# DIFFUSION ATTRIBUTION SCORE: EVALUATING TRAINING DATA INFLUENCE IN DIFFUSION MODEL

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## ABSTRACT

As diffusion models become increasingly popular, the misuse of copyrighted and private images has emerged as a major concern. One promising solution to mitigate this issue is identifying the contribution of specific training samples in generative models, a process known as data attribution. Existing data attribution methods for diffusion models typically quantify the contribution of a training sample by evaluating the change in diffusion loss when the sample is included or excluded from the training process. However, we argue that the direct usage of diffusion loss cannot represent such a contribution accurately due to the calculation of diffusion loss. Specifically, these approaches measure the divergence between predicted and ground truth distributions, which leads to an indirect comparison between the predicted distributions and cannot represent the variances between model behaviors. To address these issues, we aim to measure the direct comparison between predicted distributions with an attribution score to analyse the training sample importance, which is achieved by Diffusion Attribution Score (*DAS*). Underpinned by rigorous theoretical analysis, we elucidate the effectiveness of DAS. Additionally, we explore strategies to accelerate DAS calculations, facilitating its application to large-scale diffusion models. Our extensive experiments across various datasets and diffusion models demonstrate that DAS significantly surpasses previous benchmarks in terms of the linear datamodelling score, establishing new state-of-the-art performance. Code is available at https://anonymous.4open.science/r/Diffusion-Attribution-Score-411F.

## 1 INTRODUCTION

Diffusion models, highlighted in key studies (Ho et al., 2020; Song et al., 2021b), are advancing significantly in generative machine learning with broad applications from image generation to artistic creation (Saharia et al., 2022; Hertz et al., 2023; Li et al., 2022; Ho et al., 2022). As these models, exemplified by projects like Stable Diffusion (Rombach et al., 2022), become increasingly capable of producing high-quality, varied outputs, the misuse of copyrighted and private images has become a significant concern. A key strategy to address this issue is identifying the contributions of training samples in generative models by evaluating their influence on the generated images, a task known as data attribution. Data attribution in machine learning is essential for tracing model outputs back to influential training examples and understanding how specific data points affect model behavior. In practical applications, data attribution spans various domains, including explaining predictions (Koh & Liang, 2017; Yeh et al., 2018; Ilyas et al., 2022), curating datasets (Khanna et al., 2019; Jia et al., 2021; Liu et al., 2021), and dissecting the mechanisms of generative models like GANs and VAEs (Kong & Chaudhuri, 2021; Terashita et al., 2021), serving to enhance model transparency and explore the effect of training data on model behaviors.

Data attribution methods generally fall into two categories. The first, based on sampling (Shapley et al., 1953; Ghorbani & Zou, 2019; Ilyas et al., 2022), involves retraining models to assess how outputs change with the deletion of specific data. While effective, this method requires training thousands of models—a formidable challenge given today's large-scale models. The second approach uses approximations to assess the change in output for efficiency (Koh & Liang, 2017; Feldman & Zhang, 2020; Pruthi et al., 2020), which may compromise precision. Recent advancements have

led to innovative estimators like TRAK (Park et al., 2023), which linearizes model behavior using a kernel matrix analogous to the Fisher Information Matrix. Additionally, Zheng et al. (2024) proposed D-TRAK, which adapts TRAK to diffusion models by setting the output function as the diffusion loss. However, this setting conducts an indirect comparison between the predicted distributions since the diffusion loss is equivalent to the divergence between predicted and ground truth distributions (Ho et al., 2020). They also reported counterintuitive findings that the diffusion loss can be replaced with other output functions without modifying the form of the attribution score, and the results of these empirical designs surpass diffusion loss with theoretical designs, highlighting the need for a deeper understanding of diffusion models in data attribution.

In this paper, we first present a theoretical analysis to address the challenges associated with directly applying TRAK to diffusion models, which also explain the counter-intuitive observations noted in D-TRAK. Further, we propose a novel attribution method termed Diffusion Attribution Score (*DAS*), specifically designed for diffusion models to quantitatively assess the impact of training samples on model outputs by measuring the divergence between predicted distributions when the sample is included or excluded from the training set, which includes a self-contained derivation. Additionally, we explore several techniques to expedite the computation of DAS, such as compressing models or datasets, enabling its application to large-scale diffusion models and significantly enhancing the practicability of our approach. The primary contributions of our work are summarized as follows:

1. We offer a comprehensive analysis of the limitations when directly applying TRAK to diffusion tasks. This theoretical examination sheds light on the empirical success of D-TRAK and fosters the development of more effective attribution methods.
2. We introduce DAS, a theoretically solid metric designed to directly quantify discrepancies in model outputs, supported by detailed derivations. We also discuss various techniques, such as compressing models or datasets, to accelerate the computation of DAS, facilitating its efficient implementation.
3. DAS demonstrates state-of-the-art performance across multiple benchmarks, notably excelling in linear datamodeling scores.

## 2 Related Works

### 2.1 Data Attribution

The training data exerts a significant influence on the behavior of machine learning models. Data attribution aims to accurately assess the importance of each piece of training data in relation to the desired model outputs. However, methods of data attribution often face the challenge of balancing computational efficiency with accuracy. Sampling-based approaches, such as empirical influence functions (Feldman & Zhang, 2020), Shapley value estimators (Ghorbani & Zou, 2019; Jia et al., 2019), and datamodels (Ilyas et al., 2022), are able to precisely attribute influences to training data but typically necessitate the training of thousands of models to yield dependable results. On the other hand, methods like influence approximation (Koh & Liang, 2017; Schioppa et al., 2022) and gradient agreement scoring (Pruthi et al., 2020) provide computational benefits but may falter in terms of reliability in non-convex settings (Basu et al., 2021; Ilyas et al., 2022; Akyurek et al., 2022).

### 2.2 Data Attribution in Generative Models

The discussed methods address counterfactual questions within the context of discriminative models, focusing primarily on accuracy and model predictions. Extending these methodologies to generative models presents complexities due to the lack of clear labels or definitive ground truth. Research in this area includes efforts to compute influence within Generative Adversarial Networks (GANs) (Terashita et al., 2021) and Variational Autoencoders (VAEs) (Kong & Chaudhuri, 2021). Recently, Park et al. (2023) developed TRAK, a new attribution method that is both effective and computationally feasible for large-scale models. In the realm of diffusion models, earlier research (Dai & Gifford, 2023) has explored influence computation by employing ensembles that necessitate training multiple models on varied subsets of training data—a method less suited for traditionally trained models. In DataInf (Kwon et al., 2024), influence function using the loss gradient

and Hessian have been improved for greater accuracy and efficiency in attributing diffusion models. Journey TRAK (Georgiev et al., 2023) extends TRAK to diffusion models by attributing influence across individual denoising timesteps. Moreover, D-TRAK (Zheng et al., 2024) has revealed surprising results, indicating that theoretically dubious choices in the design of TRAK might enhance performance, highlighting the imperative for further exploration into data attribution within diffusion models. These studies are pivotal in advancing our understanding and fostering the development of instance-based interpretations in unsupervised learning contexts. Additionally, some other methods have been proposed for the data attribution in diffusion by modifying the model. Wang et al. (2023) suggest an alternative termed "customization," which involves adapting or tuning a pretrained text-to-image model through a specially designed training procedure. MONTAGE (Brokman et al., 2025) integrates a novel technique to monitor generations throughout the training via internal model representations. We give a more detail discussion about the existing data attribution methods in diffusion model in Appendix E.3.

## 3 PRELIMINARIES

### 3.1 DIFFUSION MODELS

Our study concentrates on discrete-time diffusion models, specifically Denoising Diffusion Probabilistic Models (DDPMs) (Ho et al., 2020) and Latent Diffusion Models (LDMs) which are foundational to Stable Diffusion (Rombach et al., 2022). This paper grounds all theoretical derivations within the framework of unconditional generation using DDPMs. Below, we detail the notation employed in DDPMs that underpins all further theoretical discussions.

Consider a training set $\mathbb{S} = \{\boldsymbol{z}^{(1)}, ..., \boldsymbol{z}^{(n)}\}$ where each training sample $\boldsymbol{z}^{(i)} := (\boldsymbol{x}^{(i)}, y^{(i)}) \sim \mathcal{Z}$ is an input-label pair[1]. Given an input data distribution $q(\boldsymbol{x})$, DDPMs aim to model a distribution $p_\theta(\boldsymbol{x})$ to approximate $q(\boldsymbol{x})$. The learning process is divided into forward and reverse process, conducted over a series of timesteps in the latent variable space, with $\boldsymbol{x}_0$ denoting the initial image and $\boldsymbol{x}_t$ the latent variables at timestep $t \in [1, T]$. In the forward process, DDPMs sample an observation $\boldsymbol{x}_0$ from $\mathbb{S}$ and add noise on it across $T$ timesteps: $q(\boldsymbol{x}_t|\boldsymbol{x}_{t-1}) := \mathcal{N}(\boldsymbol{x}_t; \sqrt{1-\beta_t}\boldsymbol{x}_{t-1}, \beta_t \boldsymbol{I})$, where $\beta_1, ..., \beta_T$ constitute a variance schedule. As indicated in DDPMs, the latent variable $\boldsymbol{x}_t$ can be express as a linear combination of $\boldsymbol{x}_0$:

$$\boldsymbol{x}_t = \sqrt{\bar{\alpha}_t}\boldsymbol{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, \tag{1}$$

where $\alpha_t := 1 - \beta_t, \bar{\alpha}_t := \prod_{s=1}^{t} \alpha_s$ and $\boldsymbol{\epsilon} \sim \mathcal{N}(0, \boldsymbol{I})$. In the reward process, DDPMs model a distribution $p_\theta(\boldsymbol{x}_{t-1}|\boldsymbol{x}_t)$ by minimizing the KL-divergence from data at $t$:

$$D_{\mathrm{KL}}[p_\theta(\boldsymbol{x}_{t-1}|\boldsymbol{x}_t) \| q(\boldsymbol{x}_{t-1}|\boldsymbol{x}_t, \boldsymbol{x}_0)] = \mathbb{E}_{\boldsymbol{\epsilon}\sim\mathcal{N}(0,\boldsymbol{I})}[\frac{\beta_t^2}{2\alpha_t(1-\bar{\alpha}_t)}||\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t, t)||^2], \tag{2}$$

where $\boldsymbol{\epsilon}_\theta$ is a function implemented by models $\theta$ which can be seen as a noise predictor. A simplified version of objective function for a data point $\boldsymbol{x}$ used to train DDPMs is:

$$\mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta) = \mathbb{E}_{\boldsymbol{\epsilon},t}[||\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t, t) - \boldsymbol{\epsilon}||^2]. \tag{3}$$

### 3.2 DATA ATTRIBUTION

Given a DDPM trained on dataset $\mathbb{S}$, our objective is to trace the influence of the training data on an output generated for a particular sample $\boldsymbol{z}$. This task is commonly referred to as data attribution. A common approach to data attribution involves addressing a counterfactual question: if a training sample $\boldsymbol{z}^{(i)}$ is removed from $\mathbb{S}$ and a model $\theta_{\setminus i}$ is retrained on the remaining subset $\mathbb{S}_{\setminus i}$, the influence of $\boldsymbol{z}^{(i)}$ on the sample $\boldsymbol{z}$ can be assessed by the change in the model output, computed as $f(\boldsymbol{z}, \theta) - f(\boldsymbol{z}, \theta_{\setminus i})$. The function $f(\boldsymbol{z}, \theta)$, which represents the model output, has a variety of choices, such as the direct output of the model or its loss function.

To circumvent the high computational costs associated with retraining models, some data attribution methods compute a scoring function $\tau(\boldsymbol{z}, \mathbb{S}) : \mathcal{Z} \times \mathcal{Z}^n \to \mathbb{R}^n$, which assigns scores reflecting the

[1]In text-to-image task, $\boldsymbol{z} := (\boldsymbol{x}^i, y^i)$ represents an image-caption sample, whereas in unconditional generation, it solely contains an image $\boldsymbol{z} := (\boldsymbol{x})$.

importance of each training sample in $\mathbb{S}$ for the sample $\boldsymbol{z}$ under consideration. For clarity, $\tau(\boldsymbol{z}, \mathbb{S})^{(i)}$ denotes the attribution score assigned to the influence of the individual training sample $\boldsymbol{z}^{(i)}$ on $\boldsymbol{z}$.

TRAK (Park et al., 2023) stands out as a representative data attribution method designed for large-scale models focused on discriminative tasks. TRAK defines the model output function as:

$$f_{\text{TRAK}}(\boldsymbol{z}, \theta) = \log[\hat{p}(\boldsymbol{x}, \theta)/(1 - \hat{p}(\boldsymbol{x}, \theta))], \tag{4}$$

where $\hat{p}(\boldsymbol{x}, \theta)$ represents the probability of the corresponding class $y$ for the sample $\boldsymbol{z}$. TRAK introduces an attribution function $\tau_{\text{TRAK}}(\boldsymbol{z}, \mathbb{S})^{(i)}$ designed to approximate the change in $f_{\text{TRAK}}(\boldsymbol{z}, \theta)$ following the deletion of a data point. This is expressed as:

$$\tau_{\text{TRAK}}(\boldsymbol{z}, \mathbb{S})^{(i)} := \phi(\boldsymbol{z})^\top (\boldsymbol{\Phi}^\top \boldsymbol{\Phi})^{-1} \phi(\boldsymbol{z}^{(i)}) r^{(i)} \approx f_{\text{TRAK}}(\boldsymbol{z}, \theta) - f_{\text{TRAK}}(\boldsymbol{z}, \theta_{\setminus i}), \tag{5}$$

where $r^{(i)} = 1 - \hat{p}(\boldsymbol{x}^{(i)}, \theta)$ denotes the residual for sample $\boldsymbol{z}^{(i)}$. Here, $\phi(\boldsymbol{z}) := \boldsymbol{P}^\top \nabla_\theta f_{\text{TRAK}}(\boldsymbol{z}, \theta)$ and $\boldsymbol{\Phi} := [\phi(\boldsymbol{z}^{(1)}), ..., \phi(\boldsymbol{z}^{(n)})]$ represents the matrix of stacked gradients from $\mathbb{S}$. $\boldsymbol{P} \sim \mathcal{N}(0, 1)^{d \times k}$ is a random projection matrix (Johnson & Lindenstrauss, 1984) employed to reduce the dimension of the gradient $\nabla_\theta f_{\text{TRAK}}(\boldsymbol{z}; \theta) \in \mathbb{R}^d$ to $k$. This function computes a score indicating the influence of a training sample $\boldsymbol{z}_i$ on the sample of interest $\boldsymbol{z}$ in a discriminative model setting.

To adapt TRAK in diffusion models, D-TRAK (Zheng et al., 2024) modifies the output function to align with the objective function described in Eq. 3: $f_{\text{D-TRAK}}(\boldsymbol{z}, \theta) = \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta)$, and simplifies the residual term to an identity matrix $\boldsymbol{I}$. The attribution function in D-TRAK is defined as follows:

$$\tau_{\text{D-TRAK}}(\boldsymbol{z}, \mathbb{S})^{(i)} = \phi(\boldsymbol{z})^\top (\boldsymbol{\Phi}^\top \boldsymbol{\Phi})^{-1} \phi(\boldsymbol{z}^{(i)}) \boldsymbol{I} \approx f_{\text{D-TRAK}}(\boldsymbol{z}, \theta) - f_{\text{D-TRAK}}(\boldsymbol{z}, \theta_{\setminus i}), \tag{6}$$

where $\phi(\boldsymbol{z}) := \boldsymbol{P}^\top \nabla_\theta f_{\text{D-TRAK}}(\boldsymbol{z}, \theta)$ mirrors the gradient term used in TRAK. Interestingly, D-TRAK has observed that substituting the function $f_{\text{D-TRAK}}$ with other functions can yield superior attribution performance. Examples of these include $\mathcal{L}_{\text{Square}}(\boldsymbol{z}, \theta) = \mathbb{E}_{t,\boldsymbol{\epsilon}}[||\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t, t)||^2]$ and $\mathcal{L}_{\text{Average}}(\boldsymbol{z}, \theta) = \mathbb{E}_{t,\boldsymbol{\epsilon}}[\text{Avg}(\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t, t))]$, both of which have enhanced attribution capabilities.

# 4 METHODOLOGY

In this section, we propose a novel data attribution method tailored for diffusion models to assess the impact of training samples on the generative process. Revisiting our objective, we aim to evaluate how training samples influence the generation within a diffusion model. Previous work D-TRAK, has applied TRAK designed for discriminative tasks to generative tasks, utilizing the Simple Loss as the output function. In Section 4.1, we critically analyze the shortcomings of employing a loss function as the output function in diffusion models and propose a more suitable output function.

Additionally, the attribution score is contingent upon the task setting and the selected output function. Alterations in the settings, such as the loss function and output function, significantly affect the derivation of the attribution score. Consequently, we introduce a new attribution metric called Diffusion Attribution Score (DAS), specifically developed for generative tasks in diffusion models, detailed in Section 4.2. In section 4.3, we explore methodologies for applying DAS to large-scale diffusion model and discuss strategies to expedite the computation process.

## 4.1 RETHINKING THE DESIGN OF MODEL OUTPUT FUNCTION

Reviewing the goal of data attribution within diffusion models, suppose we have a set of training data $\mathbb{S}$ and wish to measure the influence of a specific training sample $\boldsymbol{z}^{(i)}$ on a generated sample $\boldsymbol{z}^{\text{gen}}$. This task can be approached by addressing the counterfactual question: How would the model output change for $\boldsymbol{z}^{\text{gen}}$ if we removed $\boldsymbol{z}^{(i)}$ from $\mathbb{S}$ and retrained the model on the subset $\mathbb{S}_{\setminus i}$? D-TRAK proposes using $\tau_{\text{D-TRAK}}$ to approximate this difference based on the Simple Loss:

$$\begin{aligned}\tau_{\text{D-TRAK}}(\boldsymbol{z}^{\text{gen}}, \mathbb{S})^{(i)} &\approx f_{\text{D-TRAK}}(\boldsymbol{z}^{\text{gen}}, \theta) - f_{\text{D-TRAK}}(\boldsymbol{z}^{\text{gen}}, \theta_{\setminus i}) \\ &= \mathbb{E}_{\boldsymbol{\epsilon},t}[||\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t^{\text{gen}}, t) - \boldsymbol{\epsilon}||^2] - \mathbb{E}_{\boldsymbol{\epsilon},t}[||\boldsymbol{\epsilon}_{\theta_{\setminus i}}(\boldsymbol{x}_t^{\text{gen}}, t) - \boldsymbol{\epsilon}||^2],\end{aligned} \tag{7}$$

where $\boldsymbol{\epsilon}_{\theta_{\setminus i}}$ denotes the noise predictor trained on $\mathbb{S}_{\setminus i}$. However, from a distribution perspective, this approach involves an indirect comparison of KL-divergences between the model distributions and the data distribution to indicate the distances between model distributions:

$$\tau_{\text{D-TRAK}}(\boldsymbol{z}^{\text{gen}}, \mathbb{S})^{(i)} \approx D_{\text{KL}}[p_\theta(\boldsymbol{x}^{\text{gen}})||q(\boldsymbol{x}^{\text{gen}})] - D_{\text{KL}}[p_{\theta_{\setminus i}}(\boldsymbol{x}^{\text{gen}})||q(\boldsymbol{x}^{\text{gen}})]. \tag{8}$$

This indirect comparison may introduce errors when evaluating the differences between $p_\theta(\boldsymbol{x}^{\text{gen}})$ and $p_{\theta_{\setminus i}}(\boldsymbol{x}^{\text{gen}})$. For instance, both distributions might approach the data distribution $q(\boldsymbol{x}^{\text{gen}})$ from different directions, yet exhibit similar distances. To enable a more direct comparison, we propose the Diffusion Attribution Score (DAS) to assess the KL-divergence between the predicted distributions:

$$\begin{aligned}\tau_{\text{DAS}}(\boldsymbol{z}^{\text{gen}}, \mathbb{S})^{(i)} &\approx D_{\text{KL}}[p_\theta(\boldsymbol{x}^{\text{gen}})||p_{\theta_{\setminus i}}(\boldsymbol{x}^{\text{gen}})] \\ &\approx \mathbb{E}_{\boldsymbol{\epsilon},t}[||\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t^{\text{gen}}, t) - \boldsymbol{\epsilon}_{\theta_{\setminus i}}(\boldsymbol{x}_t^{\text{gen}}, t)||^2].\end{aligned} \tag{9}$$

The output function in DAS is defined as $f_{\text{DAS}}(\boldsymbol{z}, \theta) = \boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t^{\text{gen}}, t)$, which is able to directly reflect the differences between the noise predictors of the original and the retrained models. Eq. 9 also validates the effectiveness of employing $\mathcal{L}_{\text{square}}$ as the output function, which is formulated as:

$$\tau_{\text{Square}}(\boldsymbol{z}^{\text{gen}}, \mathbb{S})^{(i)} \approx \mathbb{E}_{\boldsymbol{\epsilon},t}[||\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t^{\text{gen}}, t)||^2] - \mathbb{E}_{\boldsymbol{\epsilon},t}[||\boldsymbol{\epsilon}_{\theta_{\setminus i}}(\boldsymbol{x}_t^{\text{gen}}, t)||^2]. \tag{10}$$

This approach also mitigates the influence of indirect comparisons. However, the subtraction used here does not constitute any recognized type of distance metric. Furthermore, the matrices $\boldsymbol{\epsilon}_\theta$ and $\boldsymbol{\epsilon}_{\theta_{\setminus i}}$ which retain the same high dimensionality as the input images $\boldsymbol{x}^{\text{gen}}$, should not be represented merely by scalar values, whether by average or $L^2$ norm, as this leads to a loss of dimensional information. For instance, these matrices might exhibit identical differences across various dimensions, an aspect that scalar representations fail to capture. This dimensional consistency is crucial for understanding the full impact of training data alterations on model outputs.

### 4.2 Diffusion Attribution Score

In diffusion models, the generative process involves a series of generative steps, and the model produces outputs at $T$ different timesteps. In this section, we explore methods to approximate Eq. 9 at a specific timestep $t$ without the necessity for retraining the model, which aims to evaluate the influence of training data on the model's generation. The derivation is divided into two main parts: First, we apply a Taylor expansion to linearize the output function, allowing the difference in the output function to be expressed in terms of differences in model parameters. Next, this relationship is approximated using Newton's method, forming the second part of the proof. By integrating these two components, we derive the complete formulation of DAS.

**Linearing Output Function.** Computing the output of the retrained model $\boldsymbol{\epsilon}_{\theta_{\setminus i}}(\boldsymbol{x}_t^{\text{gen}}, t)$ is computationally expensive. To enhance computational efficiency, we propose linearizing the model output function using its Taylor expansion around the optimal model parameters $\theta^*$ at convergence, simplifying the calculation as follows:

$$f_{\text{DAS}}(\boldsymbol{z}_t, \theta) \approx \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t, t) + \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t, t)^\top (\theta - \theta^*). \tag{11}$$

By substituting Eq. 11 into Eq. 9, we derive:

$$\tau_{\text{DAS}}(\boldsymbol{z}^{\text{gen}}, \mathbb{S})_t^{(i)} \approx \mathbb{E}_{\boldsymbol{\epsilon}}[||\nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{\text{gen}}, t)^\top (\theta^* - \theta^*_{\setminus i})||^2]. \tag{12}$$

The subscript $t$ indicates the attribution score for the model output at timestep $t$. Consequently, the influence of removing a sample from subset $\mathbb{S}_{\setminus i}$ on the model output can now be quantitatively evaluated through the changes in model parameters, which can be measured by the leave-one-out method, thereby significantly reducing the computational overhead.

**Estimating the model parameter.** Consider using the leave-one-out method, the variation of the diffusion model parameters can be assessed by Newton's Method (Pregibon, 1981). By removing a training sample $\boldsymbol{z}^{(i)}$, the counterfactual parameters $\theta^*_{\setminus i}$ can be approximated by taking a single Newton step from the optimal parameters $\theta^*$:

$$\theta^* - \theta^*_{\setminus i} \leftarrow -[\nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_{\setminus i_t}, t)^\top \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_{\setminus i_t}, t)]^{-1} \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_{\setminus i}, t)^\top \boldsymbol{R}_{\setminus i_t}, \tag{13}$$

where $\boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t) := [\boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{(1)}, t), ..., \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{(n)}, t)]$ represents the stacked output matrix for the set $\mathbb{S}$ at timestep $t$, and $\boldsymbol{R}_t := \text{diag}[\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t^{(i)}, t) - \boldsymbol{\epsilon}]$ is a diagonal matrix describing the residuals among $\mathbb{S}$. A detailed proof of Eq. 13 is provided in Appendix A.

Let $\boldsymbol{g}_t(\boldsymbol{x}^{(i)}) = \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{(i)}, t)$ and $\boldsymbol{G}_t(\mathbb{S}) = \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t)$. The inverse term in Eq. 13 can be reformulated as:

$$\boldsymbol{G}_t(\mathbb{S}_{\setminus i})^\top \boldsymbol{G}_t(\mathbb{S}_{\setminus i}) = \boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{G}_t(\mathbb{S}) - \boldsymbol{g}_t(\boldsymbol{x}^{(i)})^\top \boldsymbol{g}_t(\boldsymbol{x}^{(i)}). \tag{14}$$

Applying the Sherman–Morrison formula to Eq. 14 simplifies Eq. 13 as follows:

$$\theta^* - \theta^*_{\setminus i} \leftarrow \frac{[\boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{G}_t(\mathbb{S})]^{-1} \boldsymbol{g}_t(\boldsymbol{x}^{(i)}) \boldsymbol{r}_t^{(i)}}{1 - \boldsymbol{g}_t(\boldsymbol{x}^{(i)})^\top [(\boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{G}_t(\mathbb{S})]^{-1} \boldsymbol{g}_t(\boldsymbol{x}^{(i)})}, \tag{15}$$

where $\boldsymbol{r}_t^{(i)}$ is the $i$-th element of $\boldsymbol{R}_t$. A detailed proof of Eq 15 in provided in Appendix B.

**Diffusion Attribution Score.** By substituting Eq.15 into Eq.12, we derive the formula for computing the DAS at timestep $t$:

$$\tau_{\text{DAS}}(\boldsymbol{z}^{\text{gen}}, \mathbb{S})_t^{(i)} = \mathbb{E}_{\boldsymbol{\epsilon}}[|| \frac{\boldsymbol{g}_t(\boldsymbol{x}^{\text{gen}}) [\boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{G}_t(\mathbb{S})]^{-1} \boldsymbol{g}_t(\boldsymbol{x}^{(i)}) \boldsymbol{r}_t^{(i)}}{1 - \boldsymbol{g}_t(\boldsymbol{x}^{(i)})^\top [(\boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{G}_t(\mathbb{S})]^{-1} \boldsymbol{g}_t(\boldsymbol{x}^{(i)})} ||^2]. \tag{16}$$

This equation estimates the impact of training samples at a specific timestep $t$. The overall influence of a training sample $\boldsymbol{z}^{(i)}$ on the target sample $\boldsymbol{z}^{\text{gen}}$ throughout the entire generation process can be computed as an expectation over timestep $t$. However, directly calculating these expectations is extremely costly. In the next section, we discuss methods to expedite this computation.

### 4.3 Extend DAS to large-scale diffusion model

There are some difficulty in calculating Eq. 16 on large-scale diffusion model, where the calculation of inverse term is extremely costly. Besides, we need to calculate gradients for all training samples in $\mathbb{S}$ and the dimension of gradient is large. In this subsection, we discuss some techniques to speed up the calculation of Eq. 16. These methods can be roughly divided by two kinds, the first is to reduce the dimension of gradient, which accelerates the computation of inverse term. The other is to reduce the computation of gradients, by reducing the calculation of expectations and candidate training samples.

**Reducing Dimension of Gradients.** The dimension of $\boldsymbol{g}_t(\boldsymbol{x}^{(i)})$ is as large as that of the diffusion model itself, which poses a challenge in calculating the inverse term due to its substantial size. There are strategies to simplify this calculation by reducing the dimensionality of the gradient.

One effective method is to apply the Johnson and Lindenstrauss Projection (Johnson & Lindenstrauss, 1984). This technique involves multiplying the gradient vector $\boldsymbol{g}_t(\boldsymbol{x}^{(i)}) \in \mathbb{R}^p$ by a random matrix $\boldsymbol{P} \sim \mathcal{N}(0,1) \in \mathbb{R}^{p \times k} (k \ll p)$, which can preserve inner product with high probability while significantly reducing the dimension of the gradient. This projection method has been validated in previous studies (Malladi et al., 2023; Jacot et al., 2018; Park et al., 2023), demonstrating its efficacy in maintaining the integrity of the gradients while easing computational demands. With the above speed up techniques, we summarize our algorithms in Algorithm 1.

In addition to projection methods, other techniques can be employed to reduce the dimension of gradients in diffusion models. For instance, as noted by Ma et al. (2024), the up-block of the U-Net architecture in diffusion models plays a pivotal role in the generation process. Therefore, we can focus solely on the gradients of the up-block for dimension reduction purposes, optimizing computational efficiency.

Furthermore, when applying large-scale diffusion models to downstream tasks, various strategies have been proposed to fine-tune these models efficiently. One such approach is LoRA (Hu et al., 2022), which involves freezing the pre-trained model weights while utilizing trainable rank decomposition matrices. This significantly reduces the number of trainable parameters required for fine-tuning. Consequently, when attributing the influence of training samples in a fine-tuned dataset, we can selectively use the gradients of these trainable parameters to compute the DAS.

**Reducing Calculation of Expectations.** Computing $t$ times the equation specified in Eq. 16 is highly resource-intensive due to the necessity of calculating inverse terms. To simplify this process, we consider using the average gradient $\overline{\boldsymbol{g}}(\boldsymbol{x})$ and average residual $\overline{\boldsymbol{r}}$, which allows for a singular computation of Eq. 16 to evaluate the overall influence. However, during averaging, these terms may exhibit varying magnitudes across different timesteps, potentially leading to the loss of significant information. To address this, we normalize the gradients and residuals over the entire generative process before averaging:

$$\overline{\boldsymbol{g}}(\boldsymbol{x}^{(i)}) = \frac{1}{T} \sum_t \frac{\boldsymbol{g}_t(\boldsymbol{x}^{(i)})}{\sqrt{\sum_{j=1}^T [\boldsymbol{g}_j(\boldsymbol{x}^{(i)})]^2}}, \ \overline{\boldsymbol{r}}^{(i)} = \frac{1}{T} \sum_t \frac{\boldsymbol{r}_t^{(i)}}{\sqrt{\sum_{j=1}^T [\boldsymbol{r}_j^{(i)}]^2}}. \tag{17}$$

Thus, to attribute the influence of a training sample $\boldsymbol{z}^{(i)}$ on a generated sample $\boldsymbol{z}^{\text{gen}}$ throughout the entire generation process, we redefine Eq. 16 as follows:

$$\tau_{\text{DAS}}(\boldsymbol{z}^{\text{gen}}, \mathbb{S})^{(i)} = ||\frac{\overline{\boldsymbol{g}}(\boldsymbol{x}^{\text{gen}})^\top [\overline{\boldsymbol{G}}(\mathbb{S})^\top \overline{\boldsymbol{G}}(\mathbb{S})]^{-1} \overline{\boldsymbol{g}}(\boldsymbol{x}^{(i)}) \overline{\boldsymbol{r}}^{(i)}}{1 - \overline{\boldsymbol{g}}(\boldsymbol{x}^{(i)})^\top [\overline{\boldsymbol{G}}(\mathbb{S})^\top \overline{\boldsymbol{G}}(\mathbb{S})]^{-1} \overline{\boldsymbol{g}}(\boldsymbol{x}^{(i)})}||^2. \tag{18}$$

**Reducing the amount of timesteps.** The computation of Eq. 17 requires performing back propagation $T$ times to calculate the gradients, which is highly resource-intensive. The need to calculate gradients at multiple timesteps can be effectively reduced by sampling fewer timesteps. This method leverages statistical sampling techniques to estimate gradient behaviors across the generative process while significantly reducing computational overhead.

**Reducing Candidate Training Sample.** The necessity to traverse the entire training set when computing the DAS poses a significant challenge. To alleviate this, a practical approach involves conducting a preliminary screening to identify the most influential training samples. Techniques such as CLIP (Radford et al., 2021) or cosine similarity can be effectively employed to locate samples that are similar to the target. By using these methods, we can form a preliminary candidate set and concentrate DAS computations on this subset, rather than on the entire training dataset.

## 5 Experiments

In this section, we conducted comparative analyses of our method, Diffusion Attribution Score (DAS), against existing data attribution methods under various experimental settings. The primary metric used for assessing attribution performance is the Linear Datamodeling Score (LDS) (Ilyas et al., 2022). Additionally, we evaluated the effectiveness of the speed-up techniques discussed in Section 4.3. Our findings indicate that DAS significantly outperforms other methods in terms of attribution performance, confirming its capability to accurately identify influential training samples.

### 5.1 Datasets and Models

This subsection provides an overview of datasets and diffusion models utilized in our experiments. A detailed description of each dataset and model configuration is available in the Appendix E.1.

**CIFAR10 (32×32).** The CIFAR-10 dataset (Krizhevsky, 2009) contains 50,000 training samples across 10 classes. To evaluate DAS, we conducted experiments on the full CIFAR-10 dataset and a subset, CIFAR-2, with 5,000 samples randomly selected from the "automobile" and "horse" categories for computational efficiency and ablation studies. We used a 35.7M-parameter DDPM (Ho et al., 2020), generating images with a 50-step DDIM solver (Song et al., 2021a).

**CelebA (64×64).** From original CelebA dataset training and test sets (Liu et al., 2015), we extracted 5,000 training samples. Following preprocessing steps outlined by Song et al. (2021b), images were initially center cropped to 140x140 and then resized to 64x64. The diffusion model used mirrors the CIFAR-10 setup but includes an expanded U-Net architecture with 118.8 million parameters.

**ArtBench (256×256)**. ArtBench (Liao et al., 2022) is a dataset of 60,000 images across 10 artistic styles, curated for generating artwork. For our studies, we derived two subsets: ArtBench-2, with 5,000 samples from 2 classes and ArtBench-5, with 12,500 samples from five styles. We fine-tuned a Stable Diffusion model on these datasets using LoRA (Hu et al., 2022) with 128 rank and 25.5M parameters. During inference, new images were generated using a 50-step DDIM solver with a classifier-free guidance scale of 7.5 (Ho & Salimans, 2021).

### 5.2 Evaluation Method for Data attribution

Various methods are available for evaluating data attribution techniques, including the leave-one-out influence method (Koh & Liang, 2017; Basu et al., 2021) and Shapley values (Lundberg & Lee, 2017). These methods, however, present significant computational challenges in large-scale settings. To overcome these issues, the Linear Datamodeling Score (LDS) (Ilyas et al., 2022) has been developed as an effective way to assess data attribution methods.

Given a model trained $\theta$ on datset $\mathbb{S}$, LDS evaluates the effectiveness of a data attribution method $\tau$ by initially sampling a sub-dataset $\mathbb{S}' \subset \mathbb{S}$ and retraining a model $\theta'$ on $\mathbb{S}'$. The attribution-based

Table 1: LDS (%) on CIFAR-2/CIFAR-10 with timesteps (10 or 100).

| Method | CIFAR2 | | | | CIFAR10 | | | |
|---|---|---|---|---|---|---|---|---|
| | Validation | | Generation | | Validation | | Generation | |
| | 10 | 100 | 10 | 100 | 10 | 100 | 10 | 100 |
| Raw pixel (dot prod.) | 7.77±0.57 | | 4.89±0.58 | | 2.50±0.42 | | 2.25±0.39 | |
| Raw pixel (cosine) | 7.87±0.57 | | 5.44±0.57 | | 2.71±0.41 | | 2.61±0.38 | |
| CLIP similarity (dot prod.) | 6.51±1.06 | | 3.00±0.95 | | 2.39±0.41 | | 1.11±0.47 | |
| CLIP similarity (cosine) | 8.54±1.01 | | 4.01±0.85 | | 3.39±0.38 | | 1.69±0.49 | |
| Gradient (dot prod.) | 5.14±0.60 | 5.07±0.55 | 2.80±0.55 | 4.03±0.51 | 0.79±0.43 | 1.40±0.42 | 0.74±0.45 | 1.85±0.54 |
| Gradient (cosine) | 5.08±0.59 | 4.89±0.50 | 2.78±0.54 | 3.92±0.49 | 0.66±0.43 | 1.24±0.41 | 0.58±0.42 | 1.82±0.51 |
| TracInCP | 6.26±0.84 | 5.47±0.87 | 3.76±0.61 | 3.70±0.66 | 0.98±0.44 | 1.26±0.38 | 0.96±0.40 | 1.39±0.54 |
| GAS | 5.78±0.82 | 5.15±0.87 | 3.34±0.56 | 3.30±0.68 | 0.89±0.48 | 1.25±0.38 | 0.90±0.41 | 1.61±0.54 |
| Journey TRAK | / | / | 7.73±0.65 | 12.21±0.46 | / | / | 3.71±0.37 | 7.26±0.43 |
| Relative IF | 11.20±0.51 | 23.43±0.46 | 5.86±0.48 | 15.91±0.39 | 2.76±0.45 | 13.56±0.39 | 2.42±0.36 | 10.65±0.42 |
| Renorm. IF | 10.89±0.46 | 21.46±0.42 | 5.69±0.45 | 14.65±0.37 | 2.73±0.46 | 12.58±0.40 | 2.10±0.34 | 9.34±0.43 |
| TRAK | 11.42±0.49 | 23.59±0.46 | 5.78±0.48 | 15.87±0.39 | 2.93±0.46 | 13.62±0.38 | 2.20±0.38 | 10.33±0.42 |
| D-TRAK | 26.79±0.33 | 33.74±0.37 | 18.82±0.43 | 25.67±0.40 | 14.69±0.46 | 20.56±0.42 | 11.05±0.43 | 16.11±0.36 |
| DAS | **33.90±0.69** | **43.08±0.37** | **20.88±0.27** | **30.68±0.76** | **24.74±0.41** | **33.23±0.35** | **15.24±0.51** | **23.69±0.47** |

output prediction for an interested sample $\boldsymbol{z}^{\text{test}}$ is then calculated as:

$$g_\tau(\boldsymbol{z}^{\text{test}}, \mathbb{S}', \mathbb{S}) := \sum_{\boldsymbol{z}^{(i)} \in \mathbb{S}'} \tau(\boldsymbol{z}^{\text{test}}, \mathbb{S})^{(i)} \tag{19}$$

The underlying premise of LDS is that the predicted output $g_\tau(\boldsymbol{z}, \mathbb{S}', \mathbb{S})$ should correspond closely to the actual model output $f(\boldsymbol{z}^{\text{test}}, \theta')$. To validate this, LDS samples $M$ subsets of fixed size and predicted model outputs across these subsets:

$$\text{LDS}(\tau, \boldsymbol{z}^{\text{test}}) := \rho\left(\{f(\boldsymbol{z}^{\text{test}}, \theta_m) : m \in [M]\}, \{g_\tau(\boldsymbol{z}^{\text{test}}, \mathbb{S}^m; \mathcal{D}) : m \in [M]\}\right) \tag{20}$$

where $\rho$ denotes the Spearman correlation and $\theta_m$ is the model trained on the $m$-th subset $\mathbb{S}^m$.

In our evaluation, we adopt the model output function setup from D-TRAK (Zheng et al., 2024), setting $f(\boldsymbol{z}^{\text{test}}, \theta)$ as the Simple Loss described in Eq. 3 for fair comparison. Although the output functions differ between D-TRAK $f_{\text{D-TRAK}}(\boldsymbol{z}, \theta)$ and our DAS $f_{\text{DAS}}(\boldsymbol{z}, \theta)$, both methods provide consistent rankings among different subsets. We elaborate on the rationale behind the choice of model output function in LDS evaluations in Appendix D.

In the LDS setting, we set $M = 64$ following D-TRAK's methodology, with each subset containing half of the dataset. For CIFAR-2, ArtBench-2, and ArtBench-5, three models per subset are trained using different random seeds for robustness, while for CIFAR-10 and CelebA, a single model is trained per subset. A validation set, comprising 1,000 samples each from the original test set and a generated dataset, serves as $\boldsymbol{z}^{\text{test}}$ for LDS calculations. Model outputs for validation and generated samples are derived using three random seeds. Further details about LDS benchmarks are described in Appendix E.2.

### 5.3 Evaluation for Speed Up techniques

In this section, we detail the experiments conducted to evaluate the effectiveness of the speed-up techniques applied in computing the DAS. The diffusion models used in our experiments vary significantly in complexity, with parameter counts of 35.7M, 118.8M, and 25.5M respectively. These large dimensions pose considerable challenges in calculating the attribution score efficiently. To address this, we implemented speed-up techniques as discussed in Section 4.3. Due to space constraints, the results of these evaluations in this section are reported in the Appendix E.4.

**Normalization.** We evaluated the normalization of gradients and residuals, as proposed in Eq. 17, to stabilize gradient variability across timesteps and enhance computational efficiency and accuracy. By normalizing gradients and residuals across all generation processes before averaging, as detailed in Eq. 17, we observed improved performance for both DAS and D-TRAK (Table 4).

**Number of timesteps.** Computing DAS (Eq. 18) requires balancing effectiveness and computational efficiency, as more timesteps improve performance through averaging but increase back-

Table 2: LDS (%) on ArtBench-2/ArtBench-5 with timesteps (10 or 100)

| Method | ArtBench2 | | | | ArtBench5 | | | |
|---|---|---|---|---|---|---|---|---|
| | Validation | | Generation | | Validation | | Generation | |
| | 10 | 100 | 10 | 100 | 10 | 100 | 10 | 100 |
| Raw pixel (dot prod.) | 2.44±0.56 | | 2.60±0.84 | | 1.84±0.42 | | 2.77±0.80 | |
| Raw pixel (cosine) | 2.58±0.56 | | 2.71±0.86 | | 1.97±0.41 | | 3.22±0.78 | |
| CLIP similarity (dot prod.) | 7.18±0.70 | | 5.33±1.45 | | 5.29±0.45 | | 4.47±1.09 | |
| CLIP similarity (cosine) | 8.62±0.70 | | 8.66±1.31 | | 6.57±0.44 | | 6.63±1.14 | |
| Gradient (dot prod.) | 7.68±0.43 | 16.00±0.51 | 4.07±1.07 | 10.23±1.08 | 4.77±0.36 | 10.02±0.45 | 3.89±0.88 | 8.17±1.02 |
| Gradient (cosine) | 7.72±0.42 | 16.04±0.49 | 4.50±0.97 | 10.71±1.07 | 4.96±0.35 | 9.85±0.44 | 4.14±0.86 | 8.18±1.01 |
| TracInCP | 9.69±0.49 | 17.83±0.58 | 6.36±0.93 | 13.85±1.01 | 5.33±0.37 | 10.87±0.47 | 4.34±0.84 | 9.02±1.04 |
| GAS | 9.65±0.46 | 18.04±0.62 | 6.74±0.82 | 14.27±0.97 | 5.52±0.38 | 10.71±0.48 | 4.48±0.83 | 9.13±1.01 |
| Journey TRAK | / | / | 5.96±0.97 | 11.41±1.02 | / | / | 7.59±0.78 | 13.31±0.68 |
| Relative IF | 12.22±0.43 | 27.25±0.34 | 7.62±0.57 | 19.78±0.69 | 9.77±0.34 | 20.97±0.41 | 8.89±0.59 | 19.56±0.62 |
| Renorm. IF | 11.90±0.43 | 26.49±0.34 | 7.83±0.64 | 19.86±0.71 | 9.57±0.32 | 20.72±0.40 | 8.97±0.58 | 19.38±0.66 |
| TRAK | 12.26±0.42 | 27.28±0.34 | 7.78±0.59 | 20.02±0.69 | 9.79±0.33 | 21.03±0.42 | 8.79±0.59 | 19.54±0.61 |
| D-TRAK | 27.61±0.49 | 32.38±0.41 | 24.16±0.67 | 26.53±0.64 | 22.84±0.37 | 27.46±0.37 | 21.56±0.71 | 23.85±0.71 |
| DAS | **37.96±0.64** | **40.77±0.47** | **30.81±0.31** | **32.31±0.42** | **35.33±0.49** | **37.67±0.68** | **31.74±0.75** | **32.77±0.53** |

propagation costs, especially for large datasets and projection dimensions. Experiments on CIFAR-2 ($k = 4096$) show that while increasing timesteps enhances LDS results (Table 5), using 100 or even 10 timesteps achieves comparable performance to 1000 timesteps with much lower computational demands. Thus, subsequent experiments will default to 10 and 100 timesteps for optimal efficiency.

**Projection.** We applied the projection technique from Section 4.3 to reduce gradient dimensions and analyzed the impact of projection dimension $k$ on LDS performance. As Johnson & Lindenstrauss (1984) disucssed, higher projection dimensions better preserve inner products but increase computational costs. Figure 2 shows that LDS scores for both D-TRAK and DAS improve with increasing $k$ before plateauing. Based on these results, we set $k = 32768$ as the default for experiments.

**Compress Model Parameters.** We also explored techniques to reduce the gradient dimension at the model level. Specifically, we conducted experiments focusing solely on using the up-block of the U-Net to compute gradients. The results, as shown in Table 6, indicate that using only the up-block can achieve competitive performance compared to the full model configuration. Additionally, experiments on ArtBench-2 and ArtBench-5, which involved fine-tuning a Stable Diffusion model with LoRA, further demonstrated the effectiveness of this approach.

**Candidate Training Sample.** Another technique to speed up the process involves reducing the number of training samples considered. We conducted an experiment using CLIP to select the training samples most similar to the target sample. Specifically, we generated a candidate dataset comprising 1,000 training samples that CLIP identified as most similar to the generated or validated image on CIFAR-2. We then computed the attribution scores for this candidate set, assigning a score of 0 to all other samples, and calculated the LDS. The results, detailed in Table 7, validate the efficacy of this method.

### 5.4 Main Experiment

In this section, we evaluate the performance of the Diffusion Attribution Score (DAS) against existing attribution baselines that can be applied in our experimental settings, following methodologies similar to those described by Zheng et al. (2024). We limit the use of techniques to projection only, omitting others like normalization to ensure fair comparisons across different attribution methods. Our primary focus is on post-hoc data attribution methods, which are applied after the model's training is complete. These methods are categorized into similarity-based, gradient-based (without kernel), and gradient-based (with kernel) approaches, with detailed explanations provided in Appendix E.3. The outcomes are documented in Tables 1, 2, and 3, where DAS consistently outperforms existing methods across all datasets.

Compared to D-TRAK, DAS shows substantial improvements. For instance, on a validation set utilizing 100 timesteps, DAS achieves improvements of +9.33% on CIFAR-2, +8.39% on ArtBench-2, and +5.1% on CelebA. In the generation set, the gains continue with +5.01% on CIFAR-2, +5.78%

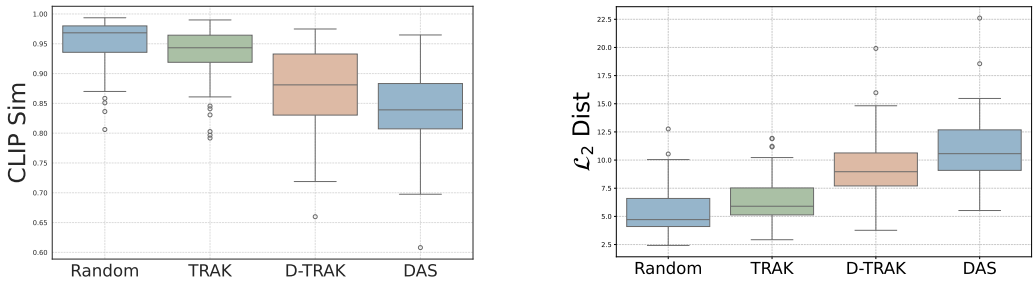


Figure 1: Boxplots of counterfactual evaluation on CIFAR2. We assess the impact of removing the 1,000 highest-scoring training samples and retraining the model using Random, TRAK, D-TRAK, and DAS. The evaluation metrics include pixel-wise $L^2$-Distance and CLIP cosine similarity between 60 generated samples and the corresponding images generated by the retrained models, sampled from the same random seed.

on ArtBench-2, and +9.21% on CelebA. Notably, DAS also achieves significant improvements on larger datasets like CIFAR10 and ArtBench5, outperforming D-TRAK by +12.07% and +10.21% on their respective validation sets.

Other methods generally underperform on larger datasets such as ArtBench5 and CIFAR10 compared to smaller datasets like CIFAR2 and ArtBench2. Conversely, our method performs better on ArtBench5 than on ArtBench2. Remarkably, our findings suggest that while with more timesteps for calculating gradients generally leads to a better approximation of the expectation $\mathbb{E}_t$, DAS, employing only a 10-timestep computation budget, still outperforms D-TRAK, which uses a 100-timestep budget in most cases, which underscores the effectiveness of our approach. Additionally, the modest improvements on CIFAR10 and CelebA may be attributed to the LDS setup for these datasets, which employs only one random seed per subset for training a model, whereas other datasets utilize three random seeds, potentially leading to inaccuracies in LDS evaluation. Another observation is that DAS performs better on the validation set than on the generation set. This could indicate that the quality of generated images may have a significant impact on data attribution performance. However, further investigation is needed to validate this hypothesis.

To more intuitively assess the faithfulness of DAS, we conduct a counter factual experiment. We use different attribution methods to identify the the top-1000 positive influencers on 60 generated images on ArtBench2 and CIFAR2. We utilize 100 timesteps and a projection dimension of $k = 32768$ to identify the top-1000 influencers. These training samples identified by each method are subsequently removed and the model is retrained. Additionally, we conduct a baseline setting where 1,000 training images are randomly removed before retraining. We re-generate the images with same random seed and compare the $L^2$-Distance and CLIP cosine similarity between the origin and counter-factual images. The result on CIFAR2 is reported in Figure 1 while the result for ArtBench2 and the detailed value of experiment is reported in Appendix G due to limited page. DAS has the best ability to identify the top influencers of generation process.

## 6 Conclusion

In this paper, we introduce the Diffusion Attribution Score (DAS) to address the existing gap in data attribution methodologies for generative models. We conducted a comprehensive theoretical analysis to elucidate the inherent challenges in applying TRAK to diffusion models. Subsequently, we derived DAS theoretically based on the properties of diffusion models for attributing data throughout the entire generation process. We also discuss strategies to accelerate computations to extend DAS to large-scale diffusion models. Our extensive experimental evaluations on datasets such as CIFAR, CelebA, and ArtBench demonstrate that DAS consistently surpasses existing baselines in terms of Linear Datamodeling Score evaluation. This paper underscores the crucial role of data attribution in ensuring transparency in the use of diffusion models, especially when dealing with copyrighted or sensitive content. Looking forward, our future work aims to extend DAS to other generative models and real-world applications to further ascertain its effectiveness and applicability.

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## A PROOF OF EQUATION 13

In this section, we provide a detailed proof of Eq. 13. We utilize the Newton Method (Pregibon, 1981) to update the parameters in the diffusion model, where the parameter update $theta'$ is defined as:

$$\theta' \leftarrow \theta + \boldsymbol{H}_{\theta_t}^{-1}(\mathcal{L}_{\text{Simple}}(\theta))\nabla_\theta \mathcal{L}_{\text{Simple}}(\theta), \tag{21}$$

Here, $\boldsymbol{H}_{\theta_t}(\mathcal{L}_{\text{Simple}}(\theta))$ represents the Hessian matrix, and $\nabla_\theta \mathcal{L}_{\text{Simple}}$ is the gradient w.r.t the Simple Loss in the diffusion model. At convergence, the model reaches the global optimum parameter estimate $\theta^*$, satisfying:

$$\boldsymbol{H}_{\theta_t}^{-1}(\mathcal{L}_{\text{Simple}}(\theta^*))\nabla_\theta \mathcal{L}_{\text{Simple}}(\theta^*) = 0. \tag{22}$$

Additionally, the Hessian matrix and gradient associated with the objective function at timestep $t$ are defined as:

$$\mathbf{H}_{\theta^*} = \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t)^\top \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t),\ \nabla_\theta \mathcal{L}(\theta^*) = \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t)^\top \boldsymbol{R}_t, \tag{23}$$

where $\boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t) := [\boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{(1)}, t), ..., \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{(n)}, t)]$ denotes a stacked output matrix on $\mathbb{S}$ at timestep $t$ and $\boldsymbol{R}_t := \text{diag}[\boldsymbol{\epsilon}_\theta(\boldsymbol{x}_t^{(i)}, t) - \boldsymbol{\epsilon}]$ is a diagonal matrix on $\mathbb{S}$ describing the residual among $\mathbb{S}$. Thus, the update defined in Eq. 21 around the optimum parameter$\theta^*$ is:

$$\theta' - \theta \leftarrow [\nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t)^\top \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t)]^{-1} \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_t, t)^\top \boldsymbol{R}_t. \tag{24}$$

Upon deleting a training sample $\boldsymbol{x}^{(i)}$ from $\mathbb{S}$, the counterfactual parameters $\theta^*_{\setminus i}$ can be estimated by applying a single step of Newton's method from the optimal parameter $\theta^*$ with the modified set $\mathbb{S}_{\setminus i}$, as follows:

$$\theta^* - \theta^*_{\setminus i} \leftarrow -[\nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_{\setminus i_t}, t)^\top \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_{\setminus i_t}, t)]^{-1} \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\mathbb{S}_{\setminus i_t}, t)^\top \boldsymbol{R}_{\setminus i_t}. \tag{25}$$

## B PROOF OF EQUATION 15

In this section, we provide a detailed proof of Eq. 15. The Sherman-Morrison formula is defined as:

$$(\boldsymbol{A} + \boldsymbol{u}\boldsymbol{v}^\top)^{-1} = \boldsymbol{A}^{-1} - \frac{\boldsymbol{A}^{-1}\boldsymbol{u}\boldsymbol{v}^\top \boldsymbol{A}^{-1}}{1 + \boldsymbol{v}^\top \boldsymbol{A}^{-1}\boldsymbol{u}}. \tag{26}$$

Let $\boldsymbol{H} = \boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{G}_t(\mathbb{S})$ and $\boldsymbol{u} = \boldsymbol{g}_t(\boldsymbol{x}^{(i)})$. Applying Eq. 26 in Eq. 14, we derive:

$$[\boldsymbol{G}_t(\mathbb{S}_{\setminus i})^\top \boldsymbol{G}_t(\mathbb{S}_{\setminus i})]^{-1} = [\boldsymbol{H} - \boldsymbol{u}\boldsymbol{u}^\top]^{-1} = \boldsymbol{H}^{-1} + \frac{\boldsymbol{H}^{-1}\boldsymbol{u}\boldsymbol{u}^\top \boldsymbol{H}^{-1}}{1 - \boldsymbol{u}^\top \boldsymbol{H}^{-1}\boldsymbol{u}}. \tag{27}$$

Additionally, we have:

$$\boldsymbol{G}_t(\mathbb{S}_{\setminus i})^\top \boldsymbol{R}_{\setminus i_t} = \boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{R}_t - \boldsymbol{g}_t(\boldsymbol{x}^{(i)})^\top \boldsymbol{r}^{(i)}{}_t = -\boldsymbol{u}^\top \boldsymbol{r}_t^{(i)}. \tag{28}$$

Applying Eq. 27 and Eq. 28 to Eq. 25, we obtain:

$$\theta^* - \theta^*_{\setminus i} = [\boldsymbol{H}^{-1} + \frac{\boldsymbol{H}^{-1}\boldsymbol{u}\boldsymbol{u}^\top \boldsymbol{H}^{-1}}{1 - \boldsymbol{u}^\top \boldsymbol{H}^{-1}\boldsymbol{u}}]\boldsymbol{u}^\top \boldsymbol{r}_t^{(i)}. \tag{29}$$

Let $\alpha = \boldsymbol{u}^\top \boldsymbol{H}^{-1}\boldsymbol{u}$. Eq. 29 simplifies to:

$$\begin{aligned}
\theta^* - \theta^*_{\setminus i} &= \boldsymbol{H}^{-1}\boldsymbol{u} \cdot (1 + \frac{\alpha}{1-\alpha})\boldsymbol{r}_t^{(i)} \\
&= \boldsymbol{H}^{-1}\boldsymbol{u} \cdot \frac{1}{1-\alpha}\boldsymbol{r}_t^{(i)} \\
&= \frac{[\boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{G}_t(\mathbb{S})]^{-1}\boldsymbol{g}_t(\boldsymbol{x}^{(i)})\boldsymbol{r}_t^{(i)}}{1 - \boldsymbol{g}_t(\boldsymbol{x}^{(i)})^\top [(\boldsymbol{G}_t(\mathbb{S})^\top \boldsymbol{G}_t(\mathbb{S})]^{-1}\boldsymbol{g}_t(\boldsymbol{x}^{(i)})}.
\end{aligned} \tag{30}$$

## C ALGORITHM OF DAS

In this section, we provide a algorithm about DAS in Algorithm 1.

**Algorithm 1** Diffusion Attribution Score

**Input:** Learning algorithm $\mathcal{A}$, Training dataset $\mathbb{S}$ of size $n$, Training data dimension $p$, Maximum timesteps for diffusion model $T$, Unet output in diffusion model $\boldsymbol{\epsilon}_\theta(\boldsymbol{x}, t)$, Projection dimension $k$, A standard gaussian noise $\boldsymbol{\epsilon} \sim \mathcal{N}(0, \boldsymbol{I})$, Normalization and average method $N$, A generated sample $\boldsymbol{z}^{\text{gen}}$
**Output:** Matrix of attribution scores $\boldsymbol{T} \in \boldsymbol{R}^n$
$\theta^* \leftarrow \mathcal{A}(\mathbb{S})$ ▷ Train a diffusion model on $\mathbb{S}$
$\boldsymbol{P} \sim \mathcal{N}(0,1)^{p \times k}$ ▷ Sample projection matrix
$\boldsymbol{R} \leftarrow 0_{n \times n}$
**for** $i = 1$ to $n$ **do**
  **for** $t = 1$ to $T$ **do**
    $\boldsymbol{\epsilon} \sim \mathcal{N}(0,1)^p$ ▷ Sample a gaussian noise
    $\boldsymbol{g}_t(\boldsymbol{x}^{(i)}) \leftarrow \boldsymbol{P}^\top \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{(i)}, t)$ ▷ Compute gradient on training set and project
    $\boldsymbol{r}_t^{(i)} \leftarrow \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{(i)}, t) - \boldsymbol{\epsilon}$ ▷ Compute residual term
  **end for**
  $\overline{\boldsymbol{g}}(\boldsymbol{x}^{(i)}) = N(\boldsymbol{g}_t(\boldsymbol{x}^{(i)}))$ ▷ Normalize projected gradient term
  $\overline{\boldsymbol{r}}^{(i)} = N(\boldsymbol{r}_t^{(i)})$ ▷ Normalize residual term
**end for**
$\overline{\boldsymbol{G}}(\mathbb{S}) \leftarrow [\overline{\boldsymbol{g}}(\boldsymbol{x}^{(i)}), ..., \overline{\boldsymbol{g}}(\boldsymbol{x}^{(n)})]^\top$
$\overline{\boldsymbol{R}} \leftarrow \text{diag}(\overline{\boldsymbol{r}}^{(1)}, ..., \overline{\boldsymbol{r}}^{(n)})$
**for** $t = 1$ to $T$ **do**
  $\boldsymbol{\epsilon} \sim \mathcal{N}(0,1)^p$ ▷ Sample a gaussian noise
  $\boldsymbol{g}_t(\boldsymbol{x}^{\text{gen}}) \leftarrow \boldsymbol{P}^\top \nabla_\theta \boldsymbol{\epsilon}_{\theta^*}(\boldsymbol{x}_t^{\text{gen}}, t)$ ▷ Compute gradient for generated sample and project
**end for**
$\overline{\boldsymbol{g}}(x^{\text{gen}}) = N(\boldsymbol{g}_t(x^{\text{gen}}))$
$\boldsymbol{T} \leftarrow ||\frac{\overline{\boldsymbol{g}}(x^{\text{gen}})^\top (\overline{\boldsymbol{G}}(\mathbb{S})^\top \overline{\boldsymbol{G}}(\mathbb{S}))^{-1} \overline{\boldsymbol{G}}(\mathbb{S}) \overline{\boldsymbol{R}}}{1 - \overline{\boldsymbol{G}}(\mathbb{S})^\top (\overline{\boldsymbol{G}}(\mathbb{S})^\top \overline{\boldsymbol{G}}(\mathbb{S}))^{-1} \overline{\boldsymbol{G}}(\mathbb{S})}||^2$ ▷ Compute attribution matrix
**return** $(T)$

## D Explanation about the choice of model output function in LDS evaluation

In the LDS framework, we evaluate the ranking correlation between the ground-truth and the predicted model outputs following a data intervention. In our evaluations, the output function used in JourneyTRAK and D-TRAK is shown to provide the same ranking as our DAS output function. Below, we provide a detailed explanation of this alignment.

Defining the function $f(\boldsymbol{z}, \theta)$ as $\mathcal{L}_{\text{Simple}}$, D-TRAK predicts the change in output as:

$$f(\boldsymbol{z}^{\text{gen}}, \theta) - f(\boldsymbol{z}^{\text{gen}}, \theta_m) = \mathbb{E}_{\boldsymbol{\epsilon}, t}[||\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\boldsymbol{z}_t^{\text{gen}}, t)||^2] - \mathbb{E}_{\boldsymbol{\epsilon}}[||\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta_m}(\boldsymbol{z}_t^{\text{gen}}, t)||^2], \tag{31}$$

where $\theta$ and $\theta_m$ represent the models trained on the full dataset $\mathbb{S}$ and a subset $\mathbb{S}_m$, respectively. With the retraining of the model on subset $\mathbb{S}_m$ while fixing all randomness, the real change can be computed as:

$$\begin{aligned} f(\boldsymbol{z}^{\text{gen}}, \theta) - f(\boldsymbol{z}^{\text{gen}}, \theta_m) =& \mathbb{E}_{\boldsymbol{\epsilon}, t}[||\boldsymbol{\epsilon}_\theta(\boldsymbol{z}_t^{\text{gen}}, t) - \boldsymbol{\epsilon}_{\theta_m}(\boldsymbol{z}_t^{\text{gen}}, t)||^2] \\ &+ 2\mathbb{E}_{\boldsymbol{\epsilon}}[(\boldsymbol{\epsilon}_\theta(\boldsymbol{z}_t^{\text{gen}}, t) - \boldsymbol{\epsilon}_{\theta_m}(\boldsymbol{z}_t^{\text{gen}}, t)) \cdot (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta_m}(\boldsymbol{z}_t^{\text{gen}}, t))]. \end{aligned} \tag{32}$$

The first expectation represents the predicted output change in DAS as well as the ground truth output, while the term $(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta_m}(\boldsymbol{z}_t^{\text{gen}}, t))$ is recognized as the error of the MMSE estimator.

In our evaluations, the sampling noise $\boldsymbol{\epsilon}$ is fixed and $\boldsymbol{\epsilon}_\theta(\boldsymbol{z}_t^{\text{gen}}, t)$ is a given value since the model $\theta$ is frozen. Therefore, the second term equals zero, following the orthogonality principle. It can be stated as a more general result that,

$$\forall f\ , \mathbb{E}[f(\boldsymbol{\epsilon}_{\theta_m}, \boldsymbol{z}^{\text{gen}}) \cdot (\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta_m}(\boldsymbol{z}^{\text{gen}}))] = 0. \tag{33}$$

The error $(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta_m}(\boldsymbol{z}^{\text{gen}}))$ must be orthogonal to any estimator $f$. If not, we could use $f$ to construct an estimator with a lower MSE than $\boldsymbol{\epsilon}_{\theta_m}(x_{gen})$, contradicting our assumption that $(\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\theta_m}(\boldsymbol{z}^{\text{gen}}))$

is the MMSE estimator. Applications of the orthogonality principle in diffusion models have been similarly proposed (Kong et al., 2024; Banerjee et al., 2005). Thus, given that we have a frozen model $\theta$, fixed sampling noise $\boldsymbol{\epsilon}$ and a specific generated sample $\boldsymbol{z}^{\text{gen}}$, the change in the output function in DAS provides the same rank as the one in JourneyTRAK and D-TRAK.

# E Implementation details

## E.1 Datasets and Models

**CIFAR10(32×32)**. The CIFAR-10 dataset, introduced by Krizhevsky (2009), consists of 50,000 training images across various classes. For the Linear Datamodeling Score evluation, we utilize a subset of 1,000 images randomly selected from CIFAR-10's test set. To manage computational demands effectively, we also create a smaller subset, CIFAR-2, which includes 5,000 training images and 1,000 validation images specifically drawn from the "automobile" and "horse" categories of CIFAR-10's training and test sets, respectively.

In our CIFAR experiments, we employ the architecture and settings of the Denoising Diffusion Probabilistic Models (DDPMs) as outlined by Ho et al. (2020). The model is configured with approximately 35.7 million parameters ($d = 35.7 \times 10^6$ for $\theta \in \mathbb{R}^d$). We set the maximum number of timesteps ($T$) at 1,000 with a linear variance schedule for the forward diffusion process, beginning at $\beta_1 = 10^{-4}$ and escalating to $\beta_T = 0.02$. Additional model specifications include a dropout rate of 0.1 and the use of the AdamW optimizer (Loshchilov & Hutter, 2019) with a weight decay of $10^{-6}$. Data augmentation techniques such as random horizontal flips are employed to enhance model robustness. The training process spans 200 epochs with a batch size of 128, using a cosine annealing learning rate schedule that incorporates a warm-up period covering 10% of the training duration, beginning from an initial learning rate of $10^{-4}$. During inference, new images are generated using the 50-step Denoising Diffusion Implicit Models (DDIM) solver (Song et al., 2021a).

**CelebA(64×64)**. We selected 5,000 training samples and 1,000 validation samples from the original training and test sets of CelebA (Liu et al., 2015), respectively. Following the preprocessing method described by Song et al. (2021b), we first center-cropped the images to 140×140 pixels and then resized them to 64×64 pixels. For the CelebA experiments, we adapted the architecture to accommodate a 64×64 resolution while employing a similar unconditional DDPM implementation as used for CIFAR-10. However, the U-Net architecture was expanded to 118.8 million parameters to better capture the increased complexity of the CelebA dataset. The hyperparameters, including the variance schedule, optimizer settings, and training protocol, were kept consistent with those used for the CIFAR-10 experiments.

**ArtBench(256×256)**. ArtBench (Liao et al., 2022) is a dataset specifically designed for generating artwork, comprising 60,000 images across 10 unique artistic styles. Each style contributes 5,000 training images and 1,000 testing images. We introduce two subsets from this dataset for focused evaluation: ArtBench-2 and ArtBench-5. ArtBench-2 features 5,000 training and 1,000 validation images selected from the "post-impressionism" and "ukiyo-e" styles, extracted from a total of 10,000 training and 2,000 test images. ArtBench-5 includes 12,500 training and 1,000 validation images drawn from a larger pool of 25,000 training and 5,000 test images across five styles: "post-impressionism," "ukiyo-e," "romanticism," "renaissance," and "baroque."

For our experiments on ArtBench, we fine-tune a Stable Diffusion model (Rombach et al., 2022) using Low-Rank Adaptation (LoRA) (Hu et al., 2022) with a rank of 128, amounting to 25.5 million parameters. We adapt a pre-trained Stable Diffusion checkpoint from a resolution of 512×512 to 256×256 to align with the ArtBench specifications. The model is trained conditionally using textual prompts specific to each style, such as "a class painting," e.g., "a romanticism painting." We set the dropout rate at 0.1 and employ the AdamW optimizer with a weight decay of $10^{-6}$. Data augmentation is performed via random horizontal flips. The training is conducted over 100 epochs with a batch size of 64, under a cosine annealing learning rate schedule that includes a 0.1 fraction warm-up period starting from an initial rate of $3 \times 10^{-4}$. During the inference phase, we generate new images using the 50-step DDIM solver with a classifier-free guidance scale of 7.5 (Ho & Salimans, 2021).

### E.2 LDS EVALUATION SETUP

For the LDS evaluation, we construct 64 distinct subsets $\mathbb{S}_m$ from the training dataset $\mathbb{S}$, each constituting 50% of the total training set size. To ensure robustness and mitigate any effects of randomness, three separate models are trained on each subset using different random seeds. The LDS is computed by measuring the Spearman rank correlation between the model outputs and the attribution-based output predictions for selected samples. Specifically, we evaluate the Simple Loss $\mathcal{L}_{\text{Simple}}(\boldsymbol{z}, \theta)$ as defined in Eq 3 for samples of interest from both the validation and generation sets. To better approximate the expectation $\mathbb{E}_t$, we utilize 1,000 timesteps, evenly spaced within the range $[1, T]$. At each timestep, three instances of standard Gaussian noise $\boldsymbol{\epsilon} \sim \mathcal{N}(0, \boldsymbol{I})$ are introduced to approximate the expectation $\mathbb{E}_{\boldsymbol{\epsilon}}$. The calculated LDS values are then averaged across the selected samples from both the validation and generation sets to determine the overall LDS performance.

### E.3 BASELINES

In this paper, our focus is primarily on post-hoc data attribution, which entails applying attribution methods after the completion of model training. These methods are particularly advantageous as they do not impose additional constraints during the model training phase, making them well-suited for practical applications (Ribeiro et al., 2016).

Following the work of Hammoudeh & Lowd (2024), we evaluate various attribution baselines that are compatible with our experimental framework. We exclude certain methods that are not feasible for our settings, such as the Leave-One-Out approach (Cook, 1977) and the Shapley Value method (Shapley et al., 1953; Ghorbani & Zou, 2019). These methods, although foundational, do not align well with the requirements of DDPMs due to their intensive computational demands and model-specific limitations. Additionally, we do not consider techniques like Representer Point (Yeh et al., 2018), which are tailored for specific tasks and models, and thus are incompatible with DDPMs. Moreover, we disregard HYDRA (Chen et al., 2021), which, although related to TracInCP (Pruthi et al., 2020), compromises precision for incremental speed improvements as critiqued by Hammoudeh & Lowd (2024).

Two works focus on diffusion model that also fall outside our framework. Dai & Gifford (2023) propose a method for training data attribution on diffusion models using machine unlearning (Bourtoule et al., 2021); however, their approach necessitates a specific machine unlearning training process, making it non-post-hoc and thus unsuitable for standard settings. Similarly, Wang et al. (2023) acknowledge the current challenges in conducting post-hoc training influence analysis with existing methods. They suggest an alternative termed "customization," which involves adapting or tuning a pretrained text-to-image model through a specially designed training procedure.

Building upon recent advancements, Park et al. (2023) introduced an innovative estimator that leverages a kernel matrix analogous to the Fisher Information Matrix (FIM), aiming to linearize the model's behavior. This approach integrates classical random projection techniques to expedite the computation of Hessian-based influence functions (Koh & Liang, 2017), which are typically computationally intensive. Zheng et al. (2024) adapted TRAK to diffusion models, empirically designing the model output function. Intriguingly, they reported that the theoretically designed model output function in TRAK performs poorly in unsupervised settings within diffusion models. However, they did not provide a theoretical explanation for these empirical findings, leaving a gap in understanding the underlying mechanics.

Our study concentrates on retraining-free methods, which we categorize into three distinct types: similarity-based, gradient-based (without kernel), and gradient-based (with kernel) methods. For similarity-based approaches, we consider Raw pixel similarity and CLIP similarity (Radford et al., 2021). The gradient-based methods without a kernel include techniques such as Gradient (Charpiat et al., 2019), TracInCP (Pruthi et al., 2020) and GAS (Hammoudeh & Lowd, 2022). In the domain of gradient-based methods with a kernel, we explore several methods including D-TRAK (Zheng et al., 2024), TRAK (Park et al., 2023), Relative Influence (Barshan et al., 2020), Renormalized Influence (Hammoudeh & Lowd, 2022), and Journey TRAK (Georgiev et al., 2023).

We next provide definition and implementation details of the baselines used in Section 5.4.

**Raw pixel.** This method employs a naive similarity-based approach for data attribution by using the raw image data itself as the representation. Specifically, for experiments on ArtBench, which

utilizes latent diffusion models (Rombach et al., 2022), we represent the images through the VAE encodings (Van Den Oord et al., 2017) of the raw image. The attribution score is calculated by computing either the dot product or cosine similarity between the sample of interest and each training sample, facilitating a straightforward assessment of similarity based on pixel values.

**CLIP Similarity.** This method represents another similarity-based approach to data attribution. Each sample is encoded into an embedding using the CLIP model (Radford et al., 2021), which captures semantic and contextual nuances of the visual content. The attribution score is then determined by computing either the dot product or cosine similarity between the CLIP embedding of the target sample and those of the training samples. This method leverages the rich representational power of CLIP embeddings to ascertain the contribution of training samples to the generation or classification of new samples.

**Gradient.** This method employs a gradient-based approach to estimate the influence of training samples, as described by Charpiat et al. (2019). The attribution score is calculated by taking the dot product or cosine similarity between the gradients of the sample of interest and those of each training sample. This technique quantifies how much the gradient (indicative of the training sample's influence on the loss) of a particular training sample aligns with the gradient of the sample of interest, providing insights into which training samples most significantly affect the model's output.

$$\tau(\boldsymbol{z}, \mathbb{S})^i = (\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta))^\top \cdot (\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}})(\boldsymbol{x}^{(i)}, \theta^*),$$
$$\tau(\boldsymbol{z}, \mathbb{S})^i = \frac{(\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta))^\top \cdot (\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^{(i)}, \theta))}{||\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta)||^\top ||\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^{(i)}, \theta)||}.$$

**TracInCP.** We implement the TracInCP estimator, as outlined by Pruthi et al. (2020), which quantifies the influence of training samples using the following formula:

$$\tau(\boldsymbol{z}, \mathbb{S})^i = \frac{1}{C} \Sigma_{c=1}^{C} (\boldsymbol{P}_c^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta^c))^\top \cdot (\boldsymbol{P}_c^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^i, \theta^c)),$$

where $C$ represents the number of model checkpoints selected evenly from the training trajectory, and $\theta^c$ denotes the model parameters at each checkpoint. For our analysis, we select four specific checkpoints along the training trajectory to ensure a comprehensive evaluation of the influence over different phases of learning. For example, in the CIFAR-2 experiment, the chosen checkpoints occur at epochs 50, 100, 150, and 200, capturing snapshots of the model's development and adaptation.

**GAS.** The GAS method is essentially a "renormalized" version of TracInCP that employs cosine similarity for estimating influence, rather than relying on raw dot products. This method was introduced by Hammoudeh & Lowd (2022) and aims to refine the estimation of influence by normalizing the gradients. This approach allows for a more nuanced comparison between gradients, considering not only their directions but also normalizing their magnitudes to focus solely on the directionality of influence.

**TRAK.** The retraining-free version of TRAK (Park et al., 2023) utilizes a model's trained state to estimate the influence of training samples without the need for retraining the model at each evaluation step. This version is implemented using the following equations:

$$\boldsymbol{\Phi}_{\text{TRAK}} = \left[\boldsymbol{\Phi}(\boldsymbol{x}^1), \cdots, \boldsymbol{\Phi}(\boldsymbol{x}^N)\right]^\top, \text{ where } \boldsymbol{\Phi}(\boldsymbol{x}) = \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta),$$
$$\tau(\boldsymbol{z}, \mathbb{S})^i = (\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta))^\top \cdot \left(\boldsymbol{\Phi}_{\text{TRAK}}{}^\top \boldsymbol{\Phi}_{\text{TRAK}} + \lambda \boldsymbol{I}\right)^{-1} \cdot \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^i, \theta),$$

where $\lambda \boldsymbol{I}$ is included for numerical stability and regularization. The impact of this term is further explored in Appendix F.

**D-TRAK.** Simliar to TRAK, as elaborated in Section 3, we adapt the D-TRAK (Zheng et al., 2024) as detailed in Eq 6. We implent the model output function $f(\boldsymbol{z}, \theta)$ as $\mathcal{L}_{\text{Square}}$. The D-TRAK is implemented using the following equations:

$$\boldsymbol{\Phi}_{\text{D-TRAK}} = \left[\boldsymbol{\Phi}(\boldsymbol{x}^1), \cdots, \boldsymbol{\Phi}(\boldsymbol{x}^N)\right]^\top, \text{ where } \boldsymbol{\Phi}(\boldsymbol{x}) = \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta),$$
$$\tau(\boldsymbol{z}, \mathbb{S})^i = (\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta))^\top \cdot \left(\boldsymbol{\Phi}_{\text{TRAK}}{}^\top \boldsymbol{\Phi}_{\text{TRAK}} + \lambda \boldsymbol{I}\right)^{-1} \cdot \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^i, \theta),$$

where $\lambda I$ is also included for numerical stability and regularization as TRAK. Additionally, the output function $f(\boldsymbol{z}, \theta)$ could be replaced to other functions.

**Relative Influence.** Barshan et al. (2020) introduce the $\theta$-relative influence functions estimator, which normalizes the influence functions estimator from Koh & Liang (2017) by the magnitude of the Hessian-vector product (HVP). This normalization enhances the interpretability of influence scores by adjusting for the impact magnitude. We have adapted this method to our experimental framework by incorporating scalability optimizations from TRAK. The adapted equation for the Relative Influence is formulated as follows:

$$\tau(\boldsymbol{z}, \mathbb{S})^{(i)} = \frac{(\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta))^\top \cdot \left( \boldsymbol{\Phi}_{\text{TRAK}}{}^\top \boldsymbol{\Phi}_{\text{TRAK}} + \lambda \boldsymbol{I} \right)^{-1} \cdot \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^{(i)}, \theta^*)}{|| \left( \boldsymbol{\Phi}_{\text{TRAK}}{}^\top \boldsymbol{\Phi}_{\text{TRAK}} + \lambda I \right)^{-1} \cdot \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^{(i)}, \theta^*) ||}.$$

**Renormalized Influence.** Hammoudeh & Lowd (2022) propose a method to renormalize influence by considering the magnitude of the training sample's gradients. This approach emphasizes the relative strength of each sample's impact on the model, making the influence scores more interpretable and contextually relevant. We have adapted this method to our settings by incorporating TRAK's scalability optimizations, which are articulated as:

$$\tau(\boldsymbol{z}, \mathbb{S})^{(i)} = \frac{(\boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}, \theta))^\top \cdot \left( \boldsymbol{\Phi}_{\text{TRAK}}{}^\top \boldsymbol{\Phi}_{\text{TRAK}} + \lambda \boldsymbol{I} \right)^{-1} \cdot \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^{(i)}, \theta)}{|| \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^{(i)}, \theta) ||}.$$

**Journey TRAK.** Journey TRAK (Georgiev et al., 2023) focuses on attributing influence to noisy images $\boldsymbol{x}_t$ at a specific timestep $t$ throughout the generative process. In contrast, our approach aims to attribute the final generated image $\boldsymbol{x}^{\text{gen}}$, necessitating an adaptation of their method to our context. We average the attributions across the generation timesteps, detailed in the following equation:

$$\tau(\boldsymbol{z}, \mathbb{S})^{(i)} = \frac{1}{T'} \Sigma_{t=1}^{T'} (\boldsymbol{P}^\top \nabla_\theta \mathcal{L}^t_{\text{Simple}}(\boldsymbol{x}_t, \theta))^\top \cdot \left( \boldsymbol{\Phi}_{\text{TRAK}}{}^\top \boldsymbol{\Phi}_{\text{TRAK}} + \lambda \boldsymbol{I} \right)^{-1} \cdot \boldsymbol{P}^\top \nabla_\theta \mathcal{L}_{\text{Simple}}(\boldsymbol{x}^{(i)}, \theta),$$

where $T'$ represents the number of inference steps, set at $50$, and $\boldsymbol{x}_t$ denotes the noisy image generated along the sampling trajectory.

### E.4 Experiments Result

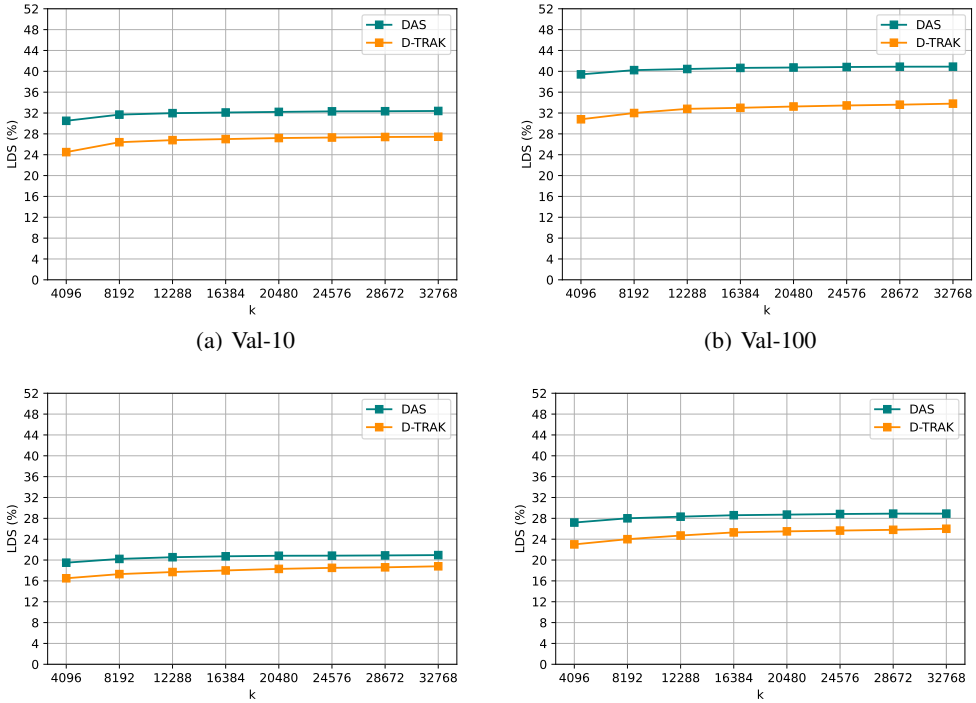
In this subsection, we present the outcomes of experiments detailed in Section 5.3. The results, which illustrate the effectiveness of various techniques designed to expedite computational processes in data attribution, are summarized in several tables and figures. These include Table 4, Table 5, Table 6, and Table 7, as well as Figure 2. Each of these displays key findings relevant to the specific speed-up technique tested, providing a comprehensive view of their impacts on attribution performance.

## F Ablation Studies

We conduct additional ablation studies to evaluate the performance differences between D-TRAK and DAS. In this section, CIFAR-2 serves as our primary setting. Further details on these settings are available in Appendix E.1. We establish the corresponding LDS benchmarks as outlined in Appendix E.2.

**Checkpoint selection** Following the approach outlined by Pruthi et al. (2020), we investigated the impact of utilizing different model checkpoints for gradient computation. As depicted in Figures 3, our method achieves the highest LDS when utilizing the final checkpoint. This finding suggests that the later stages of model training provide the most accurate reflections of data influence, aligning gradients more closely with the ultimate model performance. Determining the optimal checkpoint for achieving the best LDS score requires multiple attributions to be computed, which significantly increases the computational expense. Additionally, in many practical scenarios, access may be limited exclusively to the final model checkpoint. This constraint highlights the importance of developing efficient methods that can deliver precise attributions even when earlier checkpoints are not available.

**Value of $\lambda$** In our computation of the inverse of the Hessian matrix within the DAS framework, we incorporate the regularization parameter $\lambda$, as recommended by Hastie (2020), to ensure numerical




(a) Val-10 (b) Val-100

(c) Gen-10 (d) Val-100

Figure 2: The LDS(%) on CIFAR-2 under different projection dimension $k$. We consider 10 and 100 timesteps selected to be evenly spaced within the interval $[1, T]$, which are used to approximate the expectation $\mathbb{E}_t$. For each sampled timestep, we sample one standard Gaussian noise $\epsilon \sim \mathcal{N}(\epsilon|0, I)$ to approximate the expectation $\mathbb{E}_\epsilon$.

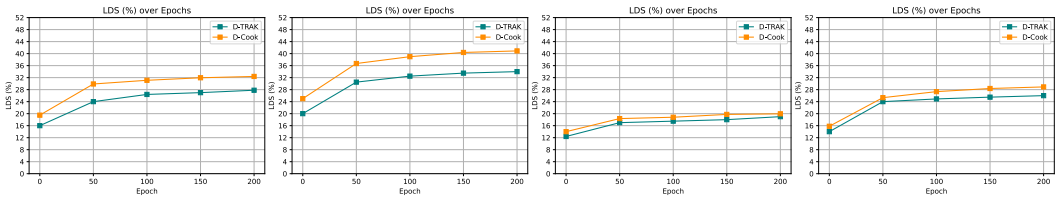



Figure 3: The LDS(%) on CIFAR-2 varies across different checkpoints. We analyze the data using 10 and 100 timesteps, evenly spaced within the interval $[1, T]$, to approximate the expectation $\mathbb{E}_t$. At each sampled timestep, we introduce one standard Gaussian noise $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ to approximate the expectation $\mathbb{E}_\epsilon$. We set the projection dimension $k = 32768$.

Table 3: LDS (%) on CelebA with timesteps (10 or 100)

| Results on CelebA | | | | |
|---|---|---|---|---|
| Method | Validation | | Generation | |
| | 10 | 100 | 10 | 100 |
| Raw pixel (dot prod.) | 5.58±0.73 | | -4.94±1.58 | |
| Raw pixel (cosine) | 6.16±0.75 | | -4.38±1.63 | |
| CLIP similarity (dot prod.) | 8.87±1.14 | | 2.51±1.13 | |
| CLIP similarity (cosine) | 10.92±0.87 | | 3.03±1.13 | |
| Gradient (dot prod.) | 3.82±0.50 | 4.89±0.65 | 3.83±1.06 | 4.53±0.84 |
| Gradient (cosine) | 3.65±0.52 | 4.79±0.68 | 3.86±0.96 | 4.40±0.86 |
| TracInCP | 5.14±0.75 | 4.89±0.86 | 5.18±1.05 | 4.50±0.93 |
| GAS | 5.44±0.68 | 5.19±0.64 | 4.69±0.97 | 3.98±0.97 |
| Journey TRAK | / | / | 6.53±1.06 | 10.87±0.84 |
| Relative IF | 11.10±0.51 | 19.89±0.50 | 6.80±0.77 | 14.66±0.70 |
| Renorm. IF | 11.01±0.50 | 18.67±0.51 | 6.74±0.82 | 13.24±0.71 |
| TRAK | 11.28±0.47 | 20.02±0.47 | 7.02±0.89 | 14.71±0.70 |
| D-TRAK | 22.83±0.51 | 28.69±0.44 | 16.84±0.54 | 21.47±0.48 |
| DAS | **29.38±0.51** | **33.79±0.23** | **28.73±0.49** | **30.68±0.31** |

Table 4: We compare D-TRAK and our methods DAS with the normalization and without normalization on CIFAR2. Besides, we also select 10, 100 and 1000 timesteps evenly spaced within the interval $[1, T]$ and calculate the average of LDS(%) among the timesteps.

| Method | Normalization | Validation | | | Generation | | |
|---|---|---|---|---|---|---|---|
| | | 10 | 100 | 1000 | 10 | 100 | 1000 |
| D-TRAK | No Normalization | 24.78 | 30.81 | 32.37 | 16.20 | 22.62 | 23.94 |
| | Normalization | 26.11 | 31.50 | 32.51 | 17.09 | 22.92 | 24.10 |
| DAS | No Normalization | 33.04 | 42.02 | 43.13 | 20.01 | 29.58 | 30.58 |
| | Normalization | 33.77 | 42.26 | 43.28 | 21.24 | 29.60 | 30.87 |

stability and effective regularization. Traditionally, $\lambda$ is set to a value close to zero; however, in our experiments, a larger $\lambda$ proved necessary. This is because we use the generalized Gauss-Newton (GGN) matrix to approximate the Hessian in the computation of DAS. Unlike the Hessian, the GGN is positive semi-definite (PSD), meaning it does not model negative curvature in any direction. The main issue with negative curvature is that the quadratic model predicts unbounded improvement in the objective when moving in those directions. Without certain techniques, minimizing the quadratic model results in infinitely large updates along these directions. To address this, several methods have been proposed, such as the damping technique discussed in (Martens, 2020). In our paper, we adopt the linear damping technique $\lambda I$ used in (Zheng et al., 2024; Georgiev et al., 2023), which has proven effective on diffusion models. We show how $\lambda$ influence the LDS result in Figure 4, Figure 5, Figure 6, Figure 7 and Figure 8.

Table 5: We compare our methods with TRAK and D-TRAK by LDS method on CIFAR-2 among different selected timesteps. The projected dimension $k = 4096$.

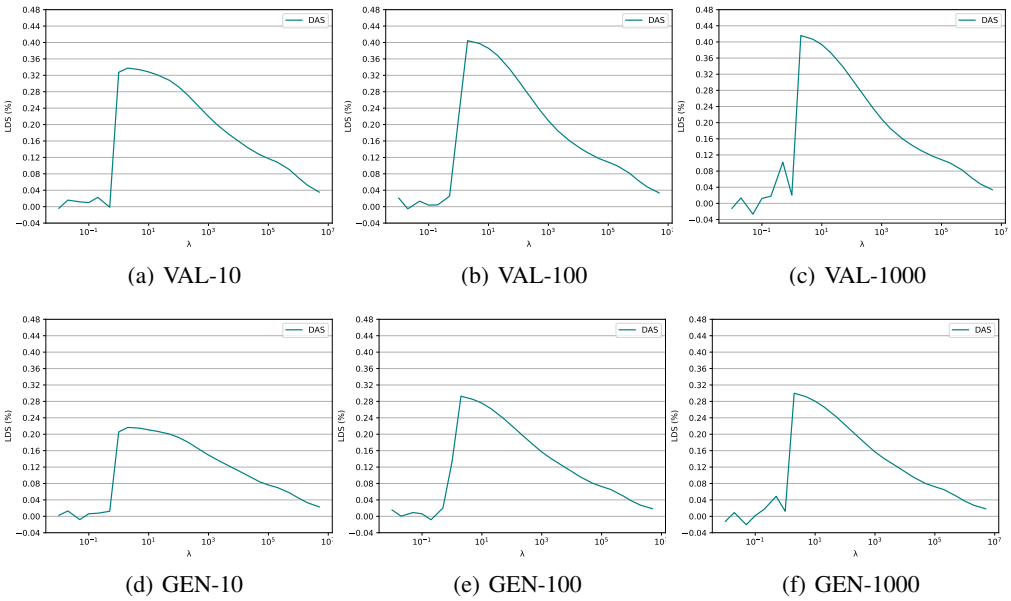
| Method | Validation | | | Generation | | |
|---|---|---|---|---|---|---|
| | 10 | 100 | 1000 | 10 | 100 | 1000 |
| TRAK | 10.66 | 19.50 | 22.42 | 5.14 | 12.05 | 15.46 |
| D-TRAK | 24.91 | 30.91 | 32.39 | 16.76 | 22.62 | 23.94 |
| DAS | **33.04** | **42.02** | **43.13** | **20.01** | **29.58** | **30.58** |

Table 6: We compute DAS only with the Up-Block gradients in U-Net and evaluate by LDS method on CIFAR-2 among different selected timesteps. The projected dimension $k = 32768$.

| Method | Validation | | Generation | |
|---|---|---|---|---|
| | 10 | 100 | 10 | 100 |
| D-TRAK | 24.91 | 30.91 | 16.76 | 22.62 |
| DAS(Up-Block) | 32.60 | 37.90 | 18.47 | 27.54 |
| DAS(U-Net) | **33.77** | **42.26** | **21.24** | **29.60** |

Table 7: We compute DAS only with the Up-Block gradients in U-Net and evaluate by LDS method on CIFAR-2 among different selected timesteps. The projected dimension $k = 32768$.

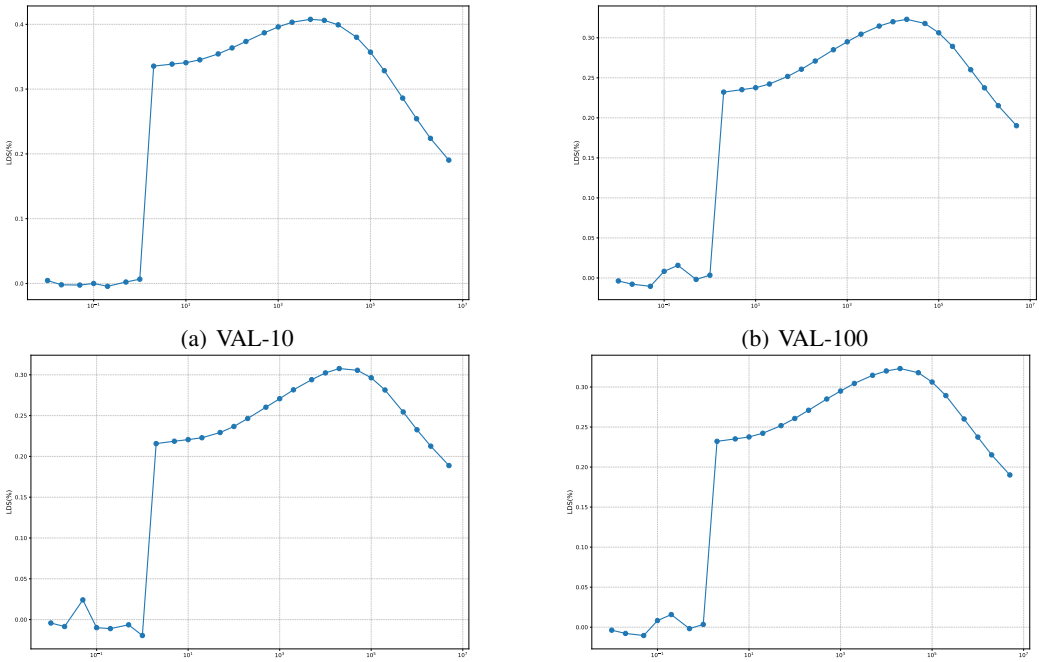
| Method | Validation | | Generation | |
|---|---|---|---|---|
| | 10 | 100 | 10 | 100 |
| D-TRAK | 24.91 | 30.91 | 16.76 | 22.62 |
| DAS(Candidate Set) | 31.53 | 37.75 | 17.73 | 23.31 |
| DAS(Entire Training Set) | **33.77** | **42.26** | **21.24** | **29.60** |




(a) VAL-10 (b) VAL-100 (c) VAL-1000

(d) GEN-10 (e) GEN-100 (f) GEN-1000

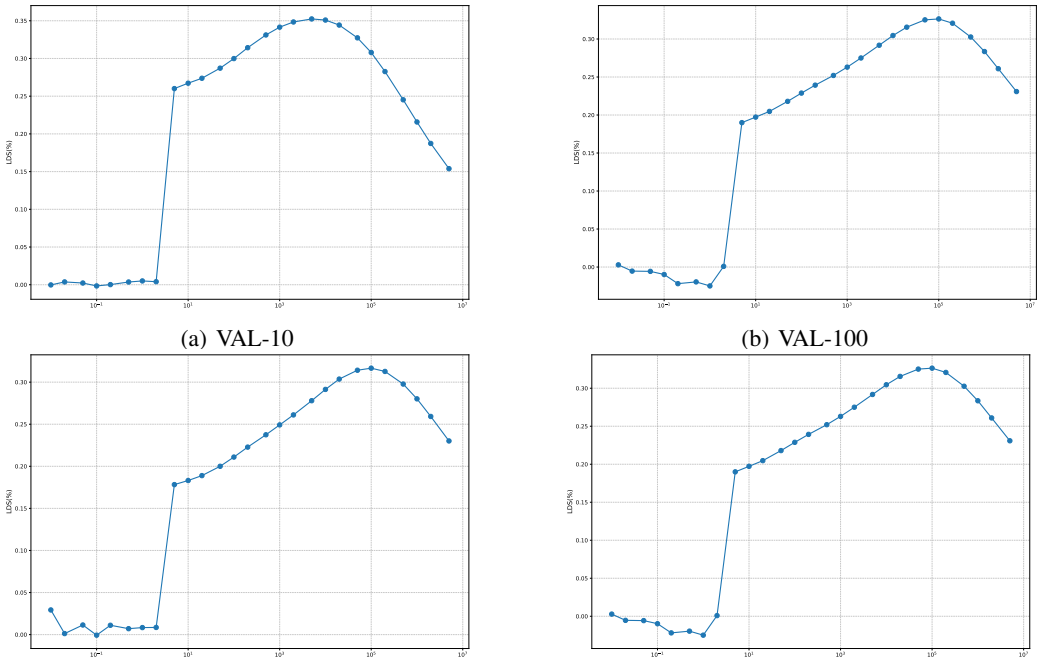
Figure 4: LDS (%) on CIFAR-2 under different $\lambda$. We consider 10, 100, and 1000 timesteps selected to be evenly spaced within the interval $[1, T]$, which are used to approximate the expectation $\mathbb{E}_t$. We set $k = 4096$.




(a) VAL-10 (b) VAL-100

(c) GEN-10 (d) GEN-100

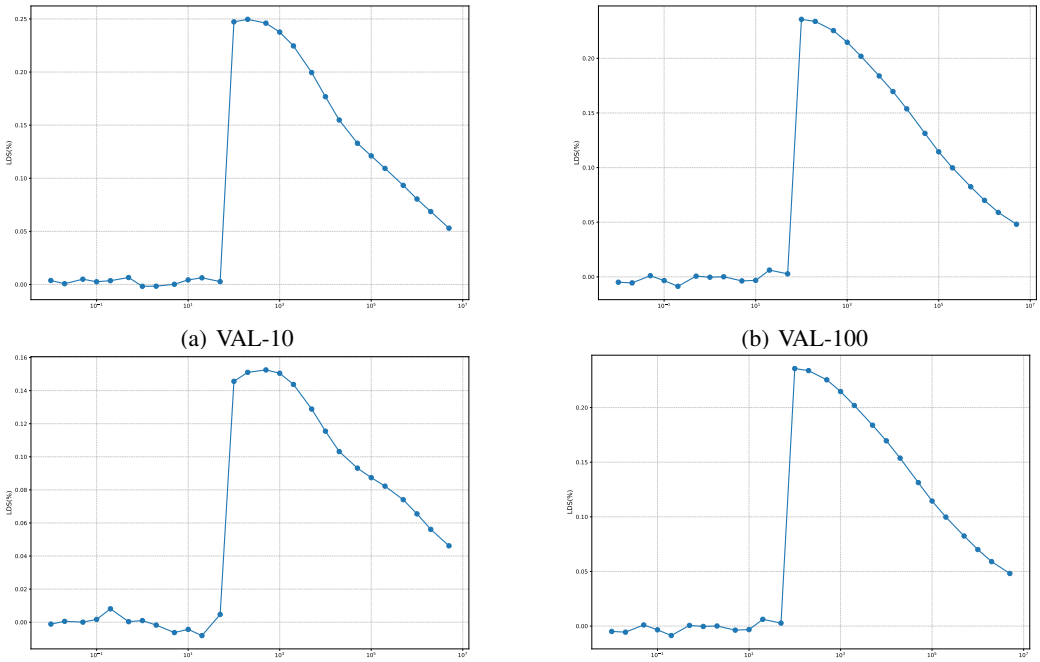
Figure 5: LDS (%) on ArtBench-2 under different $\lambda$. We consider 10 and 100 timesteps selected to be evenly spaced within the interval $[1, T]$, which are used to approximate the expectation $\mathbb{E}_t$. We set $k = 32768$.




(a) VAL-10 (b) VAL-100

(c) GEN-10 (d) GEN-100

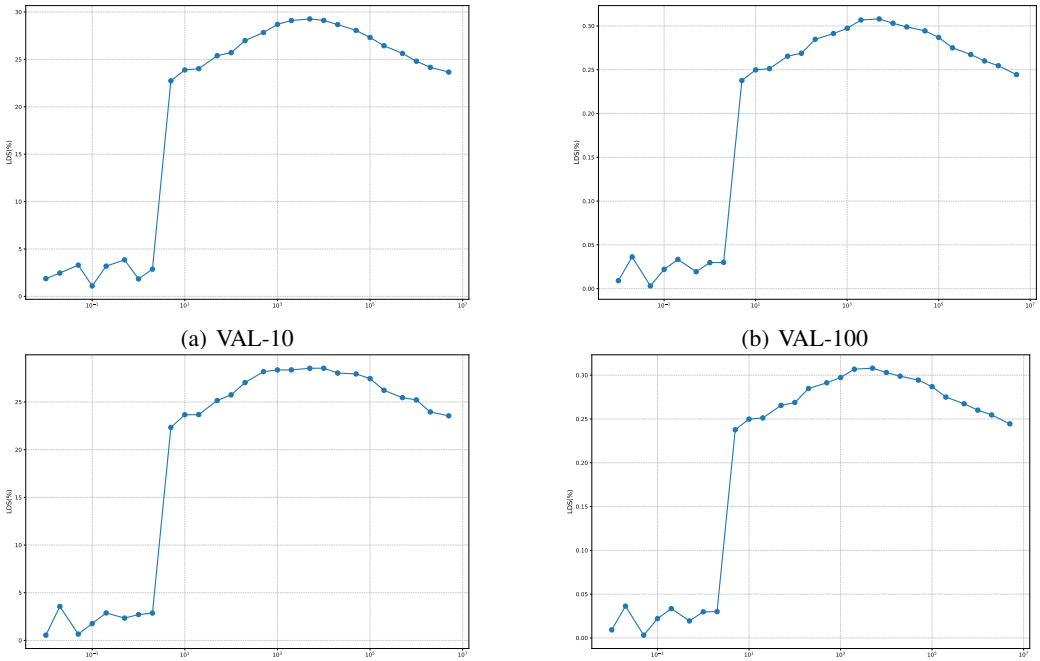
Figure 6: LDS (%) on ArtBench-5 under different $\lambda$. We consider 10 and 100 timesteps selected to be evenly spaced within the interval $[1, T]$, which are used to approximate the expectation $\mathbb{E}_t$. We set $k = 32768$.




(a) VAL-10 (b) VAL-100

(c) GEN-10 (d) GEN-100

Figure 7: LDS (%) on CIFAR-10 under different $\lambda$. We consider 10 and 100 timesteps selected to be evenly spaced within the interval $[1, T]$, which are used to approximate the expectation $\mathbb{E}_t$. We set $k = 32768$.




(a) VAL-10 (b) VAL-100

(c) GEN-10 (d) GEN-100

Figure 8: LDS (%) on CelebA under different $\lambda$. We consider 10 and 100 timesteps selected to be evenly spaced within the interval $[1, T]$, which are used to approximate the expectation $\mathbb{E}_t$. We set $k = 32768$.

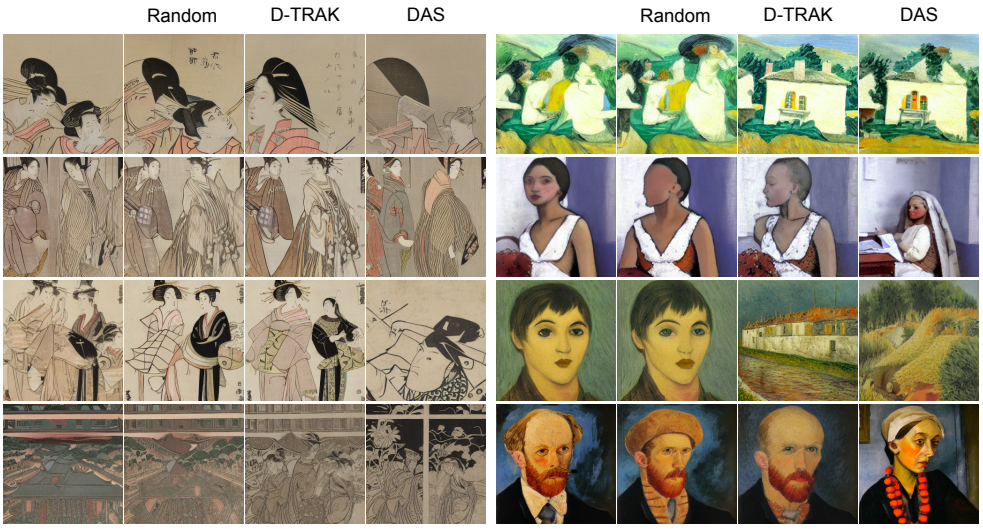
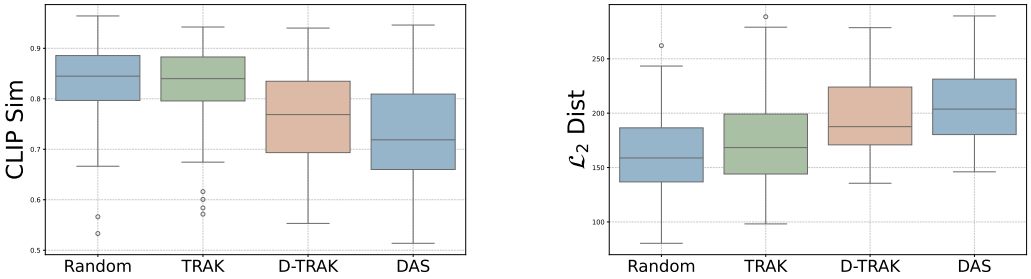



Figure 9: We conduct an visualization experiment to explore DAS effectiveness. We retrain the model after deleting 1000 most influential training samples detected by D-TRAK and DAS. We also add an baseline that randomly delete 1000 samples. From Left to Right, they are the original generated image and generated by retrained model with random deletion, D-TRAK deletion and DAS deletion.

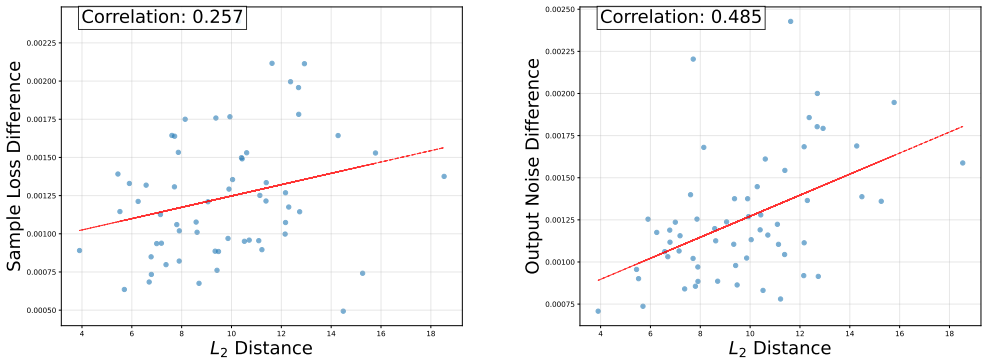



(b) ArtBench2

Figure 10: Boxplots of counterfactual evaluation on ArtBench2. We assess the impact of removing the 1,000 highest-scoring training samples and retraining the model using Random, TRAK, D-TRAK, and DAS. The evaluation metrics include pixel-wise $L^2$-Distance and CLIP cosine similarity between 60 generated samples and the corresponding images generated by the retrained models, sampled from the same random seed.

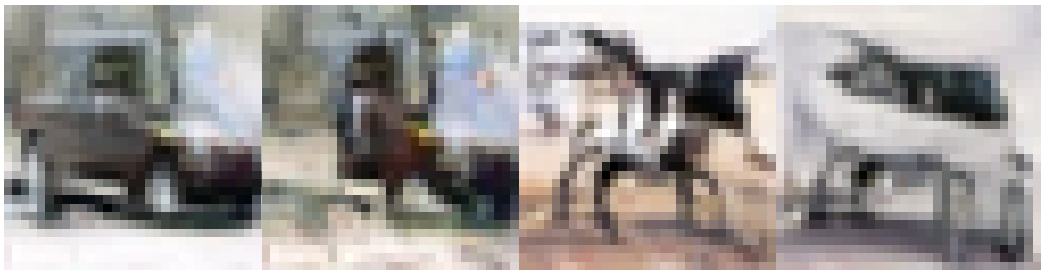
## G Counter Factual Experiment

Hu et al. (2024) discuss some limitations of LDS evaluation in data attribution. To further validate the effectiveness of DAS, we also conduct an counter-factual experiment, that 60 generate images are attributed by different attribution method, including TRAK, D-TRAK and DAS. We detect the top-1000 positive influencers identified by these methods and remove them from the training set and re-train the model. We utilize 100 timesteps and a projection dimension of $k = 32768$ to identify the top-1000 influencers for TRAK, D-TRAK and DAS. Additionally, we conduct a baseline experiment where 1000 training images are randomly removed before retraining. The experiment is conducted on ArtBench2 and CIFAR2. We generate the new images with same random seeds and compute the pixel-wise $L^2$-Distance and CLIP cosine similarity between the re-generated images and their corresponding origin images. The result is reported in Figure 1 with boxplot. For the pixel-wise $L^2$-Distance, D-TRAK yields values of 8.97 and 187.61 for CIFAR-2 and ArtBench-2, respectively, compared to TRAK's values of 5.90 and 168.37, while DAS results in values of 10.58 and 203.76. DAS achieves median similarities of 0.83 and 0.71 for ArtBench-2 and CIFAR-2, respectively, which are notably lower than TRAK's values of 0.94 and 0.84, as well as D-TRAK's values of 0.88 and 0.77, demonstrating the effectiveness of our method. An visualization illustration of the counter factual generation is in Figure 9 on ArtBench2 and a boxplot result is in Figure 10.




(a) Origin (b) Counter Factual

Figure 11: The result of Toy Experiment in Appendix H. Scatter plot showing the relationship between $L^2$ distance and two metrics: average loss difference and average noise predictor output difference. The noise predictor output difference exhibits a stronger correlation with $L^2$ distance, indicating its effectiveness in capturing image variation.



(a) Origin (b) Counter Factual (c) Origin (d) Counter Factual

Figure 12: Figures 12(a) and 12(b) represent one pair of generated images, while 12(c) and 12(d) form another pair. Despite the loss value between 12(a) and 12(b) being only 0.0007, the $L^2$ distance is 15.261. Similarly, the loss value between 12(c) and 12(d) is also 0.0007, yet the $L^2$distance is 14.485.

# H TOY EXPERIMENT

Here, we present a toy experiment to validate our theoretical claims: using the Simple Loss Value to represent changes in generated images is inadequate, as it relies on an indirect distributional comparison, as discussed in Eq. 8. Instead, we propose using changes in the noise predictor output of the diffusion model.

In the dataset, we employ 60 models, each trained after randomly deleting 1,000 samples, to generate images for 60 different seeds. The $L^2$ distance between the generated and original images is calculated to directly measure their differences. The original images are first noised to timestep $T$, and the two models are then used to denoise the latent variables at $T$. At this timestep, we compute the average loss difference and the average noise predictor output difference. For the 60 pairs of generated samples, we calculate the rank correlation between these differences and the $L^2$ distance. The results reveal that the Spearman correlation between the $L^2$ distance and the average loss difference is only 0.257, while the correlation with the average noise predictor output difference reaches 0.485. This indicates that the noise predictor output difference aligns better with the $L^2$ distance than the loss value difference. Larger noise predictor output differences correspond to larger $L^2$ distances, reflecting greater image variation. A scatter plot of these results is shown in Figure 11.

Figure 12 provides two specific examples. For images 12(a) and 12(b), as well as 12(c) and 12(d), the loss differences between the models are only 0.0007, the smallest among all pairs. However, the $L^2$ distance between 12(a) and 12(b) is 15.261, and between 12(c) and 12(d) is 14.485, making them among the top five pairs with the largest $L^2$ distances in the toy dataset. This result is unsurprising. Consider an extreme scenario: a model trained on a dataset containing cats and dogs generates an image of a cat. If all cat samples are removed, and the model is retrained, the new model would likely generate a dog image under the same random seed. In such a case, the loss values for both images might remain $v$, resulting in a loss difference of 0. This may occurs because the simple loss reflects the model's convergence for a given random seed, not the image itself. Thus, loss differences fail to capture the extent of image changes. By contrast, analyzing the differences in the model's outputs at each timestep allows us to effectively trace the diffusion model output on the images.

# I APPLICATION

Recent research has underscored the effectiveness of data attribution methods in a variety of applications. These include explaining model predictions (Koh & Liang, 2017; Ilyas et al., 2022), debugging model behaviors (Shah et al., 2023), assessing the contributions of training data (Ghorbani & Zou, 2019; Jia et al., 2019), identifying poisoned or mislabeled data (Lin et al., 2022), most influential subset selection (Hu et al., 2024) and managing data curation (Khanna et al., 2019; Liu et al., 2021; Jia et al., 2021). Additionally, the adoption of diffusion models in creative industries, as exemplified by Stable Diffusion and its variants, has grown significantly (Rombach et al., 2022; Zhang et al., 2023). This trend highlights the critical need for fair attribution methods that appropriately acknowledge and compensate artists whose works are utilized in training these models. Such methods are also crucial for addressing related legal and privacy concerns (Carlini et al., 2023; Somepalli et al., 2023).

# J LIMITATIONS AND BROADER IMPACTS

## J.1 LIMITATIONS

While our proposed Diffusion Attribution Score (DAS) showcases notable improvements in data attribution for diffusion models, several limitations warrant attention. Firstly, although DAS reduces the computational load compared to traditional methods, it still demands significant resources due to the requirement to train multiple models and compute extensive gradients. This poses challenges particularly for large-scale models and expansive datasets. Secondly, the current implementation of DAS is tailored primarily to image generation tasks. Its effectiveness and applicability to other forms of generative models, such as those for generating text or audio, remain untested and may not directly translate. Furthermore, DAS operates under the assumption that the influence of individual

training samples is additive. This simplification may not accurately reflect the complex interactions and dependencies that can exist between samples within the training data.

### J.2 Broader Impacts

The advancement of robust data attribution methods like DAS carries substantial ethical and practical implications. By enabling a more transparent linkage between generated outputs and their corresponding training data, DAS enhances the accountability of generative models. Such transparency is crucial in applications involving copyrighted or sensitive materials, where clear attribution supports intellectual property rights and promotes fairness. Nonetheless, the capability to trace back to the data origins also introduces potential privacy risks. It could allow for the identification and extraction of information pertaining to specific training samples, thus raising concerns about the privacy of data contributors. This highlights the necessity for careful handling of data privacy and security in the deployment of attribution techniques. The development of DAS thus contributes positively to the responsible use and governance of generative models, aligning with ethical standards and fostering greater trust in AI technologies. Moving forward, it is imperative to continue exploring these ethical dimensions, particularly the balance between transparency and privacy. Ensuring that advancements in data attribution go hand in hand with stringent privacy safeguards will be essential in maintaining the integrity and trustworthiness of AI systems.