# Learning to Boost Resilience of Complex Networks via Neural Edge Rewiring

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## Abstract

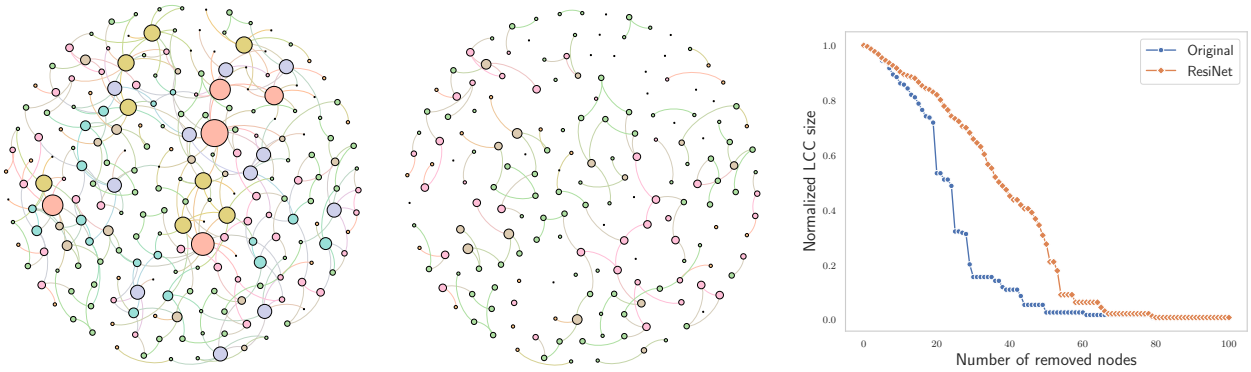
The resilience of complex networks refers to their ability to maintain functionality in the face of structural attacks. This ability can be improved by performing minimal modifications to the network structure via degree-preserving edge rewiring-based methods. Existing learning-free edge rewiring methods, although effective, are limited in their ability to generalize to different graphs. Such a limitation cannot be trivially addressed by existing graph neural networks (GNNs)-based learning approaches since there is no rich initial node features for GNNs to learn meaningful representations. In this work, inspired by persistent homology, we specifically design a variant of GNN called FireGNN, specifically designed to learn meaningful node representations solely from graph structures. We then develop an end-to-end inductive method called ResiNet, which aims to discover **resi**lient **net**work topologies while balancing network utility. ResiNet reformulates the optimization of network resilience as a Markov decision process equipped with edge rewiring action space. It learns to sequentially select the appropriate edges to rewire for maximizing resilience. Extensive experiments demonstrate that ResiNet outperforms existing approaches and achieves near-optimal resilience gains on various graphs while balancing network utility.

## 1 Introduction

Network systems, such as infrastructure systems and supply chains, are susceptible to malicious attacks, which necessitates addressing their vulnerability through the concept of *network resilience*. Network resilience serves as a metric to assess the ability of a network system to withstand failures and defend itself against attacks (Schneider et al., 2011). Figure 1 visualizes this scenario that the failures of a dozen of nodes could jeopardize the connectivity and utility of the EU power network. Maintaining network resilience is crucial in ensuring that networked systems continue to function and provide an acceptable level of utility, even when confronted with natural disasters or targeted attacks. Consequently, the study of the resilience of complex networks has found widespread applications in various fields, including ecology (Sole & Montoya, 2001), biology (Motter et al., 2008), economics (Haldane & May, 2011), and engineering (Albert et al., 2004).

To enhance network resilience, numerous learning-free optimization methods have been proposed, typically falling into heuristic-based (Schneider et al., 2011; Chan & Akoglu, 2016; Yazıcıoğlu et al., 2015; Rong & Liu, 2018) and evolutionary computation (Zhou & Liu, 2014) categories. These methods aim to improve the resilience of complex networks by making minimal modifications to graph topologies using a degree-preserving atomic operation known as *edge rewiring* (Schneider et al., 2011; Chan & Akoglu, 2016; Rong & Liu, 2018). Specifically, for a given graph $G = (V, E)$ and two existing edges $AC$ and $BD$, an edge rewiring operation alters the graph structure by removing $AC$ and $BD$ and adding $AB$ and $CD$, where $AC, BD \in E$ and $AB, CD, AD, BC \notin E$. Edge rewiring possesses several advantageous properties when compared to simple addition or deletion of edges. Firstly, it preserves node degrees, ensuring capacity constraints are not violated. Secondly, it minimizes utility degradation in terms of graph Laplacian measurement, which may not be the case with edge addition or deletion, as they can lead to significant network utility degradation (Jaume et al., 2020; Ma et al., 2021).

Despite their success, learning-free methods share the following limitations:




(a) Original EU network (b) EU network after attacks (c) Remaining LCC size under attack

Figure 1: The EU power network under the adaptive degree-based attack which removes the most critical node recursively with (a) original EU network with 217 nodes, (b) remaining EU network after a series of attacks on 40 nodes, and (c) the change of the normalized size of the largest connected component (LCC). The node size is proportional to its degree and the node color is given by DBSCAN (Ester et al., 1996).

- *Transduction.* Existing methods for selecting edges for rewiring are transductive, meaning they search for robust topologies specific to each individual graph instance. This search procedure is performed independently for each graph and does not generalize across graphs, even if the graphs only differ slightly in structure.
- *Local optimality.* Combinatorially choosing two edges to rewire in order to achieve globally optimal resilience is an NP-hard problem (Mosk-Aoyama, 2008). Previous studies primarily rely on greedy-like algorithms, resulting in local optimality in practice (Chan & Akoglu, 2016).
- *Utility Loss.* The rewiring operation in network resilience optimization may result in significant degradation of the network utility, potentially compromising the network's overall functionality.

To the best of our knowledge, there is currently no learning-based inductive method for optimizing network resilience. One of the key challenges lies in the fact that many network science tasks, including resilience optimization, often involve pure network topologies without rich node features. Learning paradigms based on Graph Neural Networks (GNNs) have demonstrated their effectiveness in solving a wide range of graph-related tasks when rich features are available in an inductive manner (Li et al., 2018; Joshi et al., 2019; Khalil et al., 2017; Nazari et al., 2018; Peng et al., 2020). However, it remains challenging to adapt these approaches to tasks that rely solely on topological structures, particularly those that require distinguishable node/edge representations for sequentially constructing a solution. For instance, Boffa et al. (2022) demonstrated a significant performance degradation of GNNs when solving the Traveling Salesman Problem (TSP) without node coordinate features. Similarly, we have empirically observed that the popular combination of GNNs and reinforcement learning (RL) fails to optimize network resilience. The RL agent gets trapped in an undesired infinite action backtracking loop without meaningful edge representations, as illustrated in Figure 2. A more detailed analysis can be found in Appendix D.

Therefore, devising a novel graph neural network (GNN) that can effectively handle network resilience optimization tasks without relying on rich features is a challenging endeavor. In this study, we address the aforementioned limitation of GNNs in modeling graphs without rich features and introduce the first *inductive* learning-based method for discovering resilient networks through successive edge rewiring operations. To achieve this, we propose a specialized variant of GNN called **Fi**lt**r**ation **e**nhanced **GNN** (FireGNN), which is solely focused on topology-oriented graph analysis. Our inspiration for FireGNN stems from persistent homology and persistence diagrams (Edelsbrunner & Harer, 2008; Aktas et al., 2019; Hofer et al., 2020; Horn et al., 2022). FireGNN generates a filtration, a series of subgraphs obtained by iteratively removing the node with the highest degree from the original graph. By doing so, FireGNN learns to aggregate node representations from each subgraph, allowing for the acquisition of meaningful representations through the proposed filtration process.

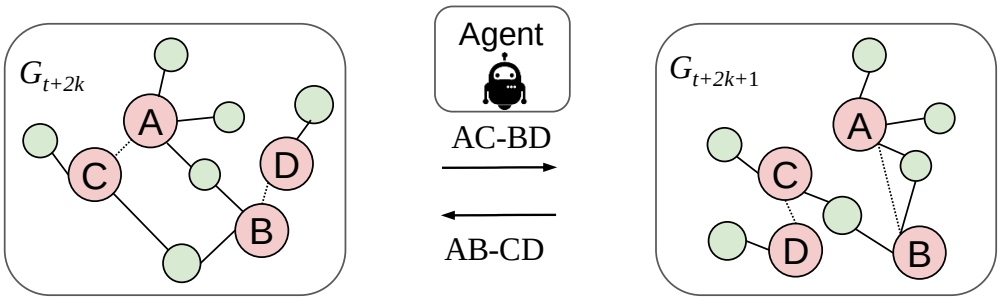



Figure 2: Action backtracking in successive edge rewirings since GNNs cannot provide distinguishable edge representations on graphs without rich features. After selecting $AC$ and $BD$ from $G_{t+2k}$ for rewiring at step $t+2k$, the agent would select $AB$ and $CD$ at step $t+2k+1$, returning back to $G_{t+2k}$ and forming a cycled action backtracking between $G_{t+2k}$ and $G_{t+2k+1}$.

The main contributions of this paper can be summarized as follows:

1) We propose ResiNet, the first learning-based method designed to enhance network resilience without relying on rich node features. ResiNet employs an inductive approach to preserve degrees while minimizing utility loss during the resilience optimization process. It formulates resilience optimization as a sequential decision-making process for neural edge rewirings. Extensive experiments demonstrate that ResiNet achieves near-optimal resilience while effectively balancing network utilities, outperforming existing approaches by a significant margin.

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## 2 Related Work

**GNNs for graph-related tasks with rich features.** GNNs are powerful tools to learn from relational data with rich features, providing meaningful representations for downstream tasks. Several successful applications using GNNs as backbones include node classification (Kipf & Welling, 2017; Hamilton et al., 2017), link prediction (Li et al., 2020a; Kipf & Welling, 2017), graph property estimation (Xu et al., 2019; Kipf & Welling, 2017; Li et al., 2020a; Bodnar et al., 2021), and combinatorial problems on graphs (e.g., TSP (Li et al., 2018; Joshi et al., 2019; Khalil et al., 2017; Hudson et al., 2022), vehicle routing problem (Nazari et al., 2018; Peng et al., 2020), graph matching (Yu et al., 2021) and adversarial attack on GNNs (Ma et al., 2021; Dai et al., 2018)). However, till now, it remains unclear how to adapt GNNs to graph tasks without rich feature (Zhu et al., 2021) like the resilience optimization task that we focus on. Current topology-based GNNs like TOGL (Horn et al., 2022) still rely on distinct node features for calculating the filtration, while our proposed FireGNN addresses this by creating a temporal-related filtration and learning to aggregate them.

**Graph rewiring** Graph rewiring is typically used in the GNN community to build novel classes of GNNs by preprocessing a given graph to overcome the problems of the over-squashing issue of training GNNs. For example, Klicpera et al. (2019) developed graph diffusion convolution (GDC) to improve GNN's performance on downstream tasks by replacing message passing with graph diffusion convolution (Topping et al., 2022) proposed an edge-based combinatorial curvature to help alleviate the over-squashing phenomenon in GNNs. To our knowledge, there is currently no inductive learning-based graph rewiring method, and graph rewiring methods rely on rich features to train GNNs better on downstream tasks. The edge rewiring operation used in our paper is a special graph rewiring operator that preserves node degree.

**Network resilience.** Modern network systems are threatened by various malicious attacks, such as the destruction of critical nodes, critical connections and critical subset of the network via heuristics/learning-based attack (Fan et al., 2020; Iyer et al., 2013; Grassia et al., 2021; Fan et al., 2020). Network resilience was proposed and proved as a suitable measurement for describing the robustness and stability of a network system under such attacks (Schneider et al., 2011). Around optimizing network resilience, various defense strategies have been proposed to protect the network functionality from crashing and preserve network's topologies to some extent. Commonly used manipulations of defense include adding additional edges (Li et al., 2019; Carchiolo et al., 2019), protecting vulnerable edges (Wang et al., 2014) and rewiring two edges (Schneider et al., 2011; Chan & Akoglu, 2016; Buesser et al., 2011). Among these manipulations, edge rewiring fits well to real-world applications as it induces fewer functionality changes to the original network and does not impose additional loads to the vertices (degree-preserving) (Schneider et al., 2011; Rong & Liu, 2018; Yazıcıoğlu et al., 2015). By now, there has been no learning-based inductive edge rewiring strategy for the resilience task.

**Extended related work.** The related work on *network utility*, *graph structure learning*, *multi-views graph augmentation for GNNs* and *deep graph generation* is deferred to Appendix A.

## 3 Problem Definition

An undirected graph is defined as $G = (V, E)$, where $V = \{1, 2, \ldots, N\}$ is the set of $N$ nodes, $E$ is the set of $M$ edges, $A \in \{0, 1\}^{N \times N}$ is the adjacency matrix, and $F \in \mathbb{R}^{N \times d}$ is the $d$-dimensional node feature matrix[1]. The degree of a node is defined as $d_i = \sum_{j=1}^{N} A_{ij}$, and a node with degree 0 is called an isolated node. Let $\mathbb{G}_G$ denote the set of graphs with the *same* node degrees as $G$.

Following the conventional setting in network science, resilience metrics used in our experiments include graph connectivity-based (Schneider et al., 2011) and spectrum-based measurements (adjacency matrix spectrum and Laplacian matrix spectrum). Utility metrics consist of global efficiency and local efficiency (Latora & Marchiori, 2001).

**Resilience metrics** Three kinds of resilience metrics are considered:

- The graph connectivity-based resilience measurement is defined as (Schneider et al., 2011)

$$\mathcal{R}(G) = \frac{1}{N} \sum_{q=1}^{N} s(q)\,,$$

  where $s(q)$ is the fraction of nodes in the largest connected remaining graph after removing $q$ nodes from $G$ according to certain attack strategy. The range of possible values of $\mathcal{R}$ is $[1/N, 1/2]$, where these two extreme values correspond to a star network and a fully connected network, respectively.

- The spectral radius ($\mathcal{SR}$) denotes the largest eigenvalue of an adjacency matrix.

- The algebraic connectivity ($\mathcal{AC}$) is the second smallest eigenvalue of the Laplacian matrix of $G$.

**Utility metrics** The global and local communication efficiency are used as two measurements of the network utility, which are widely applied across diverse applications of network science, such as transportation and communication networks (Latora & Marchiori, 2003).

The average efficiency of a network $G$ is defined inversely proportional to the average over pairwise distances (Latora & Marchiori, 2001) as

$$E(G) = \frac{1}{N(N-1)} \sum_{i \neq j \in V} \frac{1}{d(i,j)}\,,$$

where $d(i, j)$ is the length of the shortest path between a node $i$ and another node $j$.

[1]For a graph with pure topology structure, node feature matrix is not available.

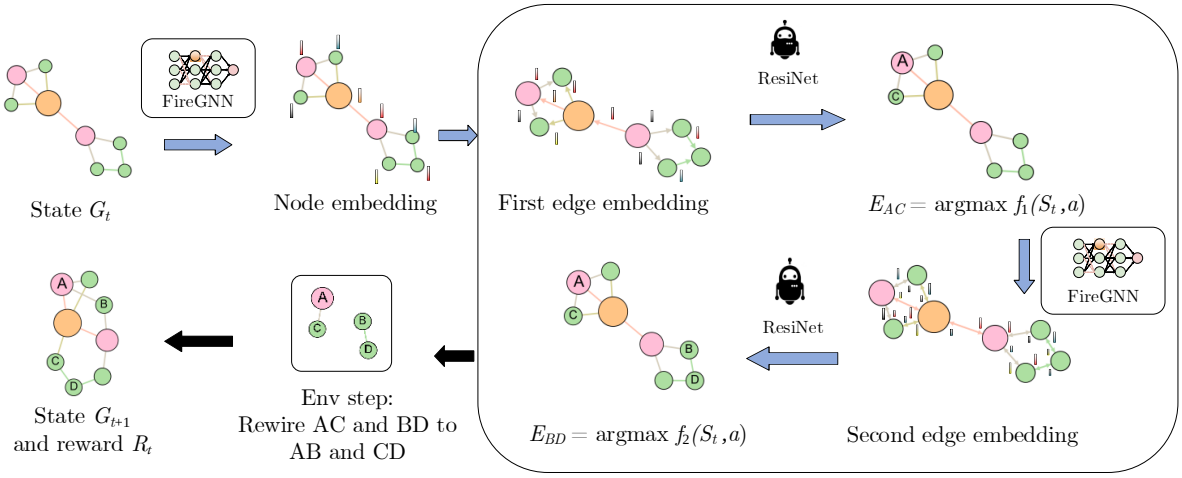



Figure 3: Overview of the architecture of ResiNet to select two edges for edge rewiring.

Based on the average efficiency, the global efficiency and local efficiency are defined as

- The global efficiency of a network $G$ is defined as (Latora & Marchiori, 2001)

$$E_{global}(G) = \frac{E(G)}{E(G^{ideal})},$$

  where $G^{ideal}$ is the "ideal" fully-connected graph on $N$ nodes and the range of $E_{global}(G)$ is [0, 1].

- The local efficiency of a network $G$ measures a local average of pairwise communication efficiencies and is defined as (Latora & Marchiori, 2001)

$$E_{local}(G) = \frac{1}{N}\sum_{i \in V} E(G_i),$$

  where $G_i$ is the local subgraph including only of a node $i$'s one-hop neighbors, but not the node $i$ itself. The range of $E_{local}(G)$ is [0, 1].

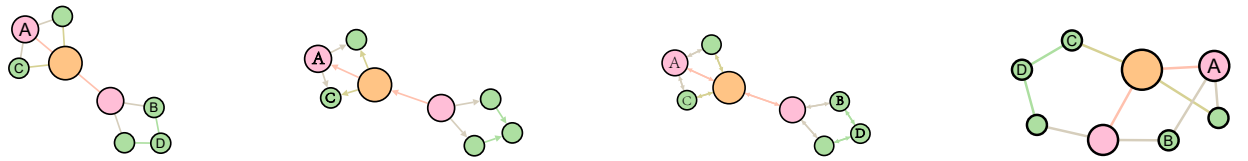
Given the network resilience metric $\mathcal{R}(G)$ (Schneider et al., 2011) and the utility metric $\mathcal{U}(G)$ (Latora & Marchiori, 2003), the objective of boosting the resilience of $G$ is to find a target graph $G^\star \in \mathbb{G}_G$, which maximizes the network resilience while balancing the network utility. Formally, the problem of maximizing the resilience of complex networks is formulated as

$$G^\star = \arg\max_{G' \in \mathbb{G}_G} \alpha \cdot \mathcal{R}(G') + (1-\alpha) \cdot \mathcal{U}(G').$$

To satisfy the constraint of preserving degree, edge rewiring is the default atomic operation for obtaining new graphs $G'$ from $G$. Combinatorially, a total of $T$ successive steps of edge rewiring has the complexity of $O(E^{2T})$.

## 4 Proposed Approach: ResiNet

In this section, we formulate the task of boosting network resilience as a reinforcement learning task by learning to select two edges and rewire them successively. We first present the graph resilience-aware environment design and describe our innovation FireGNN in detail. Finally, we present the graph policy network that guides the edge selection and rewiring process.




(a) Example graph (b) First edge selection (c) Second edge selection (d) Graph after the edge rewiring

Figure 4: The edge rewiring operation with the removal of $AC$, $BD$ and the addition of $AB$, $CD$.

### 4.1 Boosting Network Resilience via Edge Rewiring as Markov Decision Process

To satisfy the constraint of preserving the node degree, the resilience optimization of a given graph is based on edge rewiring. We formulate the network resilience optimization problem via successive edge rewiring operations into the Markov decision process (MDP) framework. The Markov property denotes that the graph obtained at time step $t+1$ relies only on the graph at time step $t$ and the rewiring operation, reducing the complexity from original $O(E^{2T})$ to $O(TE^2)$. Then we further reduce the complexity to $O(TE)$ by designing an autoregressive edge selection module shown as follows.

As shown in Figure 3, the environment performs the resilience optimization in an auto-regressive step-wise way through a sequence of edge rewiring actions. Given an input graph, the agent first decides whether to terminate or not. If not, it selects one edge from the graph to remove, receives the very edge it just selected as the auto-regression signal, and then selects another edge to remove. Four nodes of these two removed edges are re-combined, forming two new edges to be added to the graph. The optimization process repeats until the agent decides to terminate. The detailed design of the state, the action, the transition dynamics, and the reward are presented as follows.

**State.** The fully observable state is formulated as $S_t = G_t$, where $G_t$ is the current input graph at step $t$. The widely-used node degree feature cannot significantly benefit the network resilience optimization of a single graph due to the degree-preserving rewiring. Therefore, we construct node features for each input graph to aid the transductive learning and inductive learning, including

- The distance encoding strategy (Li et al., 2020b). Node degree feature is a part of it.
- The 8-dimensional position embedding originating from the Transformer (Vaswani et al., 2017) as the measurement of the vulnerability of each node under attack. If the attack order is available, we can directly encode it into the position embedding. If the attack order is unknown, node degree, node betweenness, and other node priority metrics can be used for approximating the node importance in practice. In our experiments, we used the adaptive node degree for the position embedding.

**Action.** ResiNet is equipped with a node permutation-invariant, variable-dimensional action space. Given a graph $G_t$, the action $a_t$ is to select two edges and the rewiring order. As is shown in Figure 4, the agent first chooses an edge $e_1 = AC$ and a direction $A \to C$. Then conditioning on the state, $e_1$, and the direction the agents chooses an edge $e_2 = BD$ such that $AB, CD, AD, BC \notin E$ and a direction $B \to D$. The heads of the two edges reconnect as a new edge $AB$, and so does the tail $CD$. Although $G_t$ is undirected, we propose to consider the artificial edge directions, which effectively avoids the redundancy in representing action space since $A \to C$, $B \to D$ and $C \to A$, $D \to B$ refer to the same rewiring operation. The choice of the direction of $e_1$ is randomized (this randomized bit is still an input of choosing $e_2$). Therefore, our proposed action space effectively reduces the size of the original action space by half and still leads to a complete action space. In this way, the action space is the set of all feasible pairs of $(e_1, e_2) \in E^2$, with a variable size no larger than $2|E|(|E|-1)$.

**Transition dynamics.** The formulation of the action space implies that if the agent does not terminate at step $t$, the selected action must form an edge rewiring. This edge rewiring is executed by the environment, and the graph transits to the new graph.

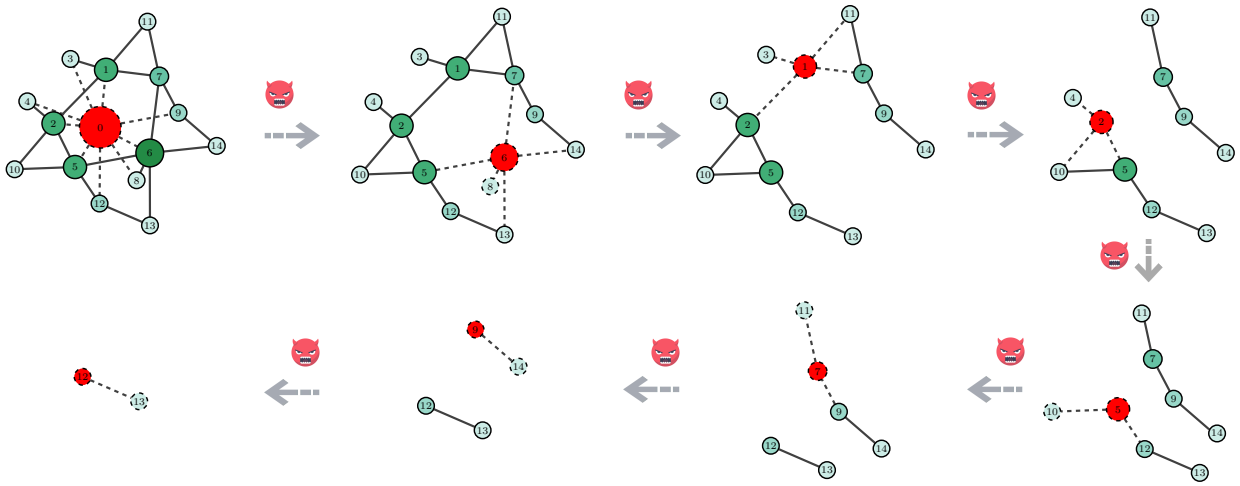


Figure 5: Filtration Process in FireGNN on BA-15. The original graph is decomposed into temporal-related subgraphs.

Note that in some other work, infeasible operations are also included in the action space (to make the action space constant through the process) (You et al., 2018; Trivedi et al., 2020). This reduces the sample efficiency and causes biased gradient estimations (Huang & Ontañón, 2020). ResiNet takes advantage of the state-dependent variable action space composed of only feasible operations.

**Reward.** ResiNet aims to optimize resilience while balancing the utility, forming a complicated and possibly unknown objective function. Despite this, by Wakuta (1995), an MDP that maximizes a complicated objective is up to an MDP that maximizes the linear combination of resilience and utility for some coefficient factor. This fact motivates us to design the reward as the step-wise gain of such a linear combination as

$$R_t = \alpha \cdot (\mathcal{R}(G_{t+1}) - \mathcal{R}(G_t)) + (1-\alpha) \cdot (\mathcal{U}(G_{t+1}) - \mathcal{U}(G_t)) \ ,$$

where $\mathcal{R}(G)$ and $\mathcal{U}(G)$ are the resilience and the utility functions, respectively. The cumulative reward $\sum_{t=0}^{T-1} R_t$ up to time $T$ is then the total gain of such a linear combination.

### 4.2 FireGNN

Motivated by graph filtration in persistent homology (Edelsbrunner & Harer, 2008), we design the **fil**t**r**ated graph **e**nhanced **GNN** termed FireGNN to model graphs without rich features, or even with only topology. For a given input graph $G$, FireGNN transforms $G$ from the static version to a temporal version consisting of a sequence of subgraphs, by repeatedly removing the node with the highest degree. Observing a sequence of nested subgraphs of $G$ grants FirGNN the capability to observe how $G$ evolves towards being empty. Then FireGNN aligns and aggregates the node, edge, and graph embedding from each subgraph, leading to meaningful representations in node, edge, and graph levels. Formally, the filtration in FireGNN is constructed as

$$\begin{aligned} G^{(k-1)} = G^{(k)} - v_k, \quad v_k &= \operatorname*{argmax}_{v_i \in G^{(k)}} \mathrm{DEGREE}(v_i) \\ (V, \emptyset) = G^{(0)} \subset G^{(1)} &\subset \cdots \subset G^{(N)} = G \\ \tilde{G} &= [G^{(0)}, G^{(1)}, \ldots, G^{(N)}] \, , \end{aligned}$$

where $G^{(k)}$ denotes the remaining graph after removing $N-k$ nodes with highest node degrees, $v_k$ denotes the node with highest degree in current subgraph $G^{(k)}$, $\mathrm{DEGREE}(\cdot)$ measures the node degree, $G^{(N)}$ is the original graph, and $G^{(0)}$ contains no edge. The sequence of the nested subgraphs of $G$ is termed the filtrated graph $\tilde{G}$. We illustrate the filtration process on a toy dataset in Figure 5.

**Node embedding.** Regular GNN only operates on the original graph $G$ to obtain the node embedding for each node $v_i$ as $h(v_i) = \phi(G^{(N)} = G)_i$ , where $\phi(\cdot)$ denotes a standard GNN model. In FireGNN, by using the top $K+1$ subgraphs in a graph filtration, the final node embedding $h(v_i)$ of $v_i$ is obtained by

$$h(v_i) = \mathrm{AGG}_N \left( h^{(N-K)}(v_i), \ldots, h^{(N-1)}(v_i), h^{(N)}(v_i) \right) ,$$

where $\mathrm{AGG}_N(\cdot)$ denotes a node-level aggregation function, $h^{(k)}(v_i)$ is the node embedding of $i$ in the $k$-th subgraph $G^{(k)}$ obtained by passing $G$ to a backbone GNN, and $K \in [N]$. In practice, $h^{(k)}(v_i)$ is discarded when calculating $h(v_i)$ if $v_i$ is isolated or not included in $G^{(k)}$.

**Edge embedding.** The directed edge embedding $h^{(k)}(e_{ij})$ of the edge from node $i$ to node $j$ in each subgraph is obtained by combining the embeddings of the two end vertices in $G^{(k)}$ as

$$h^{(k)}(e_{ij}) = m_f \left( \mathrm{AGG}_{N \to E} \left( h^{(k)}(v_i), h^{(k)}(v_j) \right) \right) ,$$

where $\mathrm{AGG}_{N \to E}(\cdot)$ denotes an aggregation function for obtaining edge embedding from two end vertices (typically chosen from `min`, `max`, `sum`, `difference`, and `multiplication`). $m_f(\cdot)$ is a multilayer perceptron (MLP) model that ensures the consistence between the dimensions of edge embedding and graph embedding.

The final embedding of the directed edge $e_{ij}$ of the filtrated graph $\tilde{G}$ is given by

$$h(e_{ij}) = \mathrm{AGG}_E \left( h^{(N-K)}(e_{ij}), \ldots, h^{(N-1)}(e_{ij}), h^{(N)}(e_{ij}) \right) ,$$

where $\mathrm{AGG}_E(\cdot)$ denotes an edge-level aggregation function.

**Graph embedding.** With the node embedding $h^{(k)}(v_i)$ of each subgraph $G^{(k)}$ available, the graph embedding $h^{(k)}(G)$ of each subgraph $G^{(k)}$ is calculated by a readout functions (e.g., `mean`, `sum`) on all non-isolated nodes in $G^{(k)}$ as

$$h^{(k)}(G) = \text{ READOUT} \left( h^{(k)}(v_i) \right) \ \forall v_i \in G^{(k)} \text{ and } d_i^{(k)} \geq 0 \, .$$

The final graph embedding of the filtrated graph $\tilde{G}$ is given by

$$h(G) = \mathrm{AGG}_G \left( h^{(N-K)}(G), \ldots, h^{(N-1)}(G), h^{(N)}(G) \right) ,$$

where $\mathrm{AGG}_G(\cdot)$ denotes a graph-level aggregation function.

### 4.3 Edge Rewiring Policy Network

Having presented the details of the graph resilience environment and FireGNN, in this section, we describe the policy network architecture of ResiNet in detail, which learns to select two existing edges for rewiring at each step. At time step $t$, the policy network uses FireGNN as the graph extractor to obtain the directed edge embedding $h(e_{ij}) \in \mathbb{R}^{2|E| \times d}$ and the graph embedding $h(G) \in \mathbb{R}^d$ from the filtrated graph $\tilde{G}_t$, and outputs an action $a_t$ representing two selected rewired edges, leading to the new state $G_{t+1}$ with reward $R_t$.

To be inductive, we adapt a special autoregressive node permutation-invariant dimension-variable action space to model the selection of two edges from graphs with arbitrary sizes and permutations. The detailed mechanism of obtaining the action $a_t$ based on edge embedding and graph embedding is presented as follows, further reducing the complexity from $O(TE^2)$ to $O(TE)$.

**Auto-regressive latent edge selection.** An edge rewiring action $a_t$ at time step $t$ involves the prediction of the termination probability $a_t^{(0)}$ and the selection of two edges ($a_t^{(1)}$ and $a_t^{(2)}$) and the rewiring order. The action space of $a_t^{(0)}$ is binary, however, the selection of two edges imposes a huge action space in $O(|E|^2)$, which is too expensive to sample from even for a small graph. Instead of selecting two edges simultaneously, we decompose the joint action $a_t$ into $a_t = (a_t^{(0)}, a_t^{(1)}, a_t^{(2)})$, where $a_t^{(1)}$ and $a_t^{(2)}$ are two existing edges which do not share any common node (recall that $a_t^{(1)}$ and $a_t^{(2)}$ are directed edges for an undirected graph). Thus the probability of $a_t$ is

$$\mathbb{P}(a_t|s_t) = \mathbb{P}(a_t^{(0)}|s_t)\mathbb{P}(a_t^{(1)}|s_t, a_t^{(0)})\mathbb{P}(a_t^{(2)}|s_t, a_t^{(0)}, a_t^{(1)}) \, .$$

**Predicting the termination probability.** The first policy network $\pi_0(\cdot)$ takes the graph embedding as input and outputs the probability distribution of the first action that decides to terminate or not as $\mathbb{P}(a_t^{(0)}|s_t) = \pi_0(h(G))$ , where $\pi_0(\cdot)$ is implemented by a two-layer MLP. Then the probability of the first subaction is given as

$$a_t^{(0)} \sim \text{Bernoulli}(\mathbb{P}(a_t^{(0)}|s_t)) \in \{0, 1\}\,.$$

**Selecting edges.** If the signal $a_t^{(0)}$ given by the agent decides to continue to rewire, two edges are then selected in an auto-regressive way. The signal of continuing to rewire $a_t^{(0)}$ is input to the selection of two edges as a one-hot encoding vector $l_c$. The second policy network $\pi_1(\cdot)$ takes the graph embedding and $l_c$ as input and outputs a latent vector $l_1 \in \mathbb{R}^d$. The pointer network (Vinyals et al., 2015) is used to measure the proximity between $l_1$ and each edge embedding $h(e_{ij})$ in $G$ to obtain the first edge selection probability distribution. Then, to select the second edge, the graph embedding $h(G)$ and the first selected edge embedding $h(e_t^{(1)})$ and $l_c$ are concatenated and fed into the third policy network $\pi_2(\cdot)$. $\pi_2(\cdot)$ obtains the latent vector $l_2$ for selecting the second edge using a respective pointer network. The overall process can be formulated as

$$\begin{aligned} l_1 &= \pi_1([h(G), l_c]) \\ \mathbb{P}(a_t^{(1)}|s_t, a_t^{(0)}) &= f_1(l_1, h(e_{ij})) \\ l_2 &= \pi_2([h(G), h(e_t^{(1)}), l_c]) \\ \mathbb{P}(a_t^{(2)}|s_t, a_t^{(1)}, a_t^{(0)}) &= f_2(l_2, h(e_{ij})), \end{aligned}$$

where $e_{ij} \in E$ and $\pi_i(\cdot)$ is a two-layer MLP model, $[\cdot, \cdot]$ denotes the concatenation operator, $h(e_t^{(1)})$ is the embedding of the first selected edge at step $t$, and $f_i(\cdot)$ is a pointer network.

## 5 Experiments

In this section, we demonstrate the advantages of ResiNet over existing non-learning-based and learning-based methods in achieving superior network resilience, inductively generalizing to unseen graphs, and accommodating multiple resilience and utility metrics. Moreover, we show that FireGNN can learn meaningful representations from graph data without rich features, while current GNNs fail. *Our implementation is available at* `https://anonymous.4open.science/r/ResiNet-F241`.

### 5.1 Experimental Settings

**Datasets.** Synthetic and real datasets including EU power network (Zhou & Bialek, 2005) and Internet peer-to-peer networks (Leskovec et al., 2007; Ripeanu et al., 2002) are used to demonstrate the performance of ResiNet in transductive and inductive settings. The details of data generation and the statistics of the datasets are presented in Appendix B.1. Following the conventional experimental settings in network science, the maximal node size is set to be around 1000 (Schneider et al., 2011), taking into account: 1) the high complexity of selecting two edges at each step is $O(E^2)$; 2) evaluating the resilience metric is time-consuming for large graphs.

**Baselines.** We compare ResiNet with existing graph resilience optimization algorithms, including learning-free and learning-based algorithms. Learning-free methods (upper half of Table 1) include the hill climbing (HC) (Schneider et al., 2011), the greedy algorithm (Chan & Akoglu, 2016), the simulated annealing (SA) (Buesser et al., 2011), and the evolutionary algorithm (EA) (Zhou & Liu, 2014). Since to our knowledge there is no previous learning-based baseline, we specifically devise five counterparts based on our method by replacing FireGNN with existing well-known powerful GNNs (DE-GNN (Li et al., 2020b), $k$-GNN (Morris et al., 2019), DIGL (Klicpera et al., 2019) and SDRF (Topping et al., 2022)) (lower half of Table 1). The classical GIN model is used as the backbone (Xu et al., 2019).

The ResiNet's training setup is detailed as follows. Our proposed FireGNN is used as the graph encoder in ResiNet, including a 5-layer defined GIN (Xu et al., 2019) as the backbone. The hidden dimensions for

Table 1: Resilience optimization algorithm under the fixed maximal rewiring number budget of 20. Entries are in the format of $X(Y)$, where 1) $X$: weighted sum of the graph connectivity-based resilience and the network efficiency improvement (in percentage); 2) $Y$: required rewiring number. ✗ means that the algorithm cannot find a solution in a reasonable time.

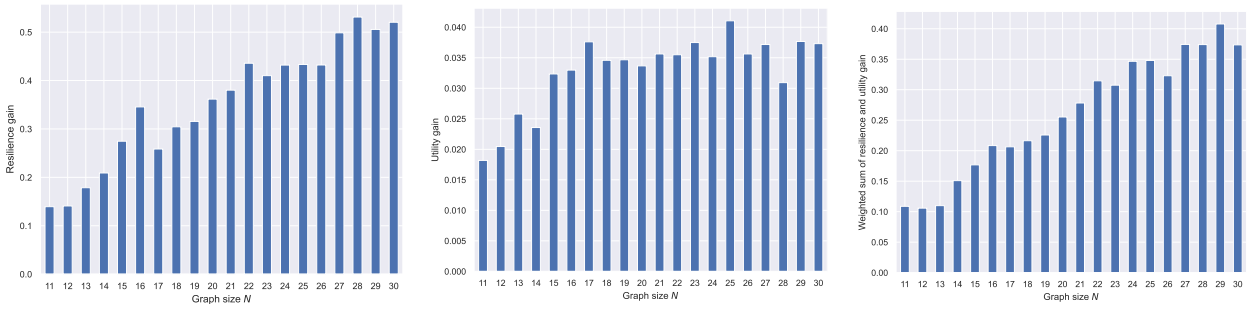
| **Method** | $\alpha$ | BA-15 | BA-50 | BA-100 | BA-500 | BA-1000 | EU | P2P-Gnutella05 | P2P-Gnutella09 |
|---|---|---|---|---|---|---|---|---|---|
| HC | 0 | 26.8 (10) | 30.0 (20) | 24.1 (20) | 6.4 (20) | 66.6 (20) | 19.8 (20) | 6.2 (20) | 8.4 (20) |
| | 0.5 | 18.6 (11.3) | 22.1 (20) | 14.9 (20) | 5.9 (20) | 16.4 (20) | 16.3 (20) | 5.2 (20) | 7.0 (20) |
| SA | 0 | 21.6 (17.3) | 11.9 (20) | 12.5 (20) | 3.8 (20) | 42.9 (20) | 14.9 (20) | 3.9 (20) | 3.7 (20) |
| | 0.5 | 16.8 (19.0) | 11.4 (20) | 13.4 (20) | 4.0 (20) | 15.4 (20) | 14.0 (20) | 6.3 (20) | 4.8 (20) |
| Greedy | 0 | 23.5 (6) | 48.6 (13) | 64.3 (20) | ✗ | ✗ | 0.5 (3) | ✗ | ✗ |
| | 0.5 | 5.3 (15) | 34.7 (13) | 42.7 (20) | ✗ | ✗ | 0.3 (3) | ✗ | ✗ |
| EA | 0 | 8.5 (20) | 6.4 (20) | 4.0 (20) | 8.5 (20) | **174.1** (20) | 8.2 (20) | 2.7 (20) | 0 (20) |
| | 0.5 | 6.4 (20) | 4.7 (20) | 2.8 (20) | 5.6 (20) | 18.7 (20) | 9.3 (20) | 3.7 (20) | 0.1 (20) |
| DE- | 0 | 13.7 (2) | 0 (1) | 0 (1) | 1.6 (20) | 41.7 (20) | 9.0 (20) | 2.2 (20) | 0 (1) |
| GNN-RL | 0.5 | 10.9 (2) | 0 (1) | 0 (1) | 2.7 (20) | 20.1 (14) | 2.1 (20) | 0 (1) | 1.0 (20) |
| $k$-GNN- | 0 | 13.7 (2) | 0 (1) | 0 (1) | 0 (1) | 8.8 (20) | 4.5 (20) | -0.2 (20) | 0 (1) |
| RL | 0.5 | 6.3 (2) | 0 (1) | 0 (1) | 0 (20) | -24.9 (20) | 4.8 (20) | -0.1 (20) | 0 (1) |
| DIGL- | 0 | 9.8 (2) | 0 (1) | 0 (1) | ✗ | ✗ | 5.9 (20) | ✗ | ✗ |
| RL | 0.5 | 6.3 (2) | 0 (1) | 0 (1) | ✗ | ✗ | 7.2 (20) | ✗ | ✗ |
| SDRF- | 0 | 9.8 (2) | 0 (1) | 0 (1) | ✗ | ✗ | 4.7 (20) | ✗ | ✗ |
| RL | 0.5 | 8.0 (2) | 0 (1) | -4.7 (20) | ✗ | ✗ | 5.3 (20) | ✗ | 55 |
| ResiNet | 0 | **35.3** (6) | **61.5** (20) | **70.0** (20) | **10.2** (20) | 172.8 (20) | **54.2** (20) | **14.0** (20) | **18.6** (20) |
| (ours) | 0.5 | **26.9** (20) | **53.9** (20) | **53.1** (20) | **15.7** (20) | **43.7** (20) | **51.8** (20) | **12.4** (20) | **15.1** (20) |

node embedding and graph embedding in each hidden layer are set to 64 and the SeLU activation function is used after each message passing propagate. Graph normalization strategy is adopted to stabilize the training of GNN (Cai et al., 2021). The jumping knowledge network (Xu et al., 2018) is used to aggregate node features from different layers of the GNN. The overall policy is trained by using the highly tuned implementation of proximal policy optimization (PPO) algorithm (Schulman et al., 2017). Several critical strategies for stabilizing and accelerating the training of ResiNet are used, including advantage normalization (Andrychowicz et al., 2021), the dual-clip PPO (the dual clip parameter is set to 10) (Ye et al., 2020), and the usage of different optimizers for policy network and value network. Additionally, since the step-wise reward range is small (around 0.01), we scale the reward by a factor of 10 to aim the training of ResiNet. The policy head model and value function model use two separated FireGNN encoder networks with the same architecture. We run all experiments for ResiNet on the platform with two GEFORCE RTX 3090 GPU and one AMD 3990X CPU.

### 5.2 Comparisons to the Baselines

In this section, we compare ResiNet to baselines in optimizing the combination of resilience and utility with weight coefficient $\alpha \in \{0, 0.5\}$. Following conventional setting, the graph connectivity-based metric is used as resilience metric (Schneider et al., 2011) and the global efficiency is used as utility metric (Latora & Marchiori, 2003).

Table 1 records the metric gain and the required number of rewiring operations of different methods under the same rewiring budget. ResiNet outperforms all baselines consistently on all datasets. Note that this performance may be achieved by ResiNet under a much fewer number of rewiring operations, such as on BA-15 with $\alpha = 0$. In contrast, despite approximately searching for all possible new edges, the greedy algorithm is trapped in a local optimum (as it maximizes the one-step resilience gain) and is too expensive to optimize the resilience of a network with more than 300 nodes. For SA, the initial temperature and the temperature decay rate need to be carefully tuned for each network. EA performs suboptimally with a limited rewiring budget due to the numerous rewiring operations required in the internal process (e.g., the crossover operator). Learning-based methods using existing GNNs coupled with distance encoding cannot learn effectively compared to our proposed ResiNet, supporting our claim about the effectiveness of FireGNN on graphs without rich features.




(a) Induction on resilience (b) Induction on utility (c) Induction on both metrics

Figure 6: The inductive ability of ResiNet on the test dataset (BA-10-30) when optimizing (a) network resilience, (b) network utility, and (c) their combination.

Table 2: The effect of the coefficient $\alpha$ on ResiNet. The result is shown as percentages.

| Dataset | Gain | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| BA-15 | Resilience | 35.3 | 35.3 | 35.3 | 33.3 | 17.6 | 17.6 | 27.5 | 17.6 | 17.6 | 17.6 | -2.0 |
| | Utility | -5.9 | -3.9 | -3.8 | -2.7 | 1.1 | 1.1 | 0 | 1.1 | 1.1 | 1.1 | 5.4 |
| | Reward | 35.3 | 34.2 | 32.9 | 29.7 | 15.2 | 14.2 | 19.7 | 11.4 | 9.2 | 6.0 | 5.4 |
| BA-50 | Resilience | 56.7 | 51.1 | 42.3 | 48.6 | 53.9 | 59.2 | 51.4 | 50.6 | 48.1 | 39.3 | -19.1 |
| | Utility | -3.6 | 3.4 | -2.1 | -4.0 | -4.2 | -4.2 | -2.6 | -2.2 | -2.1 | 0.5 | 5.5 |
| | Reward | 56.7 | 49.5 | 39.9 | 43.1 | 44.9 | 45.6 | 35.7 | 30.1 | 22.0 | 11.8 | 5.5 |
| BA-100 | Resilience | 75.4 | 74.6 | 74.8 | 76.1 | 72.8 | 72.8 | 75.1 | 75.4 | 74.9 | 71.6 | -11.8 |
| | Utility | -4.0 | -4.6 | -3.9 | -5.1 | -4.2 | -4.2 | -3.8 | -3.7 | -3.5 | -2.5 | 4.8 |
| | Reward | 75.4 | 71.9 | 69.0 | 66.4 | 59.4 | 54.3 | 49.7 | 41.8 | 31.1 | 16.7 | 4.8 |

### 5.3 Ablation Study of ResiNet

In this section, we investigate the impact of coefficient $\alpha$ of the objective on ResiNet and the effect of the filtration order $K$ on FireGNN.

To investigate the impact of the $\alpha$ in the reward function on ResiNet, we run a grid search by varying $\alpha$ from 0 to 1 and summarize the resilience gain, utility gain, and the sum of them in Table 2. Table 2 shows that when we only optimize the resilience with $\alpha = 0$, the utility will degrade. Similarly, the resilience would also decrease if we only optimize the utility with $\alpha = 1$. This suggests a general tradeoff between resilience and utility and is consistent with their definitions. However, despite this tradeoff, we can achieve resilience gain and utility gain simultaneously on BA-15 and BA-50 since the original graph usually does not have the maximum resilience or utility. This incentivizes almost every network conducts such optimization to some extent when feasible.

In FireGNN, the filtration order $K$ of FireGNN determines the total number of subgraphs involved in calculating the final node embedding, edge embedding, and graph embedding. FireGNN degenerates to existing GNNs when the filtration order $K$ is 0. Table 1 validates the effectiveness and necessity of FireGNN. Without FireGNN (other GNNs as the backbone), it is generally challenging for ResiNet to find a positive gain on graphs without rich features since ResiNet cannot learn to select the correct edges with the incorrect edge embeddings. The maximum $K$ of each dataset is recorded in Appendix Table 6, which shows that the maximum $K$ equals the around half size of the graph since we gradually remove the node with the largest degree, leading to a fast graph filtration process. For our experiments, we use the maximum of $K$ for graphs of sizes less than 50 and set $K = 3$ (1) for graphs of sizes larger than 50 (200). To validate that ResiNet is not sensitive to $K$, we run a grid search on several datasets to optimize the resilience by setting $K = 0, 1, 2, 3$. As shown in Appendix Table 4, the resilience is improved significantly with $K > 0$ and ResiNet performs well with $K = 1$ or $K = 2$.

## 6 Generalization

To demonstrate the induction of ResiNet, we first train ResiNet on two different datasets (BA-10-30 and BA-20-200), and then evaluate its performance on an individual test dataset. The test dataset is not observed during the training process and fine-tuning is not allowed. We report the averaged resilience gain for the graphs of the same size for each dataset.

The performance of ResiNet on BA-10-30 is shown in Figure 6 and the results of other datasets are deferred to Figure 9 in Appendix C. Figure 6 shows a nearly linear improvement of resilience with the increase of graph size, which is also consistent with the results in the transductive setting that larger graphs usually have a larger room to improve their resilience. Moreover, we conduct experiments to demonstrate ResiNet's generalization on optimizing different utility and resilience metrics, and the details are deferred to Appendix C.

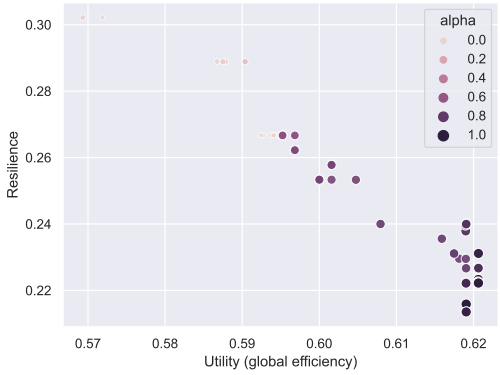



Figure 7: Pareto points obtained by ResiNet of balancing various combinations of the graph connectivity-based resilience and the global efficiency-based utility on the BA-15 dataset.

To explore the complicated objective of resilience and utility, BA-15 is taken as an example to be optimized by ResiNet to obtain the approximate Pareto frontier. The Pareto points are shown in Figure 7 to denote the optimum under different objectives. Surprisingly, the initial gain of resilience (from around 0.21 to around 0.24) is obtained without loss of the utility, which incentivizes almost every network to conduct such optimization to some extent when feasible.

## 7 Conclusion

We have proposed a learning-based inductive method, ResiNet, for the discovery of resilient network topologies with minimal changes to the graph structure. ResiNet is the first inductive method that formulates the task of boosting network resilience as an MDP of successive edge rewiring operations. Our technical innovation, FireGNN, is motivated by persistent homology as the graph feature extractor for handling graphs with only topologies available. FireGNN alleviates the insufficiency of current GNNs (including GNNs more powerful than 1-WL test) on modeling graphs without rich features. By decomposing graphs into temporal subgraphs and learning to combine the individual representations from each subgraph, FireGNN can learn meaningful representations on the resilience task to provide sufficient gradients for training an RL agent to select correct edges while current GNNs fail due to the infinite action backtracking. Our method is practically feasible as it balances the utility of the networks when boosting resilience. FireGNN is potentially general enough to be applied to solve various graph problems without rich features.

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