DEEP REINFORCEMENT LEARNING WITH ADAPTIVE COMBINED CRITICS

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ABSTRACT

The overestimation problem has long been popular in deep value learning, because function approximation errors may lead to amplified value estimates and suboptimal policies. There have been several methods to deal with the overestimation problem, however, further problems may be induced, for example, the underestimation bias and instability. In this paper, we focus on the overestimation issues on continuous control through deep reinforcement learning, and propose a novel algorithm that can minimize the overestimation, avoid the underestimation bias and retain the policy improvement during the whole training process. Specifically, we add a weight factor to adjust the influence of two independent critics, and use the combined value of weighted critics to update the policy. Then the updated policy is involved in the update of the weight factor, in which we propose a novel method to provide theoretical and experimental guarantee for future policy improvement. We evaluate our method on a set of classical control tasks, and the results show that the proposed algorithms are more computationally efficient and stable than several existing algorithms for continuous control.

INTRODUCTION

The task of deep reinforcement learning (DRL) is to learn good policies by optimizing a discounted cumulative reward through function approximation. Although a variety of control tasks have gained success through DRL Mnih et al. (2013; 2015); Van Hasselt et al. (2016); Wang et al. (2015); Schaul et al. (2015); Lillicrap et al. (2015); Mnih et al. (2016), there still exists biases caused by function approximation errors, studied in prior work Mannor et al. (2007).

Overestimation bias is rooted in Q-learning by consistently maximizing a noisy value estimate, and was originally reported to be present in the algorithms, typically, like deep Q-network (DQN) Mnih et al. (2015) which is aimed for discrete control tasks. Since DQN adopts neural networks to approximate a cumulative future reward, it is unavoidable that the noise evolved from function approximation accompanies the whole training. Besides, DQN updates its policy by choosing the action that maximizes the value function, which may bring the inaccurate value approximation that outweighes the true value, i.e., the overestimation bias. This bias further accumulates at every step via bootstrapping of temporal difference learning, which estimates the value function using the value estimate of a subsequent state.

When the overestimation bias is harmful to a certain task, it will cause instability, divergence and suboptimal policy updates. To solve this, Double deep Q-networks (DDQN) Van Hasselt et al. (2016) alters the policy update strategy by taking actions based on a relatively independent target value function instead of maximizing the original Q network. DDQN not only yields more accurate value estimates, but leads to much higher scores on several games Van Hasselt et al. (2016). However, due to the randomness of approximation errors, DDQN provides no theoretical bounds for the overestimation, and the unresolved overestimation or induced underestimation makes the performance of DDQN worse than DQN in some cases, reported in Van Hasselt et al. (2016).

For the case of high-dimensional, continuous action spaces, Deep Deterministic Policy Gradient (DDPG) Lillicrap et al. (2015) provides a model-free, off-policy actor-critic method using deterministic policy gradients. Due to the slow-changing policy in an actor-critic setting, the current and target value estimates will be too dependent to circumvent bias if the solution of DDQN to overestimation is directly under the continuous control setting. Inspired by Double Q-learning Hasselt (2010)

which employs a pair of independently trained critics to achieve a less biased value estimation, the twin delayed deep deterministic policy gradient algorithm (TD3) Fujimoto et al. (2018) proposed a clipped Double Q-learning variant choosing the lower target value as an approximate upper-bound to estimate the true current value, which favors underestimation biases that will not accumulate during training because actions with high value estimation are preferred. However, there are two problems lying in TD3. First, by taking actions towards the maximization of lower action-value (Q-value) at every step, the policy improvement cannot be guaranteed, which may cause potential suboptimal policies and instability. Second, using the same target value to update two critic networks will make them less independent.

The paper has the following contributions. First, we propose a combined value of two independent critics connected by a weight factor, and use it to update the policy instead of serving as a shared target estimate for the two critics, to avoid losing independence. Second, we propose a sign multiplier which determines whether the updated combined value has increased. The objective function for updating the weight factor is the product of the sign and the combined value evaluated by the updated policy. Third, we present a novel algorithm for continuous control, which can minimize the overestimation bias while providing guarantee for future policy improvement. And theoretical proofs show that the proposed algorithm has the property of asymptotical convergence and expected policy improvement. Fourth, we further apply the proposed algorithm to an unbiased framework to create another algorithm, which can remove the systematic bias due to the probability mismatch between the behavior policy and the target policy existing in off-policy methods. Fifth, extensive evaluations are conducted to compare the proposed algorithms with some baseline algorithms, in terms of computational efficiency, convergence rate and stability.

BACKGROUND

The task of reinforcement learning (RL) is to learn an optimal policy that maximizes a single return, i.e., the expected discounted cumulative reward of an episode. During an episode, the agent continually receives a sequence of observations by interacting with the environment before encountering an terminated state or the timeout. The return is calculated as the expected sum of future rewards. DRL combines the neural networks with RL so that the return can be approximated by the parameterized function. In DRL, the agent follows a behavior policy to determine future rewards and states, and store these observations in a memory, which will be randomly sampled over to train the network parameters and update the target Q-values. The target updates can be immediate, delayed or "soft".

Generally, the Q-value of DRL is represented as the expected discounted cumulative reward, an estimate function with respect to the state and action, which is given by

$$Q_\pi(s,a) = \mathbb{E}_{p^\pi(h|s_0,a_0)}\left[\sum_{t=0}^{\infty}\gamma^t r(s_t,a_t)|s_0=s,a_0=a\right], \tag{1}$$

where $r(s,a)$ is the immediate reward which is usually connected to the state-action pair, (s,a) is the value of initial state-action pair, and $\gamma \in (0,1)$ is the discount horizon factor for future rewards. Under the guidance of behavior policy π, $p^\pi(h|s_0,a_0)$ is the joint probability of all state-action pairs during an episode given the initial state-action pair (s_0,a_0).

When neural networks are used to approximate Q-values, the update of behavior policy π is closely related to the Q network under the setting of Markov Decision Process (MDP). The Q-value function in (1) takes the state-action pair (s,a) as input and maps it to the Q-value. The foundation for updates of network parameters is the Bellman equation Gattami (2019). Most of existing DRL algorithms adopt an independent target Q network to approximate an essential part of the target value, which can be set based on Bellman equation and organized to form the general loss function for DRL as

$$L(\omega) = \mathbb{E}_{(s,a,r,s')}\left[(r+\gamma Q(s',\mu(s')|\omega') - Q(s,a|\omega))^2\right], \tag{2}$$

where a is the action drawn from a behavior policy based on s, r and s' are the immediate reward and next state received by interacting with the MDP environment, respectively. Overall, (s,a,r,s') is the transition slot at every step. Besides, $\mu(s')$ is the target policy mapping s' to the next action a', which is normally different from the behavior policy in off-policy algorithms. Moreover, ω is the parameter of current Q network, which is normally different from the target network parameter ω'.

Once the Q networks are updated, the objective at the current iteration is to find the optimal policy $\mu(\cdot)$. In actor-critic methods, the policy is deterministic and commonly parameterized as the actor network $\mu(\cdot|\theta)$. Then the parameter θ is updated by maximizing the expected return $J(\theta) = \mathbb{E}_s\left[Q_\pi(s, \mu(s|\theta)|\omega)\right]$. For the case of continuous control, the actor parameter θ can be updated by gradient descent $\nabla_\theta J(\theta)$.

ADAPTIVE DELAYED DEEP DETERMINISTIC POLICY GRADIENT

In TD3, the clipped variant of Double Q-Learning is proposed for actor-critic to reduce the overestimation bias. Instead of using a pair of actors and critics to learn twice, TD3 upper-bounds the less biased value estimate by taking the minimum between two critic networks to offer target estimation for the update of future critics. Inspired by this work, we also adopts two critic networks and their target estimates, which are parameterized by ω, Ω, ω' and Ω', respectively. Since we plan to deal with our problem through DRL, we can substitute sampled minibatches into the loss functions of action-values (2), which are given by

$$L(\omega) = \frac{1}{N}\sum_{n=1}^{N}(r_n + \gamma Q(s'_n, a'_n|\omega') - Q(s_n, a_n|\omega))^2, \tag{3}$$

$$L(\Omega) = \frac{1}{N}\sum_{n=1}^{N}(r_n + \gamma Q(s'_n, a'_n|\Omega') - Q(s_n, a_n|\Omega))^2, \tag{4}$$

where $a'_n = \mu(s'_n|\theta')$ is the action taken from the target actor network, and (s_n, a_n, r_n, s'_n) is the n-th transition slot of the minibatch.

In TD3, the minimum of two critics is employed to serve as the target estimation value. However, updating two Q networks according to the same target estimate will make them less independent, which will further negatively affect the training efficiency. Besides, it is more reasonable to apply the clipped variant of Double Q-Learning to the procedure of actor update. Inspired by the Lagrange relaxation applied in constrained MDP problems Tessler et al. (2018), we propose a dual form of combined value function via a weight factor to determine actions reducing the harmful effect from overestimation. The dual form of combined value function for policy update is given by

$$\pi^* = \arg\min_{0\le\lambda\le1}\max\left[(1-\lambda)Q^1_\pi(s) - \lambda Q^2_\pi(s)\right], \forall s \in \chi \setminus \chi'. \tag{5}$$

where χ' be the set of transient states in continuous compact state space χ, and the weight factor λ determines the influence of two Q functions. Specifically, (5) will reduce to a normal policy evaluation with one critic when $\lambda = 0$ or $\lambda = 1$. Generally, (5) needs to be solved by a two-timescale approach that may result in a saddle point problem, i.e., on the faster timescale, the policy π or its parameter is solved by (5) when fixing λ, while on the slower timescale, λ is slowly increased until the overestimation error is minimized without losing the optimal solution to (5). Due to the potential non-convexity of the action-value function, the method may lead to a sub-optimal solution, and even cause instability when the convergence is slow.

To avoid the potential saddle point problem, we propose a two-step separation method to solve (5) for each policy update, which determines the optimal action to restrain overestimation based on the target weight factor λ' and then use the up-to-date policy to update λ. The process of this separation method is given by

$$J(\theta) = \frac{1}{N}\sum_{n=1}^{N}\left[\overline{\lambda'}Q(s_n, a_n|\omega) + \lambda' Q(s_n, a_n|\Omega)\right], \tag{6}$$

$$J(\lambda) = \frac{1}{N}\sum_{n=1}^{N}\left[\overline{\lambda}Q(s_n, a_n|\omega) + \lambda Q(s_n, a_n|\Omega)\right], \tag{7}$$

where $0 \le \lambda' \le 1$, $\overline{\lambda'} = 1 - \lambda'$, $\overline{\lambda} = 1 - \lambda$, $a_n = \mu(s_n|\theta)$. λ' is the target weight factor, which can be updated from λ by the "soft" target updates. The objective function for θ updating in (6) adopts

the target weight factor λ'. Then $(\omega', \Omega', \theta')$ are soft updated by (ω, Ω, θ), in the way of

$$\begin{aligned}
\omega'_i &\leftarrow \tau \arg\min_{\omega_i} L(\omega_i) + (1-\tau)\omega'_{i-1}, \\
\Omega'_i &\leftarrow \tau \arg\min_{\Omega_i} L(\Omega_i) + (1-\tau)\Omega'_{i-1}, \\
\theta'_i &\leftarrow \tau \arg\max_{\theta_i} J(\theta_i) + (1-\tau)\theta'_{i-1},
\end{aligned} \tag{8}$$

where $\tau < 1$ is the factor to control the speed of policy updates for the sake of small value error at each iteration, and λ' is left for further updating after θ'.

Although the underestimation bias accompanying the minimization operator is far preferable to overestimation and does not explicitly propagate through the policy update, it indeed negatively affect the policy improvement at every iteration and further brings fluctuation on algorithm convergence. The policy improvement means that the optimized objective function in (5) should steadily increase during training. The insurance of this value improvement lies in the choice of weight factor λ in (7). We provide the guarantee of policy improvement by multiplying (7) by a sign, which is given by

$$sign_i = \mathbb{I}\left(\frac{1}{N}\sum_{n=1}^{N}\left[\min\left(Q(s_n, a_{n,i}|\omega_i), Q(s_n, a_{n,i}|\Omega_i)\right) - \hat{Q}(s_n, a_{n,i-1}|\omega_{i-1}, \Omega_{i-1}, \lambda_{i-1})\right]\right), \tag{9}$$

where

$$\hat{Q}(s, a|\omega, \Omega, \lambda) = (1-\lambda)Q(s, a|\omega) + \lambda Q(s, a|\Omega), \tag{10}$$

$a_{n,i} = \mu(s_n|\theta_i)$, $a_{n,i-1} = \mu(s_n|\theta_{i-1})$, and $\mathbb{I}(x)$ is 1 when x is negative, and vice versa. The sign in (9) is actually the comparison between the minimum updated Q-value in two critics and the combined value in (6) before the policy update. And then the value of target weight factor λ' can be updated as

$$\lambda'_i \leftarrow \tau \arg\min_{\lambda_i} \left[sign_i * J(\lambda_i)\right] + (1-\tau)\lambda'_{i-1}, \tag{11}$$

where the objective function to update λ_i is the product of (7) and (9), i.e., $sign_i * J(\lambda_i)$. We organize the above procedures as the adaptive delayed deep deterministic policy gradient (AD3) algorithm, whose pseudocode is described by Algorithm 1.

Theorem 1. *AD3 algorithm asymptotically converges as the iteration $i \to \infty$ with properly chosen learning rate.*

Theorem 2. *AD3 algorithm has the property of asymptotical expected policy improvement. Specifically, when the critics tend to converge, i.e., $\exists K$, $\forall i \geq K$, $\forall \varepsilon > 0$,*

$$\left|\mathbb{E}_{(s,a)}\left[\hat{Q}(s, a|\omega_{i+1}, \Omega_{i+1}, \lambda_i) - \hat{Q}(s, a|\omega_i, \Omega_i, \lambda_i)\right]\right| < \varepsilon, \tag{12}$$

then

$$\mathbb{E}_{(s,a)}\left[\hat{Q}(s, a|\omega_{i+1}, \Omega_{i+1}, \lambda_{i+1})\right] \geq \mathbb{E}_{(s,a)}\left[\hat{Q}(s, a|\omega_i, \Omega_i, \lambda_i)\right]. \tag{13}$$

The proof of Theorem 1 and Theorem 2 can be found in the Appendix.

UNBIASED ADAPTIVE DELAYED DEEP DETERMINISTIC POLICY GRADIENT

To further improve the performance of proposed AD3 algorithm, we employ AD3 under the framework of unbiased DRL (UDRL) Zhang & Huang (2020). UDRL attempts to solve the systematic bias induced by the approximation of MDP samples. This systematic bias happens in the experience replay mechanism without importance sampling (IS) Precup et al. (2000); Hachiya et al. (2008); Mahmood et al. (2014); Thomas & Brunskill (2016); Jiang & Li (2016); Wang et al. (2016); Foerster et al. (2017); Metelli et al. (2018), because there exists mismatch between distributions of the target policy and the behavior policy. Without IS to weight the transition slots with different probabilities in commonly applied off-policy methods, the experience replay that memorizes the past

Algorithm 1 AD3 Algorithm

Input: The batch size N, the maximum of updates M, the timeout step T, and the soft update parameter τ.
Initialization: Initialize parameters $(\omega, \Omega, \theta, \lambda) \leftarrow (\omega_0, \Omega_0, \theta_0, \lambda_0)$, $(\omega', \Omega', \theta', \lambda') \leftarrow (\omega'_0, \Omega'_0, \theta'_0, \lambda'_0)$ randomly; Initialize replay buffer R, the counter $i \leftarrow 0$.
while $i < M$ **do**
  Reset randomly the initial state s_1.
  for $t = 1, T$ **do**
    Select action a_t according to the current behavior policy, i.e., $\mu(s_t|\theta_i)$ added by exploration noise;
    Execute actions a_t, get next states s_{t+1}, and immediate reward r_t;
    Store transition (s_t, a_t, r_t, s_{t+1}) in R;
    if R is full **then**
      Randomly and uniformly sample the slot (s_i, a_i, r_i, s_{i+1}) from R;
      Minimize the Q_1 loss function shown in (3) by gradient decent, and then update ω_i;
      Minimize the Q_2 loss function shown in (4) by gradient decent, and then update Ω_i;
      Maximize the expected return shown in (6) by gradient ascent, and then update θ_i;
      Minimize the product of (7) and (9) by gradient ascent, and then update λ_i;
      Execute the "soft" target updates shown in (8) to update θ'_i, ω'_i, Ω'_i, and update λ'_i according to (11);
      $i \leftarrow i + 1$;
    end if
  end for
end while

observations for random samples will accumulate the systematic errors and lower the convergence performance. When applying UDRL, the independently and identically distributed (IID) initial states are parallelly sampled to start respective transition slots at the beginning of each iteration. Then the parallel virtual agents follow the same behavior policy to complete their transition slots, which serve as the observations to synchronously train and update the shared network parameters.

In the case of unbiased AD3 (UAD3) method, the parallelly sampled IID observations should be used to train the two critic networks, an actor network and a weight factor. At each iteration, the actions are taken following the same behavior to receive rewards and next states, so that the achieved four-tuple transition slots are independent and follow the same joint probability. By this means, no IS is required in the approximations of (3), (4), (6), (7) and (9). The pseudocode of UAD3 is organized as Algorithm 2.

Algorithm 2 UAD3 Algorithm

Input: The batch size N, the maximum of updates M, and the soft update parameter τ.
Initialization: Initialize parameters $(\omega, \Omega, \theta, \lambda) \leftarrow (\omega_0, \Omega_0, \theta_0, \lambda_0)$, $(\omega', \Omega', \theta', \lambda') \leftarrow (\omega'_0, \Omega'_0, \theta'_0, \lambda'_0)$ randomly.
for $i = 1, M$ **do**
  Sample $S_i = (s_{i,1}, s_{i,2}, \cdots, s_{i,N})$ IID;
  Choose actions $A_i = (a_{i,1}, a_{i,2}, \cdots, a_{i,N})$ for S_i according to the current actor network $\mu(S_i|\theta_i)$ added by exploration noise;
  Execute actions A_i, get next states $S'_i = (s'_{i,1}, s'_{i,2}, \cdots, s'_{i,N})$ and immediate rewards $R_i = (r_{i,1}, r_{i,2}, \cdots, r_{i,N})$;
  Minimize the Q_1 loss function shown in (3) by gradient decent, and then update ω_i;
  Minimize the Q_2 loss function shown in (4) by gradient decent, and then update Ω_i;
  Maximize the expected return shown in (6) by gradient ascent, and then update θ_i;
  Minimize the product of (7) and (9) by gradient ascent, and then update λ_i;
  Execute the "soft" target updates shown in (8) to update θ'_i, ω'_i, Ω'_i, and update λ'_i according to (11);
end for

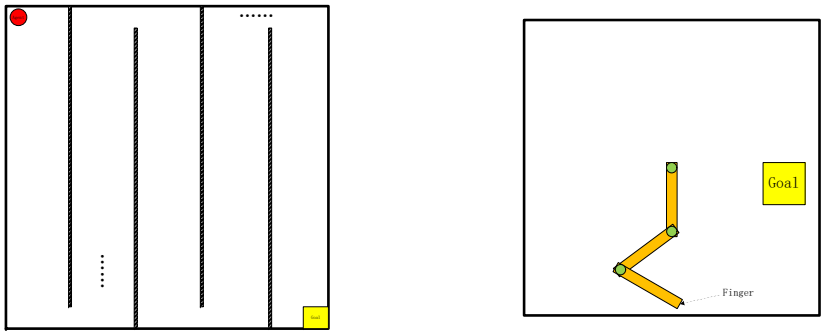
Table 1: List of hyperparameters

Hyperparameter	Value	Description
learning rate	0.001	The learning rate used by gradient descent optimizer
soft update parameter	0.01	τ used in "soft" target updates
reward discount factor	0.9	The discount horizon factor of reward
initial exploration variance	10	Initial variance of gaussian exploration noise added to actions
variance decay rate	0.9995	The exploration variance multiplied by the rate per training
batch size	200	The sample size of UAD3
mini-batch size	200	The sample size of minibatches in AD3, TD3 and DDPG
maximum episode steps	1000	The timeout steps in AD3, TD3 and DDPG

EXPERIMENTS

CONTINUOUS MAZE

One of the benchmark tasks we choose is the continuous maze which is filled with obstacles. The environment of the continuous maze problem includes infinite states and actions which is shown in Fig. 1(a). At every step, the agent is able to move towards all directions with its step size. Since the state-action space is continuous, the agent may travel through the obstacles represented by the gray grids if no effective guide is provided during the whole training. The dark solid line at the edge of Fig. 1(a) is represented as the wall to show the maze is closed except for the goal. The task of this experiment is move the agent from the $start$ to the $goal$ colored yellow with no block. This goal can be achieved by setting scores for the agent at every step. Specifically, the agent receives minus reward if it encounters any jam. If the agent reaches the $goal$, it will be rewarded 100 score. In other blank areas, the reward is set as the minus distance from the agent to the $goal$ for the purpose of stimulating the agent to fulfill the task as soon as possible.

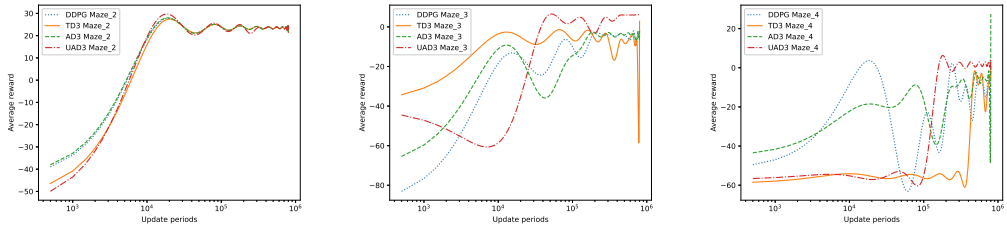


(a) (b)

Figure 1: (a) The maze environment with continuous state-action space and lines of obstacles; (b) The robot arm environment with a move and grasp task.

In this experiment, we evaluate the performance of AD3 and UAD3 algorithms, using the baseline algorithms DDPG and TD3. The hyperparameters are shown in Table 1. Every 500 iterations (update periods), an evaluation procedure is launched, which records 100 episodes and averages their results to improve accuracy, i.e., the average reward, where each episode observes the agent from the $start$ to the $goal$ and keeps the rewards along the path. Similar evaluation procedures are shared by all experiments in the context with the same cycle.

Figure 2 illustrates the average reward versus update periods of the continuous maze with barriers of different lines plotted in Fig. 1(a). Fig. 2(a) shows that AD3 converges faster than DDPG and TD3. From Figs. 2(b)-2(c), we see UAD3 robustly converges so that the agent can reach the goal and receives a positive reward, however, other algorithms diverges and fails in their missions.



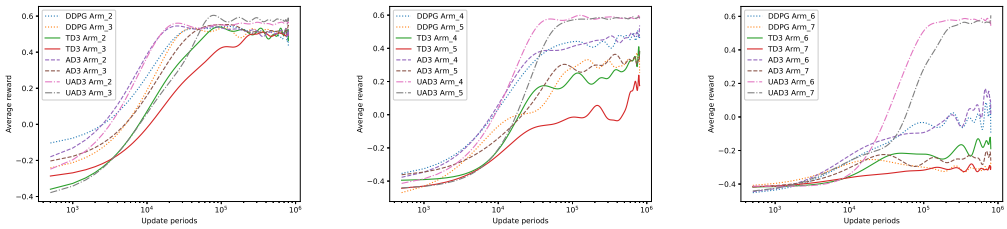
(a) (b) (c)

Figure 2: Convergence performance of the continuous maze with barriers (a) of 1 line; (b) of 2 lines; (c) of 3 lines.

ROBOT ARM

The "robot arm" experiment is a move and grasp task which is shown in Fig. 1(b). In this figure, we just show three sections to represent a general arm that may contain several more sections. The aim of this task is to move the $finger$ represented by the end of the arm to catch the $goal$. Specifically, the $finger$ should get into the yellow "goal box", and then hold on to the moving goal for required steps to fulfill the grasp task. In the experiment setting, the $goal$ is randomly moving, so the state representation includes both the positions of joints represented by the green circles and their relative positions to the goal. Besides, the number of sections determines the dimension of action space which contains the rotation angle of each section. The reward is set as the negative distance from the $finger$ to the $goal$ outside the "goal box". If the $finger$ is located within the "goal box", the reward is set as 1.

We also evaluate our proposed algorithms using the baseline algorithms DDPG and TD3 based on the hyperparameters in Table 1. Fig. 3 shows the computational performance of the robot arm with $2-7$ sections by fitting scatterplots run for 800 thousand iterations. Notably, one more section will raise the state dimension by 4, including the 2-dimensional coordinates of joints and their relative coordinates to the goal. Increased state dimension needs more time for convergence and produces lower converged value. Throughout Fig. 3, we observe UAD3 can robustly converge to a value much higher than other algorithms, which shows the best performance because higher converged value means the agent can react more promptly to the moving goal. From Figs. 3(a)-3(b), AD3 converges faster and more robustly to a higher value compared with DDPG and TD3 under the same circumstance. In Fig. 3(c), AD3 converges above the zero line with a value higher than DDPG for 6 sections, however, TD3 diverges for both 6 and 7 sections. Overall, UAD3 and AD3 have better performance than their counterparts DDPG and TD3.



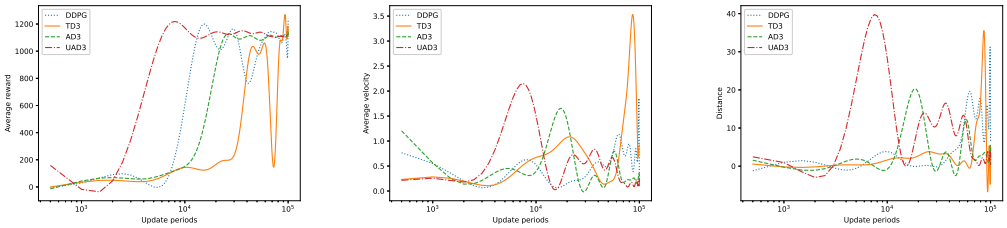
(a) (b) (c)

Figure 3: Convergence performance of the robot arm with (a) 2-3 sections; (b) 4-5 sections; (c) 6-7 sections.

CLASSICAL CONTROL ENVIRONMENT

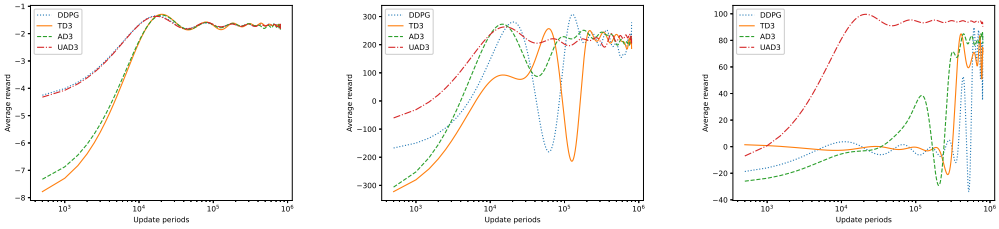
In this part, we adopt a series of classical control experiments including Pendulum, Acrobat, continuous mountain car and Cartpole as the benchmark tasks to evaluate the proposed AD3 and UAD3 algorithms based on the hyperparameters in Table 1.

It is noteworthy that we revised the rewards of Cartpole environment to make them more challenging. Specifically, the distance and average velocity of the cart are added to the rewards to stimulate the cart to go farther and faster because staying at the origin trading for stability is not enough to tell the robustness of algorithms. The results of the average reward, average velocity and distance versus update periods for Cartpole are given by Fig. 4. From Fig. 4(a), we see UAD3 and AD3 can converge much faster and more stably than DDPG and TD3. Besides, UAD3 and AD3 are able to simulate the cart to move faster and farther according to Figs. 4(b) and 4(c), respectively. Figs. 5(a)-5(c) present the results of average reward versus update periods for Pendulum, Acrobat and continuous mountain car, which further show the advantages of UAD3 and AD3 over DDPG and TD3.



(a) (b) (c)

Figure 4: (a) Average reward; (b) Average velocity; (c) Distance versus update periods in Cartpole.



(a) (b) (c)

Figure 5: Computational efficiency in (a) in Pendulum; (b) Acrobat; (c) continuous mountain car.

CONCLUSION

In this paper, we proposed the combined value of two independent critics connected by a weight factor to update the policy, which will in turn to update the combined value. We also proposed the objective function for updating the weight factor by multiplying the updated combined value by a sign, which compares the minimum updated Q-value in two critics with the combined value before updating. Based on the above two components, we present AD3 to reduce the overestimation bias and ensure future policy improvement at the same time. Furthermore, we apply AD3 to UDRL framework to eliminate the systematic bias caused by the probability mismatch between the behavior policy and the target policy in experience replay, and present UAD3. The proposed AD3 algorithm is theoretically proved to possess the property of asymptotical convergence and expected policy improvement. Evaluation results show that our proposed algorithms can boost and stabilize the convergence.

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APPENDIX

PROOF OF THEOREM 1

Proof First, using the Bellman equation as an iterative update, i.e.,

$$Q(s,a|\omega) = r(s,a) + \gamma \mathbb{E}_{s' \sim p(s'|s,a)} \left[Q(s', \mu(s')|\omega') \right], \\ \forall (s,a) \in \chi \times A, \tag{14}$$

where $p(s'|s,a)$ is the transition probability of the next state s' given the current state-action pair (s,a), and $\chi \times A$ is the state-action space.

Second, (3) and (4) describe a general model-free stochastic approximation method. According to Szepesvári (2010), if the learning rate satisfies the Robbins-Monro (RM) conditions, i.e.,

$$\sum_{t=0}^{\infty} \alpha_t = \infty, \quad \sum_{t=0}^{\infty} \alpha_t^2 < +\infty, \tag{15}$$

then the stochastic approximations which are $Q(s,a|\omega) \leftarrow r + \gamma Q(s', \mu(s')|\omega')$ and $Q(s,a|\Omega) \leftarrow r + \gamma Q(s', \mu(s')|\Omega')$ will converge to their Bellman forms, and even the commonly used constant learning rate can make algorithms converge in distribution. Besides, within the scope of the current environment DRL is dealing with, transition probabilities are normally deterministic, which means that the next state s' is unique given a specific state-action pair (s,a). In this case, the stochastic approximations are exactly equal to their Bellman forms described by (14). □

PROOF OF THEOREM 2

Proof If $sign_{i+1} \geq 0$, then

$$\begin{aligned} &\mathbb{E}_{(s,a)} \left[\hat{Q}(s,a|\omega_{i+1}, \Omega_{i+1}, \lambda_{i+1}) \right] \\ &= \mathbb{E}_{s} \left[\hat{Q}(s, \mu(s|\theta_{i+1})|\omega_{i+1}, \Omega_{i+1}, \lambda_{i+1}) \right] \\ &\approx \frac{1}{N} \sum_{n=1}^{N} \hat{Q}(s_n, \mu(s_n|\theta_{i+1})|\omega_{i+1}, \Omega_{i+1}, \lambda_{i+1}) \\ &\geq \frac{1}{N} \min \left(\sum_{n=1}^{N} Q(s_n, \mu(s_n|\theta_{i+1})|\omega_{i+1}), \sum_{n=1}^{N} Q(s_n, \mu(s_n|\theta_{i+1})|\Omega_{i+1}) \right) \\ &\geq \frac{1}{N} \sum_{n=1}^{N} \left[\hat{Q}(s_n, \mu(s_n|\theta_i)|\omega_i, \Omega_i, \lambda_i) \right] \\ &\approx \mathbb{E}_{(s,a)} \left[\hat{Q}(s,a|\omega_i, \Omega_i, \lambda_i) \right], \end{aligned} \tag{16}$$

where the approximations hold because the replay buffer can remove correlations in the observation sequence and smooth over changes in the data distribution (or unbiased approximation for UAD3), the first inequality holds based on (10), and the second inequality holds because (9) is no less than 0.

Otherwise, if $sign_{i+1} \leq 0$, then

$$
\begin{aligned}
&\mathbb{E}_{(s,a)}\left[\hat{Q}(s,a|\omega_{i+1},\Omega_{i+1},\lambda_{i+1})\right] \\
&= \mathbb{E}_{s}\left[\hat{Q}(s,\mu(s|\theta_{i+1})|\omega_{i+1},\Omega_{i+1},\lambda_{i+1})\right] \\
&\approx \frac{1}{N}\sum_{n=1}^{N}\hat{Q}(s_n,\mu(s_n|\theta_{i+1})|\omega_{i+1},\Omega_{i+1},\lambda_{i+1}) \\
&\geq \frac{1}{N}\sum_{n=1}^{N}\hat{Q}(s_n,\mu(s_n|\theta_{i+1})|\omega_{i+1},\Omega_{i+1},\lambda_{i}) \\
&= \frac{1}{N}\sum_{n=1}^{N}\hat{Q}(s_n,\mu(s_n|\theta_{i+1})|\omega_{i+1},\Omega_{i+1},\lambda'_{i}) \\
&\geq \frac{1}{N}\sum_{n=1}^{N}\hat{Q}(s_n,\mu(s_n|\theta_{i})|\omega_{i+1},\Omega_{i+1},\lambda'_{i}) \\
&= \frac{1}{N}\sum_{n=1}^{N}\hat{Q}(s_n,\mu(s_n|\theta_{i})|\omega_{i+1},\Omega_{i+1},\lambda_{i}) \\
&\approx \mathbb{E}_{s}\left[\hat{Q}(s,\mu(s|\theta_{i})|\omega_{i},\Omega_{i},\lambda_{i})\right] \\
&= \mathbb{E}_{(s,a)}\left[\hat{Q}(s,a|\omega_{i},\Omega_{i},\lambda_{i})\right],
\end{aligned}
\tag{17}
$$

where the first inequality holds due to the fact that the update of λ is to maximize (7) given a negative sign, the second inequality holds because the update of θ is to maximize (6). Although (16) and (17) are done for immediate target updates, the same conclusions can be achieved under the condition of delayed or "soft" target updates if the Q network is linear with respect to the actor and critic parameters. □