \tag{23}$$

Figure 14 shows the results. In general, using the L_1 distance yields the best performance across the robotic environments, while L_2 performs better in Craftax. This highlights the importance of this design choice and suggests that some tuning may be required to select the most effective critic function.

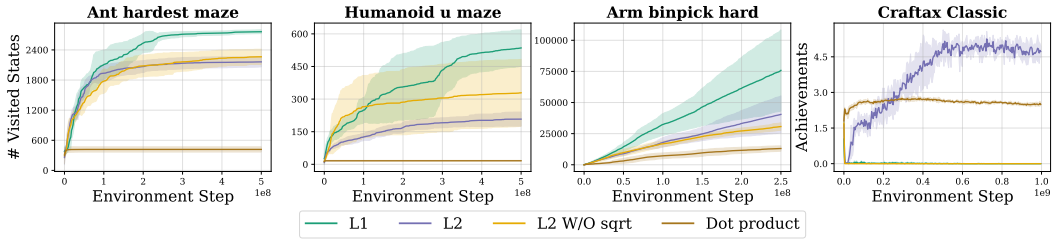

Figure 11: **Comparison of Critic function.** Overall,the L_1 distance yields the best performance across the robotic environments, while L_2 performs better in Craftax.

C.4 Contrastive Critic Architecture

In this ablation we compare two architectures of the contrastive critic, the separable architecture $(\phi_\theta(s_t, a_t), \psi_\theta(s_f))$, which is the one we use in all of our experiment, and the monolithic critic f_θ i.e., a single model that takes in triplet $f(s, a, s_f)$.

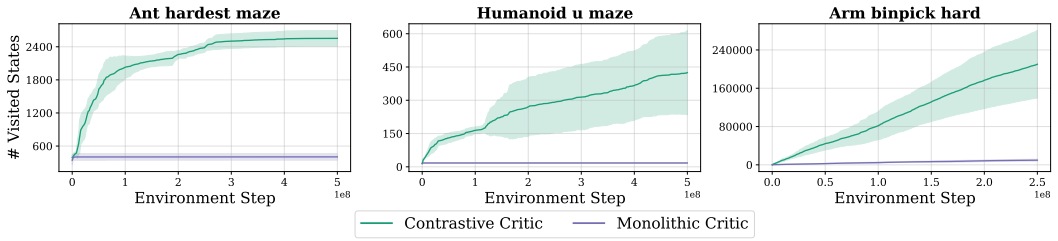

Figure 12: **Critic parameterization** Using a monolithic critic results in poor exploration performance, while using the separable architecture results in much better exploration. This shows the importance of the critic parameterization as distance function between two representations.

Importantly, these experiments show that the factorized representation parameterization is a necessary (relative to the monolithic critic) condition for effective exploration. We discuss the possible failure mode of using the monolithic critic in Section 5.2. These experiments do not demonstrate sufficiency, and we claim that the information-theoretic interpretation for a critic that fully captures the point-wise MI is still useful for analysis.

C.5 Forward vs reverse KL

As mentioned in Section 5.1, we hypothesized that the reverse KL C-TeC reward is important for exploration. As it encourages mode-seeking behavior (prioritizing unfamiliar states), to test this hypothesis we run C-TeC with the negative-forward KL reward (Equation (8)), the results shown in Appendix C.5 indicates that using the reverse is necessary for exploration.

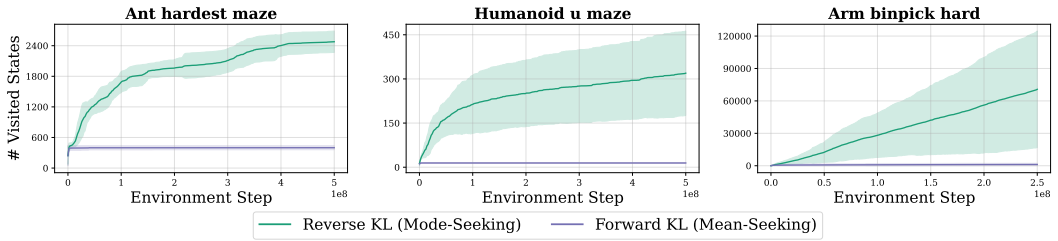

Figure 13: **Forward vs Reverse KL** C-TeC with the reverse KL reward promotes mode-seeking behavior which encourages the agent to prioritize visiting unfamiliar states resulting in much better exploration.

D Exploration in Noisy TV setting

We investigate C-TeC performance in the presence of a noisy TV state, we run this experiment on a modified grid environment from xland-minigrid (Nikulin et al., 2024) of size 256×256 Figure 15 with a noisy TV region. We did not observe any evidences of worse exploration performance namely the agent has covered all the states in the grid world, Appendix D shows the state coverage of C-TeC compared to the maximum coverage.

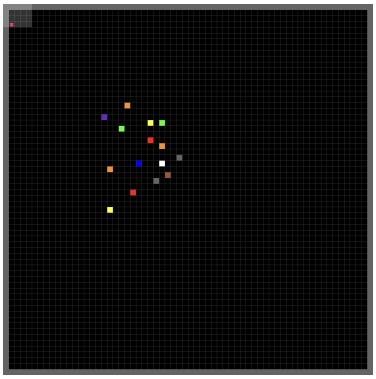

Figure 15: Xland-Minigrid (Nikulin et al., 2024) with noisy TV states indicated by the random colors.

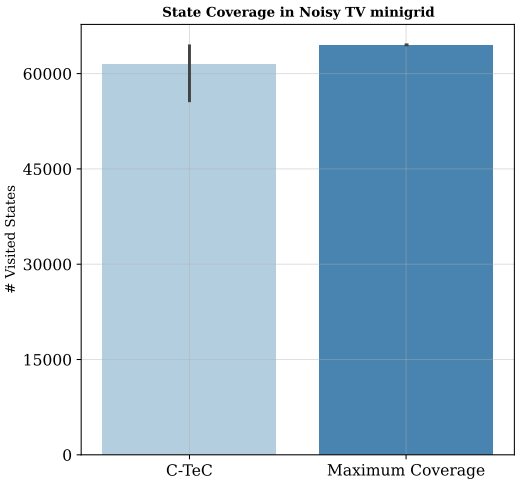

Figure 14: **C-TeC Coverage in noisy TV setting** C-TeC can effectively explore in the presence of noisy states

E No (Achievable) Trivial Fixed Points

One may ask: does C-TeC have stable fixed points? Without additional simplifications, this problem is intractable. Notably, standard analysis would fail to prove convergence due to the non-convexity/concavity of the objectives. While the zero-gradient condition for the InfoNCE objective is clear, the zero-gradient condition for the objective is not obvious due to the complex relationship between π and the state occupancy measure.

A more aggressive simplification that can simplify analysis of the global optimum is to (1) assume that the policy optimization is done directly over S_π and A_π and (2) assume the representations perfectly capture the point-wise MI. Furthermore, we assume that future states are exclusively sampled from the one-step transition dynamics and are deterministic.

In practice, these assumptions are very unrealistic; however, such simplifications have been used in prior work on unsupervised RL to give a conceptual picture of exploration methods Pitis et al. (2020). Throughout, we assume fully expressive representations that capture the point-wise MI – thus, we are strictly analyzing fixed points and fixed point-stability/achievability without taking into account representations.

Though the following analysis assumes a discrete setting (summations vs. integrals, Kronecker Deltas vs. Dirac Deltas) for cleaner math, we do not directly invoke the assumption of discreteness; the conclusions of a lack of a stable fixed point should hold in the continuous case.

With these simplifications, the InfoNCE objective reduces to:

$$\max_{\phi,\psi} \; [\log K - \mathcal{L}_{\phi,\psi}(Z_{\mathcal{T}}, F_{\mathcal{T}})] \underset{K\to\infty,\text{infinitely expressive reps}}{\longrightarrow} I\big(S_{\mathcal{T}}, A_{\mathcal{T}};\, S_f\big).$$

Because the "policy" optimization is fixed in $p(s_f \mid s, a)$, the MI $I(S_f; S_\pi, A_\pi)$ (see Eq. 9) is concave in $p_\pi(s, a)$ and $p_\pi(s_f)$ Cover & Thomas (1991). Our objective has now reduced to a constrained optimization problem with conditions $\sum_{s,a} p_\pi(s, a) = 1$ and $p_\pi(s, a) \geq 0$ for all $(s, a) \in \mathcal{S} \times \mathcal{A}$.

Consider the fixed point conditions given by the Lagrangian that is Lipschitz-continuous over the probability simplex $\Delta_{\mathcal{S}}$. Let λ and $\mu(s, a)$ denote the Lagrange multipliers for the normalization and non-negativity conditions respectively. Then, the full Lagrangian $\mathcal{L}_{\text{Lagrangian}}$ is as follows:

$$\mathcal{L}_{\text{Lagrangian}}(p_\pi, \lambda, \mu) = I(S_\pi, A_\pi; S_f) + \lambda\Big(\sum_{s,a} p_\pi(s, a) - 1\Big) - \sum_{s,a} \mu(s, a) p_\pi(s, a).$$

Note that by complementary slackness, we have $\mu(s, a)p(s, a) = 0$. Taking the functional derivative of $\mathcal{L}_{\text{Lagrangian}}$ with respect to distribution $p(s, a)$ yields the KL-divergence:

$$\frac{\delta \mathcal{L}_{\text{Lagrangian}}}{\delta p_\pi}[s, a] = D_{KL}[p_{\mathcal{T}}(s_f \mid s, a) || p_{\mathcal{T}}(s_f)] - 1 + \lambda - \mu(s, a).$$

By the complementary slackness, the distribution $p_\pi(s, a)$ is a fixed point if the KL-divergence $D_{KL}[p_{\mathcal{T}}(s_f \mid s, a) || p_{\mathcal{T}}(s_f)]$ is *constant* for any (s, a) where $p_\pi(s, a)$ has support. Any deviation would lead to a non-zero gradient at the point (s, a). In other words, all conditional trajectory future state distributions look equally "far" from the marginal.

Stationarity over *iterations* of C-TeC requires an additional condition: that the D_{KL} remains constant over all (s, a) *after* updating buffer $p_{\mathcal{T}}(s_f)$ with states encountered during the roll-out. A model of this is reweighing the marginal with the rollout probability distribution $p_\pi(s_f) = \sum_{s,a} p_\pi(s_f \mid s, a)p_\pi(s, a)$:

$$D_{KL}[p_{\mathcal{T}}(s_f \mid s, a) || p_{\text{new}}(s_f)] = D_{KL}[p_{\mathcal{T}}(s_f \mid s, a) || (1 - \alpha) \cdot p_{\mathcal{T}}(s_f) + \alpha \cdot p_\pi(s_f)] \tag{24}$$

where $0 < \alpha < 1$. Again, we assume deterministic dynamics for simplicity and that s_f is always the next state (i.e. small discount factor like the Craftax setting) so the conditional distribution does not change. We have no easy way of determining the change in $p_{\mathcal{T}}(s_f \mid s, a)$ after roll-out.

We drop subscripts on transitions and simplify:

$$\begin{aligned}
LHS &= \sum_{s_f} p(s_f \mid s, a) \log \frac{p(s_f \mid s, a)}{p_{\mathcal{T}}(s_f)} - \sum_{s_f} p(s_f \mid s, a) \log \frac{p_{\mathcal{T}}(s_f)}{(1 - \alpha) \cdot p_{\mathcal{T}}(s_f) + \alpha \cdot p_\pi(s_f)} \\
&= D_{KL}[p_{\mathcal{T}}(s_f \mid s, a) || p_{\mathcal{T}}(s_f)] - \mathbb{E}_{p(s_f|s,a)}\Big[\log \frac{p_{\mathcal{T}}(s_f)}{(1 - \alpha) \cdot p_{\mathcal{T}}(s_f) + \alpha \cdot p_\pi(s_f)}\Big] \\
&= C_{\text{old}} - \mathbb{E}_{p(s_f|s,a)}\Big[\log \frac{p_{\mathcal{T}}(s_f)}{(1 - \alpha) \cdot p_{\mathcal{T}}(s_f) + \alpha \cdot p_\pi(s_f)}\Big]
\end{aligned}$$

where C_{old} is the old constant D_{KL} across (s, a). Thus, for the LHS to also be constant across (s, a), the difference must also be constant. We assume that transitions are nontrivial (as in, $p(s_f \mid s, a) \neq p(s_f)$). This implies that the updated D_{KL} remains constant iff

$$\begin{aligned}
(1 - \alpha) \cdot p_{\mathcal{T}}(s_f) + \alpha \cdot p_\pi(s_f) &= p_{\mathcal{T}}(s_f) \\
\Rightarrow p_{\mathcal{T}}(s_f) &= p_\pi(s_f).
\end{aligned}$$

Under the assumptions of one-step, deterministic transitions and the α-reweighing of the buffer distribution, the distribution $p_\pi(s, a)$ remains a fixed point iff the roll-out future distribution and buffer future distribution are identical.

What is the stability of these fixed points? We can calculate the Jacobian of the update, where prime (') denotes the next-step $\delta p_\pi(s_f)$. The update of $\delta p_\pi(s_f)$ is as follows:

$$\begin{aligned}
\delta p'_\pi(s_f) &= \delta p_\pi(s_f) - \eta p(s_f \mid s, a)\Big[\Big(\nabla^2_{p_\pi(s,a)} I(S_\pi, A_\pi; S_f)\Big)\, \delta p_\pi\Big](s, a) && (25)\\
&= \delta p_\pi(s_f) - \eta\Big[\Big(\nabla^2_{p_\pi(s_f)} I(S_\pi, A_\pi; S_f)\Big)\, \delta p_\pi\Big](s_f) && \text{(change of vars.)}\\
&= \Big(I - \eta \nabla^2_{p_\pi(s_f)} I(S_\pi, A_\pi; S_f)\Big)\delta p_\pi(s_f), && (26)
\end{aligned}$$

We can similarly calculate the update for $\delta p_\mathcal{T}(s_f)$:

$$\begin{aligned}
\delta p'_\mathcal{T}(s_f) = \alpha\Big(I - \eta \nabla^2_{p_\pi(s_f)} I(S_\pi, A_\pi; S_f)\Big)\, \delta p_\pi(s_f) && \text{(weight new traj.)}\\
+ (1-\alpha)\delta p_\mathcal{T}(s_f). && \text{(down-weight old traj.)}
\end{aligned}$$

The linear stability analysis of p_π is as follows:

$$\begin{pmatrix} \delta p'_\pi(s_f) \\ \delta p'_\mathcal{T}(s_f) \end{pmatrix} = \underbrace{\begin{pmatrix} I - \eta H & 0 \\ \alpha\,(I - \eta H) & (1-\alpha)\, I \end{pmatrix}}_{J} \begin{pmatrix} \delta p_\pi(s_f) \\ \delta p_\mathcal{T}(s_f) \end{pmatrix}.$$

where H is the Hessian of the MI with respect to $p_\pi(s_f)$. Because the MI is concave in $p_\pi(s_f)$, the Hessian H is negative semi-definite. Thus, non-vertex fixed points on the dual probability simplex $\Delta_{\mathcal{S},\pi} \times \Delta_{\mathcal{S},\mathcal{T}}$ (where $p_\mathcal{T} = p_\pi(s_f)$) are either unstable, where at least one direction corresponds to a > 1 eigenvalue in the Jacobian, or semi-stable fixed points, where the MI is locally flat at the fixed point. Finally, fixed points at the vertices of the probability simplex (Delta functions) are uninteresting and are not observed in practice.

For an arbitrary MDP, we note that semi-stable fixed points are generally hard to achieve: a nontrivial, non-constant transition function, random roll-outs at initialization, the mixture of policies in the buffer, and randomly encountered new states prevent such semi-stable states from being easily accessible.

This analysis shows that there are no stable fixed points for standard MDPs even under aggressive simplifications, implying constantly evolving probability distributions. Future work remains to investigate the existence of dynamical steady-states and whether the reached probability distributions cover a large region of the probability simplex.

F Emergent Exploration Behavior

Figure 17 shows some of the learned behaviors of C-TeC in the `humanoid-u-maze`, where the agent learns to jump over the wall to escape the maze.

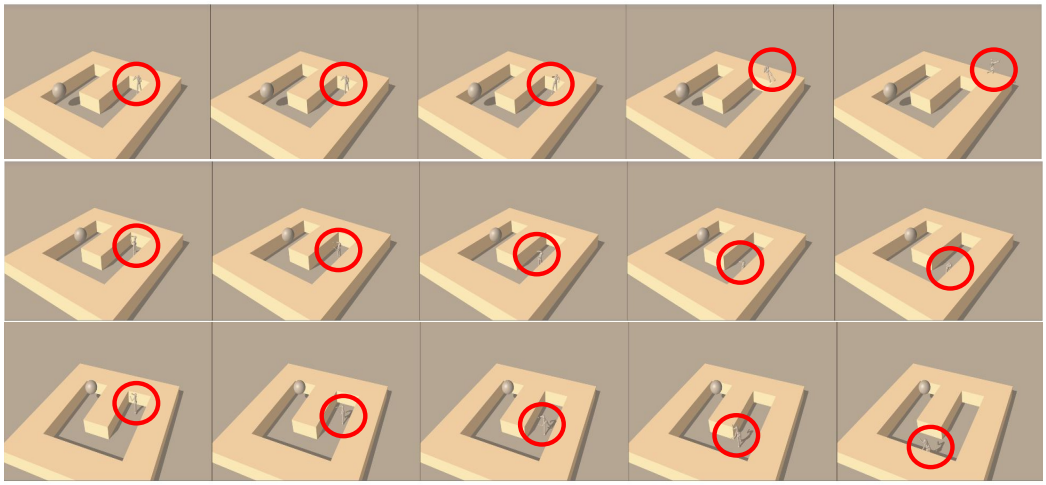

Figure 16: **Emergent Exploration Behavior in `humanoid-u-maze`.** C-TeC exhibits interesting emergent behaviors; for example, in the `humanoid-u-maze` environment, the agent learns to jump over the maze walls to escape the maze. Each row represents an independent evaluation epsidoe.

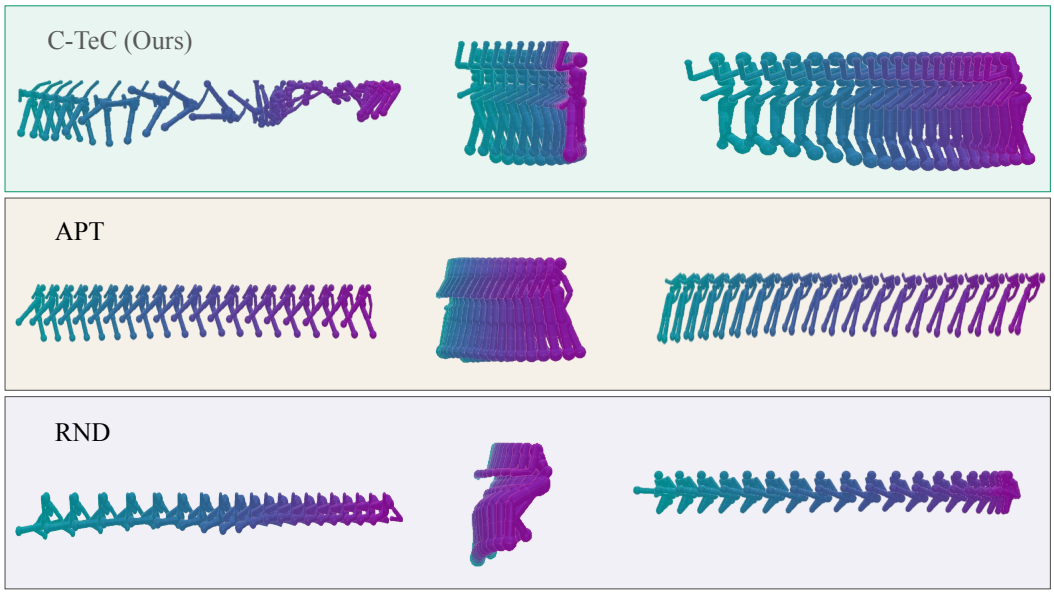

Figure 17: **Qualitative Comparison in `humanoid-u-maze`.**

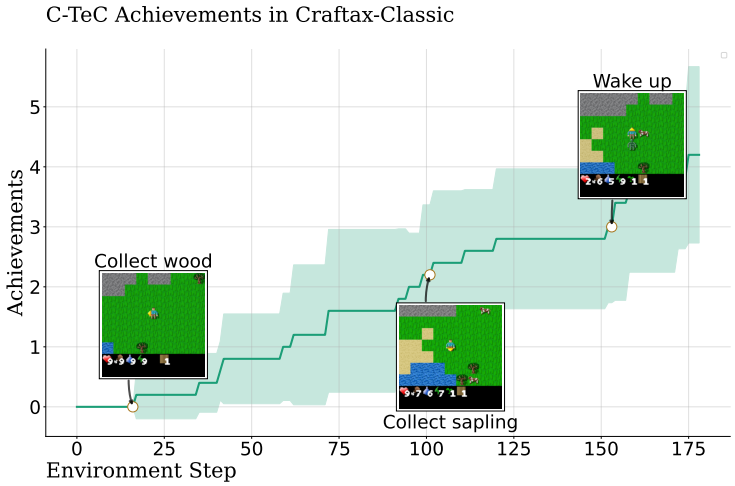

Figure 18: **C-TeC Achievements.** C-TeC unlocks interesting achievements in Craftax-Classic; the plot shows a subset of the unlocked achievements during an evaluation episode.