# RESTORATION BASED GENERATIVE MODELS

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## ABSTRACT

Denoising generative models (DGMs) have recently attracted increasing attention by showing impressive synthesis quality. DGMs are built on a diffusion process that pushes data to the noise distribution and the models learn to denoise. In this paper, we establish the interpretation of DGMs in terms of image restoration (IR). Integrating IR literature allows us to use an alternative objective and diverse forward processes, not confining to the diffusion process. By imposing prior knowledge on the loss function grounded on MAP estimation, we eliminate the need for the expensive sampling of DGMs. Also, we propose a multi-scale training, which alleviates the latent inefficiency of DGMs, by taking advantage of the flexibility of the forward process. Our model improves the quality and efficiency of both training and inference, achieving state-of-the-art performance when the number of forward steps is limited. Furthermore, we show the applicability of our model to inverse problems. We believe that our framework paves the way for designing a new type of flexible general generative model.

## 1 INTRODUCTION

Generative modeling is a prolific machine learning task that the models learn to describe how a dataset is distributed and generate new samples from the distribution. The most widely used generative models primarily differ in their choice of bridging the data distribution to a tractable latent distribution (Goodfellow et al., 2020; Kingma & Welling, 2014; Rezende et al., 2014; Rezende & Mohamed, 2015; Sohl-Dickstein et al., 2015; Chen et al., 2021a). In recent years, denoising-based generative models (DGMs) (Ho et al., 2020; Song & Ermon, 2019; Song et al., 2020b; Dockhorn et al., 2021) have drawn considerable attention by demonstrating remarkable results in terms of both high sample quality and likelihood. DGMs rely on a forward diffusion process that progressively transforms the data into Gaussian noise, and they learn to reverse the noising process. Albeit their enormous successes, their immovable diffusion process gives rise to a few limitations. To pull latent variables back to the data distribution, the denoising process requires thousands of network evaluations to sample a single instance. In addition, as DGMs couple the data and latent variables through the diffusion process, the latent and data possess the same dimension. Many follow-up studies set their mind on enhancing inference speed (Song et al., 2020a; Jolicoeur-Martineau et al., 2021; Tachibana et al., 2021) or grafting with other generative models (Xiao et al., 2022; Vahdat et al., 2021; Zhang & Chen, 2021; Pandey et al., 2022). However, most of these works still require numerous steps to generate high-fidelity samples and face latent inefficacy.

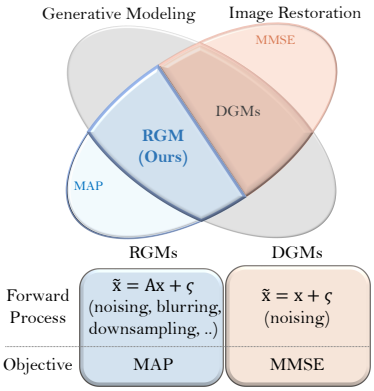


Figure 1: Comparison of DGMs and RGMs.

In this study, we focus on a different perspective. We interpret the DGMs through the lens of image restoration (IR), which is a family of inverse problems for recovering the original images from corrupted ones (Castleman, 1996; Gunturk & Li, 2018). The corruption arises in various forms, including noising (Buades et al., 2005; Rudin et al., 1992), blurring (Biemond et al., 1990), and downsampling (Farsiu et al., 2004). IR has been a long-standing problem because of its high practical value in various applications (Besag et al., 1991; Banham & Katsaggelos, 1997; Lehtinen et al., 2018; Ma et al., 2011). From an IR point of view, DGMs can be considered as IR models based

on minimum mean square error (MMSE) estimation (Zervakis & Venetsanopoulos, 1991; Laumont et al., 2022), focusing only on the denoising task. Mathematically, IR is an ill-posed inverse problem in the sense that it does not admit a unique solution and hence, leads to instability in reconstruction (Hadamard, 1902). Owing to the ill-posedness of IR, MMSE which only measures data fidelity produces impertinent results. DGMs alleviate this problem by leveraging costly Langevin sampling, and this inefficient inference scheme has been regarded as an indispensable tool in the literature of DGMs. By casting DRMs as IR models, however, the forward process need not be restricted to Gaussian noising, and ill-posedness can be detoured in ways other than Langevin dynamics.

Inspired by this observation, we propose a new flexible family of generative models, that we refer to as *restoration-based generative models (RGMs)*. First, we adopt an alternative objective; a maximum a posteriori (MAP) (Trussell, 1980; Hunt, 1977), which is predominantly used in IR. The MAP estimator compensates the ill-posedness by regularizing the data fidelity loss by a prior term. Many advent hand-crafted regularization schemes (Tikhonov, 1963; Donoho, 1995; Mallat, 1999; Baraniuk, 2007) encourage solutions to satisfy certain properties, such as smoothness and sparsity. However, for the purpose of density estimation, we implicitly parameterize a prior term as a variational regularization via GAN (Goodfellow et al., 2020) with a newly introduced random auxiliary variable. Our MAP approach retains the density estimating capability of DGMs at a much smaller computation cost. Secondly, unlike DGMs, which are buried in a Gaussian noising process, RGMs can be combined with other general degradation processes. As one instantiation, we design a multi-scale training that resolve the latent inefficiency of DGMs. Because the behavior of generative models is significantly affected by how the data distribution is transformed into a simple distribution, our approach opens the way for designing more flexible generative models. Our comprehensive empirical studies on image generation and inverse problems demonstrate that RGMs generate samples rivaling the quality of DGMs. Also, the inference of our model is several orders of magnitude faster than DGMs. In particular, our models achieve FID 2.47 on CIFAR10, with only seven number of network function evaluations (NFEs), and state-of-the-art FID on CelebA-HQ, with only four NFEs.

## 2 BACKGROUND

**Image Restoration** A common inverse problem arising in image processing, including denoising, deblurring, super-resolution, and inpainting, is the estimation of an image $\mathbf{x}$ given a corrupted image

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \xi, \tag{1}$$

where $\mathbf{A}$ is a matrix that models the degradation process, including blurring and downsampling kernels, and $\xi \sim \mathcal{N}(0, \mathbf{\Sigma})$ is an additive noise. This problem is known as image restoration (IR). The inference of the image $\mathbf{x}$ from the noised one $\mathbf{y}$ is typically ill-posed, in the sense that the inverse problem (1) has multiple valid explanations (Hadamard, 1902). In other words, the noisy $\mathbf{y}$ does not have exactly one restoration $\mathbf{x}$. This is further exacerbated when the noise level is large. To produce consistent results, most methods use the maximum a posteriori (MAP) estimator:

$$\mathbf{x}^*_{\text{MAP}} = \text{argmax}_{\mathbf{x}} \ \log p(\mathbf{x} \mid \mathbf{y}) = \text{argmin}_{\mathbf{x}} \ f(\mathbf{x}, \mathbf{y}) + \lambda g(\mathbf{x}), \tag{2}$$

where $f(\mathbf{x}, \mathbf{y}) = -\log p(\mathbf{y} \mid \mathbf{x}) = \frac{1}{2} \left\| \left(\mathbf{\Sigma}^\dagger\right)^{\frac{1}{2}} (\mathbf{A}\mathbf{x} - \mathbf{y}) \right\|_2^2$ is the data fidelity term with the pseudoinverse (Moore, 1920) $\mathbf{\Sigma}^\dagger$, $g$ is the prior term that encourages the reconstruction to satisfy some prior assumptions on $\mathbf{x}$, and a scalar $\lambda \geq 0$ controls the strength of the regularization. The regularization term $g$ is essential because it relieves the ill-posedness nature of IR by imposing the assumption about the desirable solution. Therefore, many researchers have been devoted to designing a proper $g$ (Rudin et al., 1992; Mallat, 1999; Lunz et al., 2018).

**Denoising Generative Models** Denoising-based generative models (DGMs), such as DDPM (Ho et al., 2020), and score matching with Langevin dynamics (Song et al., 2020b), have recently emerged as the forefront of image synthesis research. Starting from the data distribution, they gradually corrupt the image $\mathbf{x}_0 \sim p_{\text{data}}$ into Gaussian noise over time through a forward Markovian diffusion process;

$$q(\mathbf{x}_{1:T} \mid \mathbf{x}_0) = \prod_{t=0}^{T-1} q^{(t)}(\mathbf{x}_{t+1} \mid \mathbf{x}_t), \ \mathbf{x}_0 \sim p_{\text{data}}. \tag{3}$$

They pose the data generation as an iterative denoising procedure $p_\theta^{(t)}(\mathbf{x}_t \mid \mathbf{x}_{t+1})$, the reverse of the forward diffusion process:

$$p_\theta(\mathbf{x}_{0:T}) = p^{(T)}(\mathbf{x}_T) \prod_{t=0}^{T-1} p_\theta^{(t)}(\mathbf{x}_t \mid \mathbf{x}_{t+1}), \ \mathbf{x}_T \sim \mathcal{N}(\mathbf{0}, \mathbf{I}). \tag{4}$$

As they use linear diffusion process whose diffusion distributions $q^{(t)}(\mathbf{x}_{t+1} \mid \mathbf{x}_t)$ are modeled with conditional Gaussian distributions, we can deduces a tractable evidence lower bound (ELBO) (Sohl-Dickstein et al., 2015). The ELBO can be further simplified to the following objective (Ho et al., 2020; Song et al., 2020b):

$$\mathcal{L}(\theta) = \Sigma_{t=0}^{T} \mathbb{E}_{\mathbf{x}_0 \sim p_{\text{data}}, \mathbf{x}_t \sim q_{\sigma_t}(\mathbf{x}_t \mid \mathbf{x}_0)} \left[ \lambda(t) \left\| G_\theta(\mathbf{x}_t) - \mathbf{x}_0 \right\|_2^2 \right], \tag{5}$$

where $G_\theta$ is a neural network parametrized by $\theta$ that learns the noise by minimizing (5), and $\lambda(t) \geq 0$.

# 3 METHOD

## 3.1 DGMS ARE RESTORATION MODELS

We open this section by drawing the interpretation of DGMs in terms of restoration. DGMs use VPSDE or VESDE (Song et al., 2020b) as a forward process, and these two are known to be exchangeable with each other (Kim et al., 2022). Therefore, the rest of the paper focuses on VESDE. For a given noise level $\sigma_t$, the forward process of the VESDE is formulated as

$$\mathbf{x}_t = \mathbf{x}_0 + \xi, \ \xi \sim \mathcal{N}\left(\mathbf{0}, \sigma_t^2 \mathbf{I}\right),$$

which is the forward process (1) with identity degradation matrix $\mathbf{A} = \mathbf{I}$. As $\nabla_{\mathbf{x}_t} \log q_{\sigma_t}(\mathbf{x}_t \mid \mathbf{x}_0) = -(\mathbf{x}_t - \mathbf{x}_0)/\sigma_t^2$, the loss (5) for each forward step can be rewritten as the following minimum mean square error (MMSE) objective:

$$\mathcal{L}(\theta) = \mathbb{E}_{\mathbf{x} \sim p_{\text{data}}, \mathbf{x}_t \sim \mathcal{N}(\mathbf{x}, \sigma^2 \mathbf{I})} \left[ \| G_\theta(\mathbf{x}_t) - \mathbf{x} \|_2^2 \right]. \tag{6}$$

Therefore, DGMs are IR models which seek a denoiser $G_\theta$ for each $\sigma_t$ that minimizes the MMSE (6). MMSE loss is simple and straightforward to train, however, it confronts some apparent drawbacks. Since the MMSE only contains the recovery term, the solution is only optimized to ensure accordance with the degradation process. Therefore, it it is affected by the ill-posedness. To be precise, when $\sigma_t$ is large, (1) becomes a highly ill-posed and possess many solutions for a given observation. In this case, MMSE solution averages all these candidate solutions, resulting in an atypical reconstruction. Formally, the solution to the MMSE for a corrupted data $\mathbf{x}_t$ is

$$\mathbf{x}_{\text{MMSE}}^* = \int \mathbf{x} p_{\text{data}}(\mathbf{x}) \, p_{\sigma_t}(\mathbf{x}_t \mid \mathbf{x}) \, d\mathbf{x}.$$

Recent works (Laumont et al., 2022; Kawar et al., 2021) have endeavored to resolve this problem by stochastic sampling, however, they suffer from notoriously low efficiency as they roll out thousands of trajectories. In a similar manner, DGMs utilize a sampling scheme that requires thousands of steps. In summary, there are two limitations of DGMs from the IR perspective:

1. The degradation process is restricted to Gaussian noising.
2. The inference efficiency is intrinsically low due to the MMSE estimator.

In the following sections, we show how to cope with these two limitations.

## 3.2 MAP ESTIMATION FOR GENERATION

As alluded in Section 3.1, DGMs can be regarded as MMSE grounded IR models, specialized in denoising. This observation brings us a new perspective on the design of a family of flexible generative models. As an alternative to MMSE, we propose a new generative model with the MAP estimation (2):

$$\mathbb{E}_{\mathbf{x} \sim p_{\text{data}}, \mathbf{y} \sim \mathcal{N}(\mathbf{x}, \sigma^2 \mathbf{I})} \left[ \frac{1}{2\sigma^2} \| G_\theta(\mathbf{y}) - \mathbf{y} \|_2^2 + \lambda g(G_\theta(\mathbf{y})) \right], \tag{7}$$

where the second term delivers the prior knowledge of data distribution. MAP estimation has been adopted as a standard approach for high-dimensional imaging problems and is known to be more relevant than MMSE in many applications (Saha et al., 2009; Bigdeli et al., 2019; Chen, 2016). By leveraging prior information on the solution, MAP approaches alleviate the ill-posedness of the inverse problem (1), without use of costly sampling of MMSE estimation. Therefore, carefully crafting the relevant prior term is crucial in MAP estimation. We now show how one can execute an appropriate prior term for density estimation while alleviating the ill-posedness.

**Alleviation of ill-posedness** Unlike the general denoising task, it is necessary to bridge the image to the Gaussian noise to learn the data distribution. As the noise level increases, a single distorted observation has a tremendous amount of solutions, which indicates that the ill-posedness deepens. This in turn degrades the expressiveness of the model for estimating the distribution. Therefore, it is difficult for the regularization term to remedy all ill-posedness on its own. We further offload the ill-posedness by imposing a priori knowledge by introducing a random auxiliary variable $\mathbf{z} \sim \mathcal{N}(\mathbf{z} \mid \mathbf{0}, \mathbf{I})$. Importantly, $\mathbf{z}$ enables us to obtain several solutions for a heavily degraded $\mathbf{x}_t$ through the guidance provided by $\mathbf{z}$. This eventually helps to pull the noised observations back to the data distribution, allowing for a rich representation for the density.

**Implicit Prior Knowledge** For density estimation, the knowledge about the data distribution should be properly encoded in the prior term $g$ of (7). The explicit density function with determined marginal distribution is intractable, but it can be learned with the aid of well-developed generative models. That is, we can design $g$ by integrating various existing generative models. In this paper, we learn an implicit representation of the data density by adopting generative adversarial network (GAN), which has shown promising results in many generative tasks, and use generator loss as a relevant prior term $g$. Our MAP objective in conjunction with the GAN prior is given by:

$$\mathcal{L}(G_\theta) = \mathbb{E}_{\mathbf{x}\sim p_{\text{data}}, \mathbf{y}\sim\mathcal{N}(\mathbf{x},\sigma^2\mathbf{I}), \mathbf{z}\sim\mathcal{N}(0,I)} \left[ \frac{1}{2\sigma^2} \|G_\theta(\mathbf{y},\mathbf{z}) - \mathbf{y}\|_2^2 + \lambda g_\phi(G_\theta(\mathbf{y},\mathbf{z})) \right], \tag{8}$$

where the first term is the data fidelity term, $g_\phi(\mathbf{x}) = \log(1 - D_\phi(\mathbf{x})) - \log D_\phi(\mathbf{x})$ is a learnable prior term that is trained coupled with a discriminator $D_\phi$, and $\lambda \geq 0$ is a hyperparameter. Contrary to conventional MAP whose prior term is pre-defined, our approach tries to learn the prior term by coordinating with the denoiser $G_\theta$ through the discriminator. This end-to-end training allows our discriminative learning method to deliver promising performance. Note that when the discriminator is nearly optimal, the loss (8) with $\lambda = 1$ agrees with

$$\mathbb{E}_{bx\sim p_{\text{data}}, \mathbf{y}\sim\mathcal{N}(\mathbf{x},\sigma^2\mathbf{I})} \left[ D_{KL}(q_\theta(\mathbf{x}|\mathbf{y}) || p(\mathbf{x}|\mathbf{y})) \right] + \mathcal{H}(q_\theta), \tag{9}$$

where $\mathcal{H}$ denotes an entropy, which corresponds to training the model to learn the posterior distribution. The overall training procedure combined with all $\sigma \in \{\sigma_k\}_{k=1}^T$ is outlined in Algorithm 1.

**Small Denoising Steps** A major downside of DGMs is their slow sampling procedure, which requires hundreds to thousands of denoising steps to obtain a single image. By adopting MAP estimation and parametrizing the prior distribution through GAN, our model provides an avenue to offload the time-consuming sampling scheme and enables significantly small denoising steps. For small degradation we can obtain a restored image in one shot. But, as our restoration starts from the Gaussian noise, the data distribution is not completely estimated. Therefore, we perform the generation iteratively. In our experiments on CIFAR10, we generate a high-quality sample in four denoising steps, whereas DGMs use $T = 1000, 2000$ steps.

### 3.3 Extension to General Restoration

In Section 3.2, we proposed a denoising generative model using MAP estimation. However, from the IR perspective, it is not necessary to restrict it to denoising ($\mathbf{A} = \mathbf{I}$) and it can be generalized to any family of degradation matrices $\mathbf{A}$ and noise factors $\mathbf{\Sigma}$ in (1). Utilizing the general forward process, we can learn the generative model by generalizing the loss function (8) as follows:

$$\mathcal{L}(G_\theta) = \mathbb{E}_{\mathbf{x}\sim p_{\text{data}}, \mathbf{y}\sim\mathcal{N}(\mathbf{A}\mathbf{x},\mathbf{\Sigma}), \mathbf{z}\sim\mathcal{N}(\mathbf{0},\mathbf{I})} \left[ \frac{1}{2} \left\| \left(\mathbf{\Sigma}^\dagger\right)^{\frac{1}{2}} (\mathbf{A} \cdot G_\theta(\mathbf{y},\mathbf{z}) - \mathbf{y}) \right\|_2^2 + \lambda g(G_\theta(\mathbf{y},\mathbf{z})) \right]. \tag{10}$$

Therefore, RGM has an flexible structure that can permeate any forward process, and aids in designing a new generative model. Here, we propose a new model established upon super-resolution (SR).

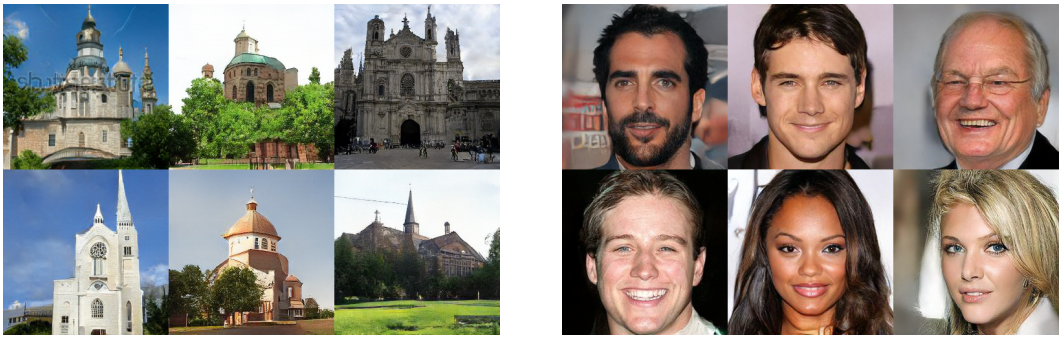

Figure 2: Generated samples on LSUN Church (left) and CelebA-HQ (right).

**Multi-scale RGM** Most DGMs maintain the image size during the diffusion process by adding noise to individual pixels. Consequently, they are very inefficient because they require a latent as much as dimension of pixel space that is much larger than the submanifold of the image space. Motivated by this, we take $\mathbf{A}$ as a block averaging filter that averages out $2 \times 2$ pixel values. Halving the number of pixels at each coarsening step allows us to train a more expressive generative model with enhanced latent efficiency. Moreover, multi-scale training has proven to be an effective strategy for synthesizing large scale images (Denton et al., 2015; Karras et al., 2017b; Reed et al., 2017). Therefore, our model produces strikingly realistic images by progressively extracting spatial information.

## 4 Experiments

This section evaluates the performance of the proposed RGMs in terms of the sample quality and likelihood on several benchmark datasets, including CIFAR10 (Krizhevsky et al., 2009) ($32 \times 32$ unconditional), CelebA-HQ (Liu et al., 2015) ($256 \times 256$), and LSUN Church (Yu et al., 2015) ($256 \times 256$). We also show the capability of our models for solving inverse problems.

**Setup** We implement two models: RGM-D is a model trained with the Gaussian noising forward process, which is mainly used by DGMs. We also consider a model (introduced in Section 3.3) whose forward degradation matrix is a downscaling filter, called RGM-SR. We parametrize our GAN generator based on the UNet-like structure (Ronneberger et al., 2015) which was successfully used in NCSN++ architecture (Xiao et al., 2022). The Models are trained with Adam (Kingma & Ba, 2014) in all experiments. The internal details of the implementation can be found in Appendix B.

### 4.1 2D Toy Example

We first employ a two-dimensional example to validate the importance and effectiveness of prior knowledge of our MAP estimation. We adopt a mixture of Gaussian with eight components (Grathwohl et al., 2018) as a base distribution. In Figure 3, we depict the benefits of our MAP estimation over the MMSE approach. As illustrated in the top row, we diffuse the data distribution through four different noise levels. Each row from the second row to the bottom represents the learned distribution of our MAP estimation, MAP without the auxiliary variable $\mathbf{z}$, and MMSE. First, the figures in the bottom row show the failure of MMSE, where the modes of the distribution are connected and then missed. This tendency exacerbates as the noise level increases. Since the MMSE fails to reconstruct the data distribution even with a small rise in the noise level, the MMSE does not yield a satisfactory generative model with a small number of diffusion steps. Consequently, MMSE approaches, such as DGMs, require a large number of steps to stably recover the data distribution. On the other hand, by adding the prior term to this MMSE objective, the MAP estimator generates samples from the multimodal distribution significantly

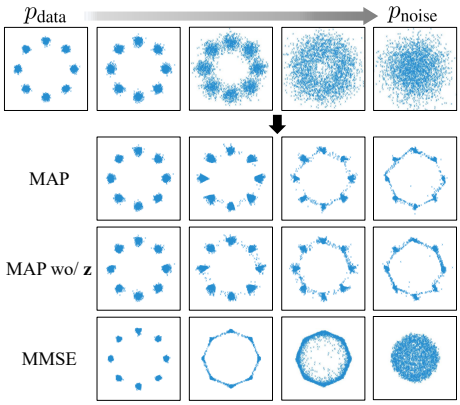


Figure 3: Comparison of recovering density by MMSE versus MAP.

Table 1: Results on unconditional generation of CIFAR10.

| Class | Model | FID (↓) | IS (↑) | NFE (↓) |
|---|---|---|---|---|
| **RGM** | RGM-D | 3.08 | 9.14 | 4 |
| | RGM-SR | 2.47 | 9.68 | 7 |
| **DGM** | DDPM (Ho et al., 2020) | 3.21 | 9.46 | 1000 |
| | NCSN (Song & Ermon, 2019) | 25.3 | 8.87 | 1000 |
| | Score SDE (VE) (Song et al., 2020b) | 2.20 | 9.89 | 2000 |
| | Score SDE (VP) (Song et al., 2020b) | 2.41 | 9.68 | 2000 |
| | Probability Flow (VP) (Song et al., 2020b) | 3.08 | 9.83 | 140 |
| | LSGM (Vahdat et al., 2021) | 2.10 | 9.87 | 147 |
| | DDIM (50 steps) (Song et al., 2020a) | 4.67 | 8.78 | 50 |
| | FastDDPM (T=50) (Kong & Ping, 2021) | 3.41 | 8.98 | 50 |
| | Recovery EBM (Gao et al., 2020) | 9.58 | 8.30 | 180 |
| | VDM (Kingma et al., 2021) | 4.00 | - | 1000 |
| | UDM (Kim et al., 2021) | 2.33 | 10.1 | 2000 |
| | Gotta Go Fast (Alexia Jolicoeur-Martineau, 2021) | 2.44 | - | 1000 |
| | Subspace Diffusion (Jing et al., 2022) | 2.17 | 9.94 | $\geq 1000$ |
| | CLD (Dockhorn et al., 2021) | 2.25 | - | 2000 |
| | DEIS (Zhang & Chen, 2022) | 3.37 | 9.74 | 15 |
| | DDGAN (Xiao et al., 2021a) | 3.75 | 9.63 | 4 |
| | StyleGAN2+ES-DDPM (Lyu et al., 2022) | 5.52 | - | 101 |
| **GAN** | SNGAN+DGflow (Ansari et al., 2020) | 9.62 | 9.35 | 25 |
| | AutoGAN (Gong et al., 2019) | 12.4 | 8.60 | 1 |
| | TransGAN (Jiang et al., 2021) | 9.26 | 9.02 | 1 |
| | StyleGAN2 w/o ADA (Karras et al., 2020) | 8.32 | 9.18 | 1 |
| | StyleGAN2 w/ ADA (Karras et al., 2020) | 2.92 | 9.83 | 1 |
| **VAE** | NVAE (Vahdat & Kautz, 2020) | 23.5 | 7.18 | 1 |
| | Glow (Kingma & Dhariwal, 2018) | 48.9 | 3.92 | 1 |
| | PixelCNN (Van Oord et al., 2016) | 65.9 | 4.60 | 1024 |
| | VAEBM (Xiao et al., 2020) | 12.2 | 8.43 | 16 |

better (See the third row). This demonstrates the effectiveness of the prior knowledge of MAP. By imposing the prior knowledge, the MAP objective accurately estimates the density, which allows distribution recovery with a much smaller number of forward processes than the MMSE approach. Furthermore, we can observe the effect of the auxiliary variable $\mathbf{z}$ by comparing the second and third rows in Figure 3. MAP with $\mathbf{z}$ has higher sample quality. The effect of $\mathbf{z}$ is further amplified for more complex distributions, such as image data (See Figure 5). This synthetic experiment validates that our MAP approach in conjunction with the random auxiliary variable $\mathbf{z}$ enables accurate and efficient generative modeling.

### 4.2 Image Generation

We compare the performance of our method with several existing baselines. For quantitative comparison, we use Fréchet Inception Distance (FID) and Inception Score (IS) as the evaluation metrics. We also report the number of network function evaluations (NFE). For DGMs and RGMs, NFE value and real inference time are proportional. Following (Song et al., 2020b; Dockhorn et al., 2021), we focus on the widely used CIFAR10 unconditional image generation benchmark and also validate the performance of RGMs on large-scale CelebA-HQ-256 images. Table 1 summarizes the quantitative evaluations on CIFAR10, and the results on CelebA-HQ is reported in Table 2. Qualitative performance is depicted in Figure 2.

**Results** We can see that our model achieves the state-of-the-art FID score on CelebA-HQ-256. On CIFAR10, our models are comparable to the best existing DGMs and GAN models. Although the best denoising models obtain better results than ours on CIFAR10, they use a much larger number of denoising steps (e.g. ScoreSDE with VESDE requires 2000 steps). Notably, our RGM-SR achieves FID 2.47 and IS 9.68 with only seven steps, which is state-of-the-art sampling FID performance when NFE is limited. The overall results confirm that our MAP estimation immediately eliminates the need for an expensive sampling scheme while still maintaining the density estimating capability of DGMs. Interestingly, RGM-SR outperforms RGM-D by a large margin. This improved performance may be attributed to the increase in NFE; however, the FID of RGM-D with $T = 8$ reported in Table 7 confirms that it is not. In addition, the results show that the SR-based model estimate the density quite well even with far fewer latent variables than RGM-D. More uncurated images can be founded in Appendix C.4.

Table 2: Results on generation of CelebA-HQ-256.

| Class | Model | FID (↓) | NFE (↓) |
|---|---|---|---|
| **RGM** | RGM-D | **7.15** | 4 |
| **DGM** | Score SDE (VP) (Song et al., 2020b) | 7.23 | 4000 |
| | Probability Flow (Song et al., 2020b) | 128.13 | 335 |
| | LSGM (Vahdat et al., 2021) | 7.22 | 23 |
| | UDM (Kim et al., 2021) | 7.16 | 2000 |
| | DDGAN (Xiao et al., 2021a) | 7.64 | 4 |
| **GAN** | PGGAN (Karras et al., 2017a) | 8.03 | 1 |
| | Adv. LAE (Pidhorskyi et al., 2020) | 19.2 | 1 |
| | VQ-GAN (Esser et al., 2021) | 10.2 | 1 |
| | DC-AE (Parmar et al., 2021) | 15.8 | 1 |
| **VAE** | NVAE (Vahdat & Kautz, 2020) | 29.7 | 1 |
| | VAEBM (Xiao et al., 2020) | 20.4 | 1 |
| | NCP-VAE (Aneja et al., 2021) | 24.8 | 1 |

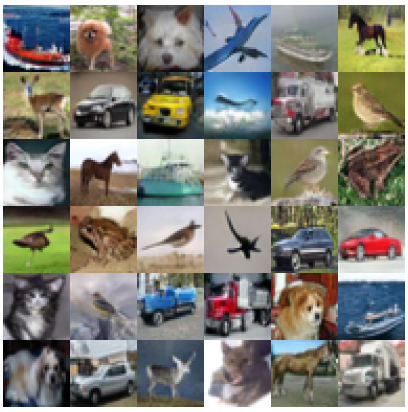

Figure 4: CIFAR10 generated samples.

### 4.3 Ablation Studies

This section is devoted to ablation analyses which show that all parts of our MAP objective, the data fidelity term, and the prior term together coupled with the auxiliary variable, and the regularization parameter, each play an important role in our performance of density estimation.

**On the role of $\mathbf{z}$** We include experimental results on CIFAR10, which demonstrate how the auxilliary variable $\mathbf{z}$ alleviates the ill-posedness of the inverse problem. By noising the upper-left image $\mathbf{x}_0$, we obtain the forward trajectory $\{\mathbf{x}_k\}_{k=1}^4$. The figures on the right are restored images of $\mathbf{x}_t$ by RGM-D together with four different $\mathbf{z}$. We can see that a reconstruction is almost unique when the noise level is small. But, as the noise level increases, a single $\mathbf{x}_t$ has various reconstructions. It is evident that assigning $\mathbf{z}$ helps generate different denoised images from a heavily degraded $\mathbf{x}_t$ through the guidance provided by $\mathbf{z}$. However, one might think that the ill-posedness is detoured by multi-step training using multiple $\sigma_t$ rather than through $\mathbf{z}$. This claim can be refuted using the result of RGM-D without $\mathbf{z}$ reported in Table 3. We observe the significant difference in FIDs of RGM-D with and without $\mathbf{z}$ under the same number of denoising steps, which indicates the effectiveness of $\mathbf{z}$.

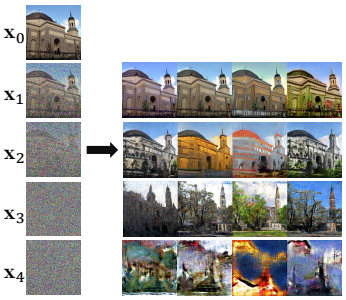


Figure 5: Study on effect of $\mathbf{z}$.

**On the effect of Varying $\lambda$** We investigate the sensitivity of the regularization parameter $\lambda$ in (8). Since it controls the relative importance between the fidelity term and the prior term, $\lambda$ is a trade-off hyperparameter that determines how much regularizes the joint distribution of $p_t$ and $p_{t+1}$. In Figure 6, we present FID scores measured on CIFAR10 with the same number of degradation steps ($T = 4$) and varying $\lambda$. We can see that our models are quite robust with respect to $\lambda$. An empirically observed sweet spot of $\lambda$ is $d/10 \leq \lambda \leq d$ for the image size $d$, in which FID is no longer improved outside this threshold. For small $\lambda$, the models put a lot of effort to recover the degradation, which hinders estimating data distribution. Choosing a large $\lambda$ also results in performance degeneration. There is another point that draws our attention. When $\lambda = \infty$, that is, when there is no fidelity term on the objective, FID scores completely deteriorate. In this case, the models are trained by the vanilla GAN loss. From the perspective of GAN, our MAP objective adds the fidelity term to the GAN loss function. We further observe that with the help of the fidelity term, our model enhances the mode-collapsing resiliency of GAN (See Appendix C.1). These results validate that the performance of RGMs owes to both fidelity and prior term. and the reliable regularization parameter is determined to balance these two terms.

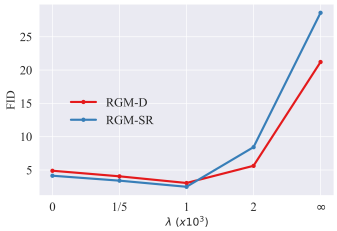


Figure 6: Study on effect of $\lambda$.

**On the scheduling** Since the forward process determines the way of connecting the data and latent distributions, it significantly affects the performance of models. The first important one is forward step scheduling, which is directly related to NFE. In Table 3, we ablate the effect of the

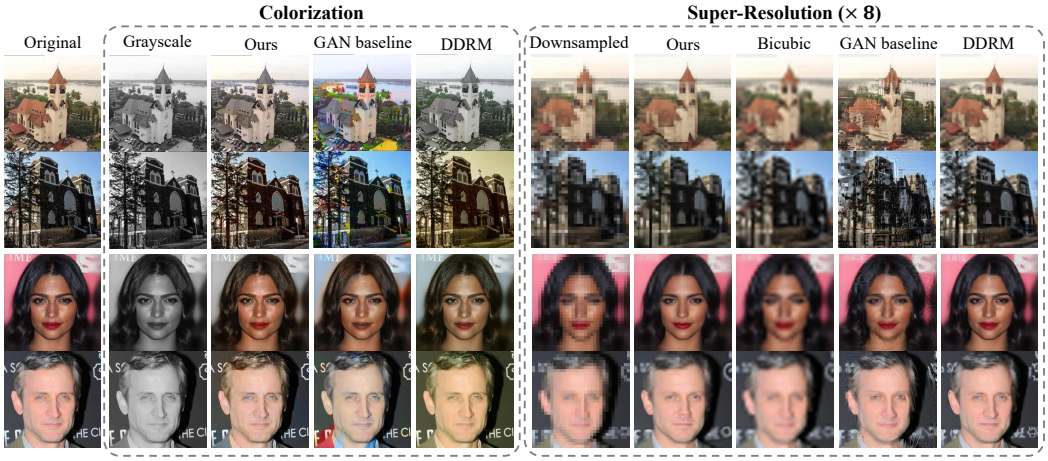


Figure 7: Colorization (left) and super-resolution (right) results on LSUN and CelebA-HQ datasets.

number of forward steps. When $T = 1$, it may be difficult for the model to directly approximate the data distribution from the Gaussian noise. This is reflected in the poor FID score. We also study the scheduling method of the forward process of the SR model. Unlike the diffusion process, the image is corrupted by a downsampling filter together with additive noise. In this case, the model (indicated by RGM-SR (naive) in the Table 3) is demanded to conduct upsampling and denoising at the same time. To make it more effective, we explore another forward scheduling that separates the downsampling and noising process and performs them alternatively. The model to which this scheduling is applied is the RGM-SR. The results demonstrate that our forward scheduling of RGM-SR improves the quality of the generated images.

Table 3: Ablation Studies.

| Model | FID (↓) |
|---|---|
| RGM-D ($\lambda = \infty$) | 32.5 |
| RGM-D ($T = 1$) | 14.6 |
| RGM-D ($T = 4$) | 3.04 |
| RGM-SR (naive) | 3.17 |
| RGM-SR | 2.47 |

### 4.4 Inverse Problems

While our model was originally devised to generate images, we further show the applicability of our models to inverse problems. Recently, a promising approach in imaging inverse problems is to leverage a learned denoiser as an alternative to the proximal operator of splitting algorithms (Romano et al., 2017; Hurault et al., 2021). Such methodology is referred to as Plug-and-Play (PnP) algorithms (Venkatakrishnan et al., 2013). In a similar spirit, we utilize the trained RGMs as a modular part of the PnP algorithms to solve various inverse problems. In this section, we testify our RGM-D in solving two inverse problems; super-resolution and colorization, by plugging our model into Douglas-Rachford Splitting (DRS) algorithm (Lions & Mercier, 1979). More details can be found in Appendix B.3.

**Results** We compare the performance of our model with current-leading models: We compare our model with DDRM (Kawar et al., 2022), which solves inverse problems with a pre-trained DDPM by a posterior sampling scheme. As a GAN baseline, we adopt StyleSwin (Pan et al., 2021) and reconstruct the image by optimizing over the latent vector (Zhang et al., 2022b). We also consider bicubic interpolation as a baseline for super-resolution. We observe that our model is capable of reconstructing faithful and realistic images, as evident in Figure 7. Compared with baselines, our model produces high-quality reconstructions across all the datasets. In particular, our model shows promising performance for colorization. These results show the applicability of RGMs to PnP prior, and this will bring a range of potential applications, including image segmentation, conditional generation, and other imaging inverse problems. Additional quantitative and qualitative results are provided in Appendix C.2.

**Comparison of RGM-D and RGM-SR** We investigate the effect of the degradation process used during training on the performance of solving inverse problems. We compare the reconstruction performance of RGM-D and RGM-SR that are trained on different degradation processes by applying both models to denoising and super-resolution (SR) tasks on CIFAR10. Qualitative results are presented in Figure 7. We can see that the RGM-SR that is trained based on SR actually performs the

SR task better. Also, we can observe a similar tendency for denoising. The results confirm that the degradation process used in training actually helps in solving the corresponding inverse problem.

Table 4: Quantitative comparison of RGM-D and RGM-SR on image reconstruction.

| Model | Super-Resolution | | | | Denoising | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | ($\times 2$) | | ($\times 4$) | | ($\sigma = 10/255$) | | ($\sigma = 20/255$) | | ($\sigma = 40/255$) | |
| | PSNR | SSIM | PSNR | SSIM | PSNR | SSIM | PSNR | SSIM | PSNR | SSIM |
| RGM-D | 26.63 | 0.88 | 20.84 | 0.58 | **30.11** | **0.93** | **26.57** | **0.86** | **24.23** | **0.80** |
| RGM-SR | **27.42** | **0.90** | **21.14** | **0.59** | 29.41 | 0.92 | 25.87 | 0.84 | 23.53 | 0.77 |

## 5 Related Work

In recent years, denoising-based generative models (Ho et al., 2020; Song & Ermon, 2019; Song et al., 2020b) have emerged as a class of density estimation models, first sparked by (Sohl-Dickstein et al., 2015). They define a sampling process as the reverse of a forward diffusion process that maps data to Gaussian noise by consecutively adding a small portion of the noise to the input data. As they faithfully estimate the data distribution and generate high-fidelity samples, they have rapidly been applied to various domains such as conditional generation (Lee et al., 2022; Ho et al., 2022a), audio synthesis(Kong et al., 2021; Popov et al., 2021), medical imaging (Song et al., 2021; Chung & Ye, 2022), video generation (Ho et al., 2022b; Yang et al., 2022), and 3D point cloud generation (Lyu et al., 2021). Their major drawback is slow and expensive inference. Many studies have been dedicated to circumventing this downside by developing a fast numerical solver (Jolicoeur-Martineau et al., 2021; Zhang & Chen, 2022; Tachibana et al., 2021; Liu et al., 2022) or using an alternative noising process such as non-Markovian (Song et al., 2020a), a second-order Langevin dynamics (Dockhorn et al., 2021), and non-linear diffusion processes (De Bortoli et al., 2021; Chen et al., 2021b). Another line of work improves sampling efficiency by incorporating it into other generative models, including GAN (Xiao et al., 2022; Lyu et al., 2022), and VAE (Vahdat & Kautz, 2020). On a side note, all the aforementioned models use the Gaussian noising process as the forward process.

Recently, the literature has begun to replace the additive Gaussian noising process with other transforms. Breaking away from the diffusion process, (Rissanen et al., 2022) proposed a forward blurring process inspired by heat dissipation. They suggest a new generation process, but they specialize in the proposed blurring process and cannot be incompatible with other degradation processes. Possibly the closest study to our work is Cold Diffusion (Bansal et al., 2022) which generalizes the diffusion process to arbitrary image transformations. It seems to use a general transform similar to our models, but Cold Diffusion only uses deterministic degradation processes by entirely removing additive Gaussian noise, which hinders its density estimation performance. Also, they use the MMSE objective, still requiring an array of several forward steps.

## 6 Conclusion and Future Work

In this study, we presented a general framework for modeling efficient generative models through the lens of IR. Compared to DGMs whose both forward and reverse processes are fixed to thousands of Gaussian steps, our approach provides more flexible models that eliminate expensive sampling and can enjoy versatile forward processes. We eliminated the usage of slow sampling by taking on MAP estimation and incorporating implicit prior information through GAN. In addition,we propose a multi-scale method as an example of the usability of various forward processes. The experimental results showed that the image quality obtained was on par with the leading DGMs, and we achieved state-or-the-art performance using a limited number of forward steps. We hope that this work provides a broad view of modeling useful generative models.

Our model has two degrees of freedom: One is how to parametrize the prior knowledge, and the other is the choice of the forward process. This opens up interesting directions for future research. In addition to the GAN we used, the prior term can be constructed in different ways. For example, the variational regularization can be explicitly parametrized via VAE. It could also be interesting to explore other degradation transformations. Future work could include the comprehensive design of a convergence guaranteed PnP algorithm for application to various inverse problems. We leave these further extensions to future work.

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