Λ-DARTS: MITIGATING PERFORMANCE COLLAPSE BY HARMONIZING OPERATION SELECTION AMONG CELLS

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Paper under double-blind review

ABSTRACT

Differentiable neural architecture search (DARTS) is a popular method for neural architecture search (NAS), which performs cell-search and utilizes continuous relaxation to improve the search efficiency via gradient-based optimization. The main shortcoming of DARTS is performance collapse, where the discovered architecture suffers from a pattern of declining quality during search. Performance collapse has become an important topic of research, with many methods trying to solve the issue through either regularization or fundamental changes to DARTS. However, the weight-sharing framework used for cell-search in DARTS and the convergence of architecture parameters has not been analyzed yet. In this paper, we provide a thorough and novel theoretical and empirical analysis on DARTS and its point of convergence. We show that DARTS suffers from a specific structural flaw due to its weight-sharing framework that limits the convergence of DARTS to saturation points of the softmax function. This point of convergence gives an unfair advantage to layers closer to the output in choosing the optimal architecture, causing performance collapse. We then propose two new regularization terms that aim to prevent performance collapse by harmonizing operation selection via aligning gradients of layers. Experimental results on six different search spaces and three different datasets show that our method (Λ-DARTS) does indeed prevent performance collapse, providing justification for our theoretical analysis and the proposed remedy.

1 INTRODUCTION

With the growth of the popularity of deep learning models, neural architecture design has become one of the most important challenges of machine learning. Neural architecture search (NAS), a now prominent branch of AutoML, aims to perform neural architecture design in an automatic way (Elsken et al., 2019; Ren et al., 2021). Initially, the problem of NAS was addressed through reinforcement learning or evolutionary algorithms by the seminal works of (Zoph & Le, 2017; Baker et al., 2017; Stanley & Miikkulainen, 2002; Real et al., 2017). But despite the recent advancements in efficiency (Baker et al., 2018; Liu et al., 2018; Cai et al., 2018), these methods remain impractical and not widely accessible. One-shot methods aim to address this impracticality by performing the architecture search in a one-shot and end-to-end manner (Liu et al., 2019; Guo et al., 2020; Pham et al., 2018; Brock et al., 2018; Bender et al., 2018). Another technique for increasing the efficiency of search is cell-search (Zoph et al., 2018), which performs NAS for a set of cells stacked on top of each other according to a pre-defined macro-architecture. Differentiable neural architecture search (DARTS) (Liu et al., 2019) is a one-shot method that performs cell-search using gradient descent and continuous relaxation. It performs the cell-search using a weight-sharing framework. As a result of these innovations, DARTS is one of the most efficient methods of architecture search (Elsken et al., 2019).

One of the most severe issues of DARTS is the performance collapse problem (Zela et al., 2020), which states that the quality of the architecture selected by DARTS gradually degenerates into an architecture solely consisting of skip-connections. This issue is usually attributed to gradient vanishing (Zhou et al., 2020) or large Hessians of the loss function (Zela et al., 2020), and mostly addressed by heavily modifying DARTS (Chu et al., 2021; Wang et al., 2021; Gu et al., 2021; Ye et al., 2022).

In this paper, we argue that these theories fail to grasp the main reason behind the performance collapse: the conditions imposed on the convergence of DARTS by the weight-sharing framework. Specifically, we first define a measure to show the correlation between the gradient of each layer corresponding to the architecture parameters - dubbed *layer alignment* (Λ). Then, following a careful analysis of the convergence conditions of DARTS and the effects of weight-sharing, we find that:

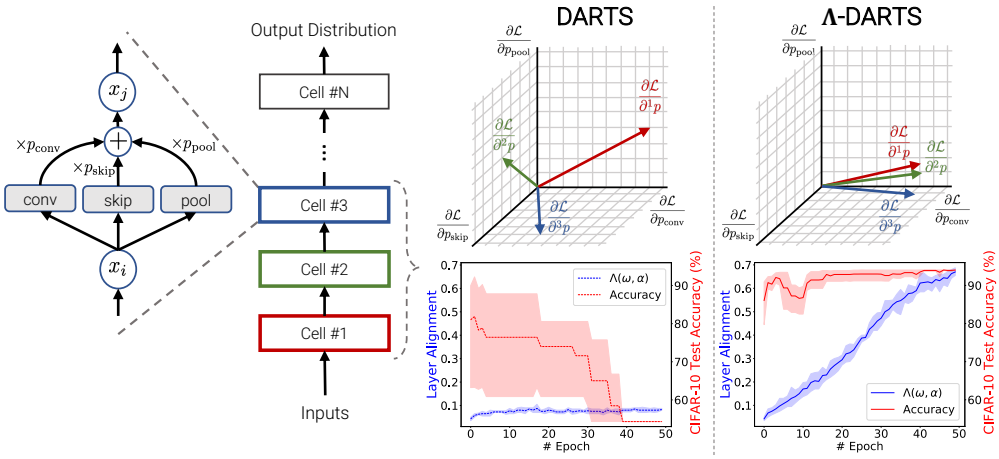


Figure 1: An illustration of the differentiable formulation for NAS by DARTS, and the effects of layer alignment over the trajectory of the performance of the discovered architectures by DARTS and Λ-DARTS. The experiments are performed on the NAS-Bench-201 search space, averaged over 4 runs with a 95% confidence interval.

1. Due to the value of Λ, DARTS can only achieve convergence by reaching the saturation point of the softmax function, where the normalized architecture parameters become almost one-hot vectors. This convergence point does not depend directly on the loss function, giving an unfair advantage to the layers that do not suffer from gradient vanishing.
2. Low value for Λ means that the optimal architecture corresponding to each layer varies wildly, with layers that are closer to the output - and therefore not suffering from gradient vanishing - mainly preferring non-parametric operations.
3. The aforementioned issues are direct contributors to the performance collapse problem. Furthermore, increasing the depth of the search model or the number of iterations used for the search increases the severity of performance collapse.

In Figure-1, we can see an illustration of DARTS and the concept of layer alignment (which we will define in section-3.2), and observe a clear correlation between the layer alignment and the performance of the discovered architecture on the NAS-Bench-201 search space (Dong & Yang, 2020). In this paper, we will use a combination of analytical and empirical evidence to support the claim that this relationship is in fact a causal relationship, resulting from the weight-sharing framework used in DARTS.

Following these conclusions, we introduce a regularization term that aims to alleviate the problem of performance collapse by increasing the layer alignment. Then, through a comprehensive empirical examination, we show that our method - dubbed Λ-DARTS - significantly improves the performance of DARTS on various search spaces and datasets without any modification to the algorithm of DARTS or the structure of the cells. Specifically, our method achieves an average accuracy of 96.57% and 83.85% on the CIFAR-10 and CIFAR-100 datasets on the DARTS search space, improving upon the current state-of-the-art by 0.06% and 0.39%, respectively. To the best of our knowledge, our work is the first to investigate DARTS from the point-of-view of its *wight-sharing framework* and *convergence conditions*.

2 Related Work

DARTS (Liu et al., 2019) proposed a continuous and differentiable search space through weighting a fixed set of operations to make NAS more scalable. It trains a super-graph with gradient descent and chooses the sub-graph consisted of weightiest operation edges. Its simplicity made it very popular and many variations emerged to address its theoretical and empirical setbacks:

Overlooked effects of design decisions in DARTS. (Wang et al., 2021) and (Gu et al., 2021) claim that DARTS chooses the final architecture with regard to operation weights, whereas these magnitudes do not necessarily correlate with the performance. (Wang et al., 2021) tries to measure operation impact directly and DOTS (Gu et al., 2021) tries to decouple topology from operation during search. IDARTS (Xue et al., 2021) posits that the innate relationship between the architecture's parameters is ignored by the gradient descent method used in DARTS and formulates their optimization as a bi-linear problem.

Generalization ability. The differentiable NAS methods are often executed on a smaller dataset of a certain task to reduce the required computational resources. AdaptNAS (Li et al., 2020), MixSearch (Liu et al., 2021), P-DARTS (Chen et al., 2021b), and DrNAS (Chen et al., 2021a) show that the discovered architecture does not always perform as well as on the proxy task on another more challenging dataset. They solve this issue by providing the network with domain information, making the search network progressively deeper, and reformulating the search algorithm.

Performance collapse. Zela et al. (2020); Chu et al. (2021) point out the performance collapse problem of DARTS. They show that DARTS consistently discovers networks with mainly skip-connection operations, which causes severe performance declination. Zela et al. (2020) monitors the eigenvalues of the Hessian of the architecture parameters and performs early stopping. Based on their work, SmoothDARTS (Chen & Hsieh, 2020) performs Hessian regularization to smooth the architecture Hessian. PC-DARTS (Xu et al., 2020) reduces the number of output channels of each operation, for the sake of lower memory consumption and a larger batch size during search. iDARTS (Zhang et al., 2021) focuses on improving the optimization of the architecture parameters by estimating the descent direction more accurately. DARTS- (Chu et al., 2021) argues that skip-connection operations have a potentially sharp loss curvature that leads to their advantage during search, and tries to solve it by adding auxiliary skip-connections to cell structure. β-DARTS (Ye et al., 2022) replaces the ℓ^2-regularization on the architecture parameters with a new regularization term to alleviate the unfair advantage of skip-connections. In this paper, we provide a novel analysis of DARTS from the point-of-view of its convergence, diagnosing an overlooked issue of its weight-sharing system that sends convergence down an irrelevant path and may be the root cause of performance collapse. We propose a solution for the matter without modifying the core formulation of DARTS.

3 An Analysis on the Convergence of DARTS

We start with an analysis of the weight-sharing framework used for cell-search in DARTS, and the required conditions for its convergence. We then empirically investigate whether DARTS can satisfy any of these conditions, and show that it is only capable of reaching convergence under one of them. Finally, using these observations, we will show that this type of convergence causes performance collapse, providing a motivation for Λ-DARTS. For the uninitiated reader, we provide a brief explanation of DARTS and the weight-sharing framework used by it for cell-search in Appendix-A.1.

3.1 The convergence of DARTS

Assuming the primary loss function $\mathcal{L}(.,.)$ to be differentiable, we can write the necessary and sufficient condition for optimality of the architecture parameters, (α) as (Boyd & Vandenberghe, 2014):

$$\nabla_{\alpha}\mathcal{L}_{val}(\omega,\alpha^*)=0. \tag{1}$$

Let $\boldsymbol{p}=\sigma(\alpha)$ be the softmax normalized architecture parameters, where $\sigma(.)$ corresponds to the softmax function. Then we can re-write (1) as:

$$\boldsymbol{J}_{\sigma}(\alpha^*)\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*)=0, \tag{2}$$

where $\boldsymbol{J}_{\sigma}(\alpha)=\text{diag}(\sigma(\alpha))-\sigma(\alpha)\sigma(\alpha)^T$ corresponds to the Jacobian of the softmax function on all edges, and diag(.) being the diagonal matrix. Note that given the weight-sharing framework, we can write $\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*)=\frac{\partial\mathcal{L}}{\partial p}(\omega,\alpha^*)$ as the sum of gradients received by each layer:

$$\boldsymbol{J}_{\sigma}(\alpha^*)\left(\sum_{\ell=1}^{L}\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*)\right)=0. \tag{3}$$

Now note that $\boldsymbol{J}_{\sigma}(.)$ is a block-diagonal matrix, with each block corresponding to one of the edges in the architecture. Therefore, $\boldsymbol{J}_{\sigma}(.)$ is a positive semi-definite matrix with its null-space containing vectors of the following form:

$$\boldsymbol{v}=\begin{bmatrix}\boldsymbol{v}_1^T & \boldsymbol{v}_2^T & ... & \boldsymbol{v}_{|E|}^T\end{bmatrix}^T, \tag{4}$$

where the vector $\boldsymbol{v}$ consists of concatenated vectors $\boldsymbol{v}_i\in\mathbb{R}^{|E|}$ of the form $\boldsymbol{v}_i^T\in\{c_i\cdot[1 \quad 1 \quad ... \quad 1]^T \mid c_i\in\mathbb{R}\}$ (Gao & Pavel, 2017). Considering the diverse set of operations used in DARTS, it is not an unfair assumption that $\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)$ won't be in the form presented in (4). In fact, we have seen empirical evidence to support this claim, which will be discussed in Section-3.2. So assuming that $\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)$ does not belong to the null space of $\boldsymbol{J}_{\sigma}(\alpha)$, satisfying (1) can only be achieved in two ways:

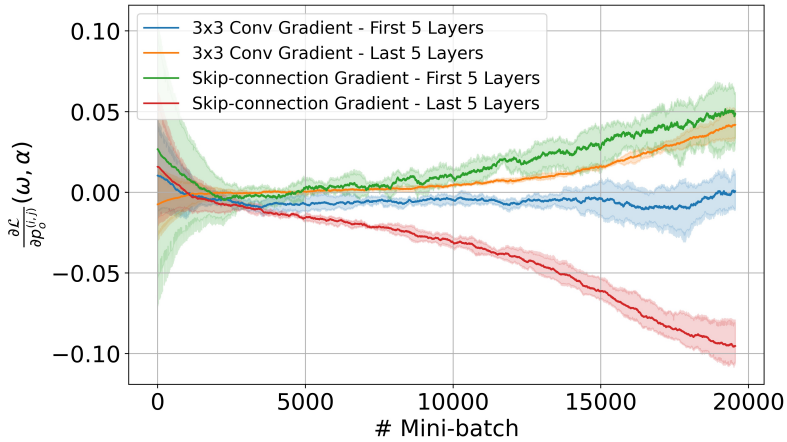


Figure 2: The exponential average (with decay rate 0.999) of the sum of the gradients of the 3×3 convolution and skip-connection operations for the first 5 layers and the last 5 layers averaged over all 6 edges of the cell on the NAS-Bench-201 search space, averaged over 4 runs with a 95% confidence interval. Note that negative values in the gradient mean the value of the parameter corresponding to the operation will be increasing.

1. $\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*)$ approaching 0.
2. The Jacobian $\boldsymbol{J}_\sigma(\alpha^*)$ approaching 0.

3.2 The layer alignment issue

Assuming that $\forall \ell: \quad \nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*) \neq 0$, a necessary condition for $\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*)$ to be able to approach zero is that for each ℓ we have $\exists \ell' \neq \ell: \quad \nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*) \not\perp \nabla_{\ell'\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*)$. Because otherwise, we can write $\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*)^T \nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*) = \|\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha^*)\|_2^2 \neq 0$. Empirically, we noticed that this is not the case in DARTS. More specifically, we define the function Λ - which we dub *layer alignment* - as:

$$\Lambda(\omega,\alpha) = \frac{1}{\binom{L}{2}} \sum_{\ell<\ell'} \frac{\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha)^T \nabla_{\ell'\boldsymbol{p}}\mathcal{L}(\omega,\alpha)}{\|\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2 \|\nabla_{\ell'\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2}. \tag{5}$$

We noticed that regardless of the search space, the layer alignment is always near zero, i.e. the layer gradients are almost orthogonal. In Figure 1, we can see the value of $\Lambda(.,.)$ on the NAS-Bench-201 search space (Dong & Yang, 2020) for both DARTS and Λ-DARTS.

As a result of this orthogonality, we expect that under the aforementioned assumptions DARTS is incapable of reaching convergence the first way, since it is likely incapable of satisfying the necessary condition for it. So we will propose a lower-bound on the value $\|\nabla_\alpha \mathcal{L}(\omega,\alpha)\|_2^2$, which formalizes our claim:

Proposition 1 *Let $\nabla_\alpha \mathcal{L}(\omega,\alpha)$ be orthogonal to the null space of $\boldsymbol{J}_\sigma(\alpha)$, and assuming we have L layers of cells in our search network, we have $\forall \ell \neq \ell' \quad \nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha) \perp \nabla_{\ell'\boldsymbol{p}}\mathcal{L}(\omega,\alpha)$. Let $\lambda_i(\alpha) > 0$ be the set of eigenvalues corresponding to $\boldsymbol{J}_\sigma(\alpha)$. Then the square norm of the gradient $\nabla_\alpha \mathcal{L}(\omega,\alpha)$ can be bounded by:*

$$\|\nabla_\alpha \mathcal{L}(\omega,\alpha)\|_2^2 \geq \min_{i,\lambda_i(\alpha)\neq 0} \lambda_i(\alpha)^2 \cdot L \cdot \min_\ell \|\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2. \tag{6}$$

We provide a proof for this proposition in Appendix-A.3. This proposition shows that for any value of $\min_\ell \|\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2$, we have a large enough value for L that prevents the right-hand-side of (6) to become too small. As a result, for a deep enough search network, the only way for α to reach convergence is through reaching the softmax saturation point, wherein we have near zero eigenvectors corresponding to $\boldsymbol{J}_\sigma(\alpha)$, and the normalized architecture parameters become close to one-hot vectors. This phenomenon has been observed in the literature (Bi et al., 2019; Dong & Yang, 2020), where DARTS usually converges to a softmax saturation point with all of the edges selecting the skip-connection or none operations, depending on the search space. In Figure-3, we can see this effect, where the absolute changes in the softmax normalized architecture weights of DARTS during search is rapidly growing, compared to Λ-DARTS which reaches a plateau during search. Note that converging to a softmax saturation point is not an inherently bad outcome, because it corresponds to near zero optimality gap when using continuous relaxation (Boyd

& Vandenberghe, 2014). But as we will see, this characteristic gives an advantage to layers closer to the output in selecting the optimal architecture, contributing to performance collapse (Zela et al., 2020).

3.3 THE PERFORMANCE COLLAPSE

Another issue caused by low layer alignment is that the optimal architectures corresponding to each layer are going to be vastly different. In Figure 2 we can see the exponential average of the sum of the gradients for convolution and skip-connection operations in the first 5 layers (furthest from the output of the network) and the last 5 layers (closest to the output of the network) averaged over all edges in a search cell on the NAS-Bench-201 search space (Dong & Yang, 2020). As evident, the optimal cell corresponding to the shallower layers (closer to the input) mostly contains convolution operations, while the optimal cell corresponding to the deeper layers (closer to the output) mostly contains skip-connection operations. We attribute this observation to skip-connections having a proclivity towards allowing the gradients to flow further into the network, while convolutions provide more complex features for upper layers. Furthermore, due to gradient vanishing, the gradients corresponding to the skip-connection operations on average have larger magnitudes, with the magnitude of the layer gradients gradually becoming smaller the deeper we go. As a result, $\nabla_\alpha \mathcal{L}(\omega,\alpha)$ will have much smaller negative values corresponding to the skip-connection operations.

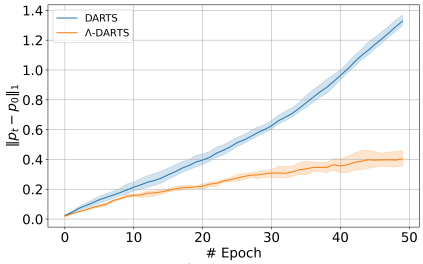


Figure 3: The ℓ^1 norm of changes in softmax-normalized architecture weights, averaged over 4 runs with a 95% confidence interval. The search is performed on CIFAR-10.

Using these observations, we can clearly bridge the analytical gap between the performance collapse problem and the layer alignment. Due to the weight-sharing structure, the value of the function $\Lambda(\omega,\alpha)$ is close to zero. As a result, DARTS can only achieve convergence in the softmax saturation points. Furthermore, given the low layer alignment, the optimal architecture from the perspective of the layers are going to be extremely varied, with deeper layers having a strong inclination towards skip-connections. This gives skip-connections a clear advantage due to the larger magnitude of their gradients. So as the search progresses, the architecture parameters corresponding to the skip-connection operations become larger than the convolution operations, eventually resulting in a saturation point that overwhelmingly selects skip-connection operations (Bi et al., 2019; Chu et al., 2021). As the depth of the search network increases, vanishing gradient becomes more severe, further exacerbating the issue (He et al., 2016). This is evident when comparing the results of DARTS on its original search space and the NAS-Bench-201 search space, which utilizes deeper search networks (Dong & Yang, 2020). Similarly, increasing the number of iterations used for the search results in further exacerbation of the performance collapse problem (Bi et al., 2019), which can be attributed to the severity of vanishing gradient increasing as the loss function approaches its optimal point (He et al., 2016). These observations motivate us to alleviate both of these issues caused by low layer alignment through increasing the value of $\Lambda(\omega,\alpha)$.

4 THE PROPOSED METHOD TO MITIGATE PERFORMANCE COLLAPSE

In the previous segment, we saw that the main contributor to the performance collapse problem is low layer alignment (i.e. $\Lambda(\omega,\alpha)\approx 0$). Now we will introduce two regularization terms based on this value to solve performance collapse. Furthermore, we will introduce a method to estimate the gradient corresponding to the regularization terms that grow linearly with the size of the network.

4.1 THE REGULARIZATION TERM

For the sake of simplicity, let's assume we aim to search for a single cell. To increase the value of $\Lambda(\omega,\alpha)$, we can directly use the function itself, since it is a differentiable function. So we define our main regularization term as the function $\Lambda(\omega,\alpha)$, which we aim to maximize.

In our preliminary experiments, we noticed that in case the dimension of α is large, $\Lambda(\omega,\alpha)$ only concentrates on the operations with relatively larger gradients. This means that while the angle between the gradients of the layers is reduced, only the gradient of a small portion of the operations will have the same sign, resulting in lower performance. So given ${}^{\ell}g = \nabla_{{}^{\ell}\boldsymbol{p}}\mathcal{L}(\omega,\alpha)$ and ${}^{\ell'}g = \nabla_{{}^{\ell'}\boldsymbol{p}}\mathcal{L}(\omega,\alpha)$, we will also introduce another regularization term that aims to increase the correlation between the signs

of the gradients of each layer:

$$\Lambda_{\pm}(\omega,\alpha)=\frac{1}{\binom{L}{2}}\sum_{\ell<\ell'}\frac{{}^{\ell}g^{T}\,{}^{\ell'}g}{|{}^{\ell}g|^{T}|{}^{\ell'}g|}, \tag{7}$$

where $|.|$ corresponds to the element-wise absolute value function. Note that while the optimal point of $\Lambda(\omega,\alpha)$ corresponds to $\nabla_{\ell}\boldsymbol{p}\mathcal{L}(\omega,\alpha)=c\nabla_{\ell'}\boldsymbol{p}\mathcal{L}(\omega,\alpha)$, where c is a constant, the optimal point of $\Lambda_{\pm}(\omega,\alpha)$ corresponds to $\text{sign}(\nabla_{\ell}\boldsymbol{p}\mathcal{L}(\omega,\alpha))=\text{sign}(\nabla_{\ell'}\boldsymbol{p}\mathcal{L}(\omega,\alpha))$, where $\text{sign}(.)$ is the element-wise signum function.

Given one of the regularization terms above, the bi-level optimization problem of Λ-DARTS will become:

$$\begin{aligned}\min_{\alpha}\quad & \mathcal{L}^{val}(\omega^{*}(\alpha),\alpha)\\ \text{s.t.}\quad & \omega^{*}(\alpha)=\underset{\omega}{\text{argmin}}\ \mathcal{L}^{train}(\omega,\alpha)-\lambda\Lambda^{train}(\omega,\alpha).\end{aligned} \tag{8}$$

Similar to most variants of DARTS, we use a linear scheduling for λ (Zela et al., 2020; Chen & Hsieh, 2020): i.e. $\lambda_t=\lambda\times\frac{t}{T}$, where t is the number of current epoch and T is the total number of epochs used for search.

4.2 Estimating the gradients of the regularization terms

Assuming the gradients corresponding to the operations are non-zero, both $\Lambda(\omega,\alpha)$ and $\Lambda_{\pm}(\omega,\alpha)$ are differentiable. So their gradient with respect to ω can be written as:

$$\nabla_{\omega}\Lambda(\omega,\alpha)=\frac{1}{\binom{L}{2}}\sum_{\ell=1}^{L}\nabla^{2}_{\omega,{}^{\ell}p}\mathcal{L}(\omega,\alpha)\sum_{\ell'\neq\ell}\delta^{\ell,\ell'}, \tag{9}$$

where we have:

$$\begin{aligned}\delta^{\ell,\ell'}_{\Lambda}&=\left(\boldsymbol{I}-\frac{{}^{\ell}g\,{}^{\ell}g^{T}}{\|{}^{\ell}g\|_2^2}\right)\frac{{}^{\ell'}g}{\|{}^{\ell}g\|_2\|{}^{\ell'}g\|_2},\\ \delta^{\ell,\ell'}_{\Lambda_{\pm}}&=\left(\boldsymbol{I}-\frac{{}^{\ell}g^{T}\,{}^{\ell'}g}{|{}^{\ell}g|^{T}|{}^{\ell'}g|}\text{diag}(\text{sign}({}^{\ell}g))\text{diag}(\text{sign}({}^{\ell'}g))\right)\frac{{}^{\ell'}g}{|{}^{\ell}g|^{T}|{}^{\ell'}g|},\end{aligned} \tag{10}$$

with $\boldsymbol{I}$ corresponding to the identity matrix. Note that calculating $\nabla_{\omega}\Lambda(\omega,\alpha)$ will be of $O(L\cdot|\omega|\cdot|\alpha|)$ due to the existence of second derivative matrices for each layer. So a direct calculation of (9) is out of the question. But similar to (Liu et al., 2019), we can use finite difference approximation to estimate $\nabla_{\omega}\Lambda(\omega,\alpha)$.

Let $\mathcal{L}(\omega,\boldsymbol{P})$ be the loss function as a function of ω and $\boldsymbol{P}\in\mathbb{R}^{L\times|\alpha|}$, where:

$$\boldsymbol{P}=[\sigma(\alpha)\quad\sigma(\alpha)\quad\ldots\quad\sigma(\alpha)]^{T} \tag{11}$$

corresponds to the softmax normalized architecture weights used for each layer in the search network. Furthermore, let $\Delta\in\mathbb{R}^{L\times|\alpha|}$ be:

$$\Delta=\left[\textstyle\sum_{\ell'\neq1}\delta^{1,\ell'}\quad\sum_{\ell'\neq2}\delta^{2,\ell'}\quad\ldots\quad\sum_{\ell'\neq L}\delta^{L,\ell'}\right]^{T}. \tag{12}$$

Then we can write:

$$\nabla_{\omega}\Lambda(\omega,\alpha)=\frac{1}{\binom{L}{2}}\left[\frac{\nabla_{\omega}\mathcal{L}(\omega,\boldsymbol{P}+\epsilon\Delta)-\nabla_{\omega}\mathcal{L}(\omega,\boldsymbol{P}-\epsilon\Delta)}{2\epsilon}\right]+O(\epsilon^{2}), \tag{13}$$

where we set $\epsilon=\frac{\epsilon_0}{\|\Delta\|_F}$. The cost of using this approximation technique is two extra forward-backward pass, which is of $O(|\omega|+|\alpha|)=O(|\omega|)$. So we have eliminated the $L\cdot|\alpha|$ factor from the complexity of Λ-DARTS, which significantly reduces the computational complexity, while completely eliminating the memory overhead.

5 Experimental Results and Discussions

In this section, we will show the effectiveness of Λ-DARTS by performing experiments on the NAS-Bench-201 (Dong & Yang, 2020) and DARTS (Liu et al., 2019) search spaces. We will then demonstrate the

Table 1: Comparison with state-of-the-art on the NAS-Bench-201 search space. Mean and standard deviation of the accuracy were calculated over 4 independent runs. The search is performed on the CIFAR-10 dataset. The best performance is boldface. (1st): first-order, (2nd): second-order

Method	**CIFAR-10**		**CIFAR-100**		**ImageNet16-120**	
	Valid	Test	Valid	Test	Valid	Test
DARTS (1st) (Liu et al., 2019)	39.77±0.00	54.30±0.00	15.03±0.00	15.61±0.00	16.43±0.00	16.32±0.00
DARTS (2nd) (Liu et al., 2019)	39.77±0.00	54.30±0.00	15.03±0.00	15.61±0.00	16.43±0.00	16.32±0.00
GDAS (Dong & Yang, 2019)	89.89±0.08	93.61±0.09	71.34±0.04	70.70±0.30	41.59±1.33	41.71±0.98
SNAS (Xie et al., 2019)	90.10±1.04	92.77±0.83	69.69±2.39	69.34±1.98	42.84±1.79	43.16±2.64
DSNAS (Hu et al., 2020)	89.66±0.29	93.08±0.13	30.87±16.40	31.01±16.38	40.61±0.09	41.07±0.09
PC-DARTS (Xu et al., 2020)	89.96±0.15	93.41±0.30	67.12±0.39	67.48±0.89	40.83±0.08	41.31±0.22
iDARTS (Zhang et al., 2021)	89.86±0.60	93.58±0.32	70.57±0.24	70.83±0.48	40.38±0.59	40.89±0.68
DARTS- (Chu et al., 2021)	91.03±0.44	93.80±0.40	71.36±1.51	71.53±1.51	44.87±1.46	45.12±0.82
β-DARTS (Ye et al., 2022)	**91.55±0.00**	**94.36±0.00**	**73.49±0.00**	**73.51±0.00**	**46.37±0.00**	**46.34±0.00**
Λ-DARTS - $\Lambda(\omega,\alpha)$	**91.55±0.00**	**94.36±0.00**	**73.49±0.00**	**73.51±0.00**	**46.37±0.00**	**46.34±0.00**
Optimal (Dong & Yang, 2020)	91.61	94.37	73.49	73.51	46.77	47.31

robustness of Λ-DARTS by performing more experiments on the reduced DARTS search spaces proposed by (Zela et al., 2020) on different datasets. Finally, by performing an ablation study, we will analyze the effects of the hyper-parameters of Λ-DARTS over the performance of DARTS. For all experiments, we report the mean and the standard deviation of Λ-DARTS for 4 independent searches with different random seeds, as well as the result of the best performing architecture, except for the experiments on the reduced DARTS search spaces, wherein only the best performance is reported. In all experiments, unless explicitly mentioned otherwise, the search is performed on CIFAR-10. For details of the experiment settings, we direct the reader to Appendix-A.2.

5.1 NAS-BENCH-201 SEARCH SPACE

In order to demonstrate the effectiveness of Λ-DARTS in a reproducible setting, we used the NAS-Bench-201 search space for our first set of experiments. In Table-1 we can see the results of Λ-DARTS compared to other DARTS-based methods. We omit from reporting the results of $\Lambda_{\pm}(\omega,\alpha)$ here, since it performed similar to $\Lambda(\omega,\alpha)$. As evident, Λ-DARTS performs exceptionally well compared to the baselines, surpassing all but one of them in terms of the mean and standard deviation of the performance. Compared to the current state-of-the-art (Ye et al., 2022), Λ-DARTS discovers the same architecture in all 4 experiments. Considering how close to the optimal the performance of Λ-DARTS is, we attribute the slightly less than optimal performance to the high variance of the CIFAR-10 dataset. Furthermore, we note that the method proposed in (Ye et al., 2022) converges to its optimal architecture very early in the search process (the first 10 epochs), which may indicate a premature convergence to a sub-optimal architecture.

In comparison, Λ-DARTS does not reach its optimal performance until late in the search procedure, where the performance of the search model is close to optimal and the correlation between the gradient of the layers is very high. In fact, in the DARTS search space we will show further evidence for this claim, where Λ-DARTS is capable of surpassing the performance of (Ye et al., 2022) on the DARTS search space by a large margin on two datasets. In Figure-1, we can see a clear correlation between the layer alignment and the accuracy of the architecture on CIFAR-10, which reaches its optimal point at around the 40th search epoch. Comparing the results to DARTS, we see that the performance of the discovered architecture declines consistently, eventually reaching an extremely low point.

5.2 DARTS SEARCH SPACE

In order to show that Λ-DARTS is effective in mitigating the performance collapse problem in a complex search space, we used the original search space of DARTS proposed in (Liu et al., 2019) for our second experiment. In Table-2, we can see the results of Λ-DARTS compared to the baselines. Λ-DARTS is able to achieve state-of-the-art results on the CIFAR-10 and CIFAR-100 datasets, surpassing the baseline models by a large margin on the more complex dataset of CIFAR-100. Furthermore, Λ-DARTS achieves comparable results on ImageNet, with about 0.1% difference in accuracy compared to the state-of-the-art results on the same macro-architecture. Furthermore, compared to (Ye et al., 2022), we see that Λ-DARTS achieves better performance on two of the three datasets, supporting our claim that β-DARTS may be converging to sub-optimal architectures due to pre-mature convergence. Interestingly, compared to most baselines, Λ-DARTS does not select unnecessarily complex architectures, with the number of parameters averaging around 3.5 on $\Lambda(\omega,\alpha)$ and 3.6 on $\Lambda_{\pm}(\omega,\alpha)$ on CIFAR-10. This means that the architectures are much more efficient in terms of performance, further proving the superiority of Λ-DARTS. This is

Table 2: Comparison with state-of-the-art on the DARTS search space. The first block contains methods that report the best performing architecture, while the second block contains methods that report the average of multiple searches (except for results on ImageNet, where always the best performance is reported). Our reported mean and standard deviation of the accuracy are calculated over 4 independent runs. We perform the search on the CIFAR-10 dataset. The best performance is boldface on each block. (1st): first-order, (2nd): second-order, (avg): average performance, (best): best performance.
† Different search space used for ImageNet.

Method	CIFAR-10		CIFAR-100		ImageNet	
	Params (M)	Test Acc (%)	Params (M)	Test Acc (%)	Params (M)	Test Acc (%)
NASNet-A (Zoph et al., 2018)	3.3	97.35	3.3	83.18	5.3	74.0
DARTS (1st) (Liu et al., 2019)	3.4	97.00±0.14	3.4	82.46	-	-
DARTS (2nd) (Liu et al., 2019)	3.3	97.24±0.09	-	-	4.7	73.3
SNAS (Xie et al., 2019)	2.8	97.15±0.02	2.8	82.45	4.3	72.7
GDAS (Dong & Yang, 2019)	3.4	97.07	3.4	81.62	5.3	74.0
P-DARTS (Liu et al., 2018)	3.4	**97.50**	3.6	82.51	5.1	75.3
PC-DARTS (Xu et al., 2020)	3.6	97.43±0.07	3.6	**83.10**	5.3	**75.8**
DrNAS (Chen et al., 2021a)	4.0	97.46±0.03	-	-	5.2	**75.8**
R-DARTS (Zela et al., 2020)	-	97.05±0.21	-	81.99±0.26	-	-
P-DARTS (Liu et al., 2018)	3.3±0.21	97.19±0.14	-	-	-	-
SDARTS-ADV (Chen & Hsieh, 2020)	3.3	97.39±0.02	-	-	5.4	74.8
DOTS (Gu et al., 2021)	3.5	**97.51±0.06**	4.1	**83.52±0.13**	5.2	75.7
DARTS+PT (Wang et al., 2021)	3.0	97.39±0.08	-	-	4.6	74.5
DARTS- (Chu et al., 2021)†	3.5±0.13	97.41±0.08	3.4	82.49±0.25	4.9	**76.2**
β-DARTS (Ye et al., 2022)	3.8±0.08	97.49±0.07	3.8±0.08	83.48±0.03	5.4	75.8
Λ-DARTS - $\Lambda(\omega,\alpha)$ (avg)	3.5±0.13	97.48±0.11	3.6±0.13	83.47±0.20	-	-
Λ-DARTS - $\Lambda(\omega,\alpha)$ (best)	3.5	97.59	3.6	83.64	5.1	75.4
Λ-DARTS - $\Lambda_{\pm}(\omega,\alpha)$ (avg)	3.6±0.13	**97.57±0.05**	3.6±0.1	**83.85±0.38**	-	-
Λ-DARTS - $\Lambda_{\pm}(\omega,\alpha)$ (best)	3.8	**97.65**	3.8	**84.20**	5.2	**75.7**

Table 3: Comparison with state-of-the-art on the reduced DARTS search spaces. The test error rate (%) of the best performing architecture from 4 independent runs is reported. The best performance is boldface.

Dataset	Search Space	DARTS	PC-DARTS	R-DARTS	SDARTS-RS	DARTS+PT	Λ-DARTS
CIFAR-10	S1	3.84	3.11	**2.78**	**2.78**	3.50	2.83
	S2	4.85	3.02	3.31	2.75	2.79	**2.56**
	S3	3.34	2.51	2.51	2.53	2.49	**2.38**
	S4	7.20	3.02	3.56	2.93	2.64	**2.46**
CIFAR-100	S1	29.46	24.69	24.25	23.51	24.48	**22.79**
	S2	26.05	22.48	22.44	22.28	23.16	**21.68**
	S3	28.90	21.69	23.99	21.09	22.03	**21.03**
	S4	22.85	21.50	21.94	21.46	20.80	**20.65**
SVHN	S1	4.58	2.47	4.79	**2.35**	2.62	2.39
	S2	3.53	2.42	2.51	2.39	2.53	**2.37**
	S3	3.41	2.41	2.48	2.36	2.42	**2.31**
	S4	3.05	2.43	2.50	2.46	2.42	**2.34**

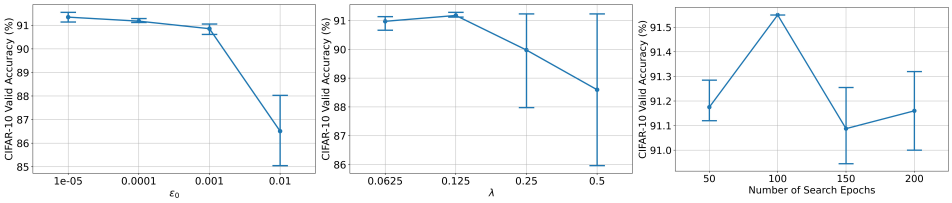
especially the case on the ImageNet search space, where our model is well within the boundaries of the mobile setting introduced in (Liu et al., 2019).

Comparing $\Lambda(\omega,\alpha)$ with $\Lambda_{\pm}(\omega,\alpha)$, we see that the second function achieves a much better performance. Comparing the discovered cells, we noticed that $\Lambda_{\pm}(\omega,\alpha)$ usually discovers deeper cells with more parametric operations, a quality that highly correlates with the performance of the model. This observation can be attributed to the large dimension of α on this search space. In Appendix-A.4 we can see the best architectures discovered by Λ-DARTS using both methods.

5.3 Reduced search spaces

In order to show the effectiveness of Λ-DARTS in alleviating the performance collapse problem even in the most extreme circumstances, we used the reduced search spaces proposed in (Zela et al., 2020) for further experimentation. In Table-3 we can see the results of Λ-DARTS compared to the baselines. Out of 12 different experiments performed, Λ-DARTS performs better than the baselines in 10 of them, with its performance in the other two experiments being comparable to the baselines. Following these observations, we can claim that our model is robustly stepping out of the performance collapse trap without any modifications to the original formulation of DARTS. In Appendix-A.4 we can see the architectures discovered by Λ-DARTS on these search spaces and datasets.

Comparing the baselines, we can see that while DARTS+PT performs better on the S4 search space, SDARTS performs mostly better on the other search spaces. Furthermore, we see similar inconsistencies in PC-DARTS and R-DARTS, where they both perform well on S3 but usually not as well on the other



(a) Effects of ϵ_0. (b) Effects of λ. (c) Effects of number of epochs.

Figure 4: An analysis of the effects of hyper-parameters on Λ-DARTS. We report the mean and 95% confidence interval of CIFAR-10 validation accuracy, calculated over 4 runs. (a) Effects of ϵ_0 over the accuracy of the approximation and the performance of discovered architectures. (b) Effects of λ over the performance of the discovered architectures. (c) Effects of the number of search epochs on the performance of the discovered architectures. The search is performed on CIFAR-10.

search spaces. On the other hand, Λ-DARTS consistently performs well across all search spaces and datasets, which further shows the robustness of our method.

5.4 Ablation Study

In order to assess how the hyper-parameters affect Λ-DARTS, we conducted an ablation study on ϵ_0 and λ. Furthermore, to show that Λ-DARTS has solved the performance collapse issue, we perform the search for longer epochs (starting from 50, up to 200 epochs) following (Chu et al., 2021; Ye et al., 2022). In Figure 4a, we can see the effect of ϵ_0 over the performance of the discovered architecture. We observe that smaller values of ϵ_0 correlate positively with the quality of the discovered architecture. Considering that lower values of ϵ_0 correspond to lower approximation error in (13), this is expected behavior. But we note that due to the problem of round-off error, we expect this trend to reverse for smaller values of ϵ_0 (Shen et al., 2020). In fact, we can see that the variance of the performance is already starting to increase for the smallest value of ϵ_0.

Figure 4b shows the effect of λ over the performance of the discovered architecture. Clearly, the model is not very sensitive to the value of λ, with the best performing value and the worst performing value having a difference of about 2% in their mean accuracy. Furthermore, the best performing value at $\lambda = 0.125$ corresponds to an average length normalized dot-product of about 0.7, while the worst performing value at $\lambda = 0.5$ is around 0.9. So we are well within the realm of diminishing returns for larger λs, and as a result of increased emphasis on the regularization term in the Pareto-optimal point, we observe negative effects from the main objective being sidetracked in the form of lower quality architectures.

We can see the performance of Λ-DARTS for different numbers of search epochs in Figure-4c. Unlike other methods that try to address the issue of performance collapse for larger number of epochs (Bi et al., 2019; Chu et al., 2021), we see a negligible drop in performance (about 0.5%) when comparing the best performing and the worst performing setting. This is a clear indication that the models have achieved convergence at around 100 epochs, reaching a local-minima that corresponds to near-optimal architectures. In fact, we can observe further proof for the convergence of Λ-DARTS in Figure-3, where clearly the absolute changes in the softmax normalized architecture weights has reached a plateau in 50 epochs of search, resulting in a concave curvature.

6 Conclusion

In this paper, we use a set of analytical and empirical tools to show that the main reason behind performance collapse in DARTS is the lack of correlation between the gradient of its layers, caused by the multi-layer structure and the use of weight-sharing for cell-search. Based on the aforementioned analysis, we propose Λ-DARTS which manages to achieve better or comparable performance when compared to state-of-the-art baselines. Here, we focused our discussion and experiments on DARTS, however the analysis and ideas are extendable to any other type of differentiable NAS model that performs cell search using weight sharing, which we plan to study in future works. Furthermore, we point out to the cost of estimating the gradient of the regularization terms proposed here as another potential subject for further research.

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A Appendix

A.1 Review on differentiable neural architecture search (DARTS)

A.1.1 Continuous relaxation

For simplicity, let's assume we are searching for a single cell. Let the vector $\alpha \in \mathbb{R}^{|E|\cdot|\mathcal{O}|}$ be the set of parameters corresponding to the architecture, where E is the set of all the possible edges in the computational graph of the cell, and $\mathcal{O}$ is a set of pre-defined operations. The goal is to find the optimal topology and operations in the continuous space using gradient-descent, and then transfer it from the continuous space to the discrete space using the argmax operator to get a near-optimal architecture.

Specifically, let x_j be the output of the jth node in the computational graph; We can write x_j as:

$$x_j = \sum_{i<j} \sum_{o \in \mathcal{O}} p_o^{(i,j)} o(x_i), \tag{14}$$

where $p_o^{(i,j)}$ is the softmax normalized architecture weights: $p_o^{(i,j)} = \frac{\exp(\alpha_o^{(i,j)})}{\sum_{o' \in \mathcal{O}} \exp(\alpha_{o'}^{(i,j)})}$. DARTS then solves the following bi-level optimization problem to find the optimal value for α:

$$\begin{aligned} \min_{\alpha} \quad & \mathcal{L}^{val}(\omega^*(\alpha), \alpha) \\ \text{s.t.} \quad & \omega^*(\alpha) = \operatorname*{argmin}_{\omega} \mathcal{L}^{train}(\omega, \alpha), \end{aligned} \tag{15}$$

where $\mathcal{L}^{train}(.,.)$ and $\mathcal{L}^{val}(.,.)$ correspond to the loss function calculated on the train and validation set. There are two methods proposed by the original paper to solve the problem above using gradient descent. The first method - dubbed first-order - performs gradient descent on α and ω in an interleaved fashion, while the second method - dubbed second-order - considers the dependency between $\omega^*(\alpha)$ and α when calculating the descent direction corresponding to α. In this paper, we focus on the first-order method since it is more widely used (Elsken et al., 2019). For further information, we direct the reader to the original paper (Liu et al., 2019).

A.1.2 Weight-sharing for cell-search

In order to perform cell-search, DARTS utilizes a weight-sharing framework. Specifically, in a cell-search scheme, L layers of the same cell are stacked on top of each other according to a pre-defined macro architecture. These cells share the same α - i.e. share the same architecture parameter - but have different parameters corresponding to the operations. So they have the same structure, but different operation weights. As a result, we can write the gradient corresponding to the architecture parameter as the sum of the gradients received by each layer.

In order to define this weight-sharing framework concretely, let ${}^{\ell}x_j$ correspond to the output of the jth node of the architecture graph produced by the ℓth layer. Then we can write the gradient received by $p_o^{(i,j)}$ from the ℓth layer as:

$$\frac{\partial \mathcal{L}}{\partial^{\ell} p_o^{(i,j)}} = \frac{\partial \mathcal{L}}{\partial^{\ell} x_j} \cdot o({}^{\ell}x_i). \tag{16}$$

In other words, the gradient received by $p_o^{(i,j)}$ from the ℓth layer corresponds to the dot product of the output of the ith node in ℓ^{th} layer and the gradient of the jth node in ℓ^{th} layer. Since we have L layers in the macro-architecture, we can write the gradient corresponding to $p_o^{(i,j)}$ as the sum of the gradients received by each layer:

$$\frac{\partial \mathcal{L}}{\partial p_o^{(i,j)}} = \sum_{\ell=1}^{L} \frac{\partial \mathcal{L}}{\partial^{\ell} p_o^{(i,j)}}. \tag{17}$$

Now, by writing the gradient for all edges and operations for p in vector form, we have:

$$\nabla_{\boldsymbol{p}} \mathcal{L}(\omega,\alpha) = \sum_{\ell=1}^{L} \nabla_{\ell \boldsymbol{p}} \mathcal{L}(\omega,\alpha). \tag{18}$$

A.2 Experiment settings

A.2.1 NAS-Bench-201 search space

NAS-Bench-201 provides a DARTS-like cell-search framework, where we search for a single cell containing 3 nodes and 5 operations. The performance of the discovered architecture on CIFAR-10, CIFAR-100, and ImageNet16-120 datasets can be attained by querrying the database provided by (Dong & Yang, 2020). We used the implementation of (Dong & Yang, 2020) for our search. The operation parameters (ω) are optimized using stochastic gradient descent with Nestrov momentum. The learning rate is set to 0.025, gradually reduced to 0.001 using cosine scheduling. The weight decay and momentum are set to 0.0005 and 0.9, respectively.

For architecture parameters (α), we use Adam with the learning rate set to 10^{-4} and the weight decay rate set to 0.001. The momentum values β_1 and β_2 are set to 0.5 and 0.999, respectively. We perform the search on CIFAR-10 for 100 epochs, and we set $\epsilon_0 = 0.0001$ and $\lambda = 0.125$ for this experiment.

A.2.2 DARTS search space

The search space of DARTS contains two types of cells: Normal and Reduction, corresponding to layers with stride of 1 and 2, respectively. Each cell has 4 intermediate nodes and 7 operations. Here, we only apply the regularization term to the normal cell, since the performance collapse is not usually associated with the reduction cell (Zela et al., 2020). We used the implementation provided by (Wang et al., 2021; Chen & Hsieh, 2020). The operation parameters (ω) are optimized using stochastic gradient descent with momentum. The learning rate is set to 0.025, gradually reduced to 0.001 using cosine scheduling. The weight decay and momentum are set to 0.0003 and 0.9, respectively.

For architecture parameters (α), we use Adam with the learning rate set to 3×10^{-4} and the weight decay rate set to 0.001. The momentum values β_1 and β_2 are set to 0.5 and 0.999, respectively. We perform the search on CIFAR-10 for 50 epochs, and we set the value for λ to 0.25 for $\Lambda(\omega,\alpha)$ and 0.125 for $\Lambda_{\pm}(\omega,\alpha)$. In both cases, we set the value of ϵ_0 to 0.0001.

A.2.3 Reduced search spaces

The four search spaces proposed by (Zela et al., 2020) contain some subset of the operations used in the DARTS search space, except for the fourth search space which contains convolutions and random noise as operations. Otherwise, the cells have the same structure as in the DARTS search space.

We perform the search and evaluation on three different datasets: CIFAR-10, CIFAR-100, and SVHN. For CIFAR-10, the search setting is the same as in Section-5.2. For SVHN and CIFAR-100, we increase

the batch size and the value of ϵ_0 to 96 and 0.001, respectively. The evaluation setting for these datasets is the same as in (Zela et al., 2020). For all experiments, we only report the results of $\Lambda(\omega,\alpha)$, since both methods perform similarly.

A.2.4 Ablation Study

For this experiment, we used the exact same setting as in Section-5.1, except for the architecture learning rate. For the experiments on ϵ_0 and λ, the architecture learning rate is set to 3×10^{-4}. For the experiments on the number of epochs, the architecture learning rate is set to 3×10^{-4} when performing the search for 50 epochs, and reduced to 10^{-4} when performing the search for more than 50 epochs.

For experiments corresponding to ϵ_0, we set $\lambda=0.125$, while for experiments corresponding to λ, we set $\epsilon_0=0.0001$, and for experiments corresponding to the number of epochs, we set $\epsilon_0=0.0001$ and $\lambda=0.125$.

A.3 Proof of Proposition 1

Proof: Using simple linear algebra, we can re-write the squared norm of the gradient $\nabla_\alpha\mathcal{L}(\omega,\alpha)$ as:

$$\|\nabla_\alpha\mathcal{L}(\omega,\alpha)\|_2^2=\|\boldsymbol{J}_\sigma(\alpha)\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2=\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)^T\boldsymbol{J}_\sigma(\alpha)^T\boldsymbol{J}_\sigma(\alpha)\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha). \tag{19}$$

Now using the assumption that $\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)$ is the sum of vectors orthogonal to the null-space of the Jacobian matrix and the fact that $\boldsymbol{J}_\sigma(\alpha)^T\boldsymbol{J}_\sigma(\alpha)$ is a positive semi-definite matrix, we can give the following lower-bound for this term (Meyer, 2000):

$$\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)^T\boldsymbol{J}_\sigma(\alpha)^T\boldsymbol{J}_\sigma(\alpha)\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\geq\min_{i,\lambda_i(\alpha)\neq0}\lambda_i^2\cdot\|\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2. \tag{20}$$

Since we assume the gradient $\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)$ can be written as the sum of orthogonal layer gradients, its norm can be lower bounded by the smallest gradient (Meyer, 2000):

$$\|\nabla_{\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2=\|\sum_{\ell=1}^{L}\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2 \tag{21}$$

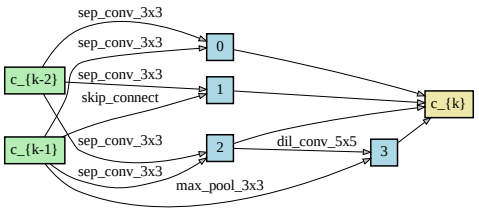
$$=\sum_{\ell=1}^{L}\|\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2 \tag{22}$$

$$\geq L\cdot\min_{\ell}\|\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2, \tag{23}$$

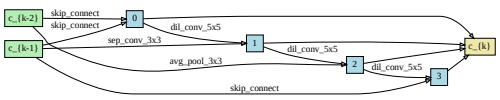
where 22 follows from the orthogonality. So using (20) and (23), we can get:

$$\|\nabla_\alpha\mathcal{L}(\omega,\alpha)\|_2^2\geq\min_{i,\lambda_i(\alpha)\neq0}\lambda_i^2\cdot L\cdot\min_{\ell}\|\nabla_{\ell\boldsymbol{p}}\mathcal{L}(\omega,\alpha)\|_2^2. \tag{24}$$

A.4 Discovered Architectures

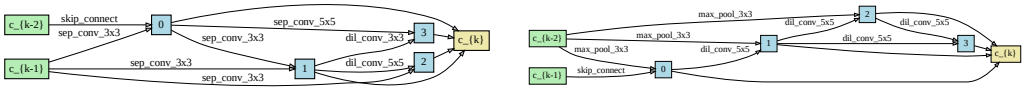


(a) Normal Cell



(b) Reduction Cell

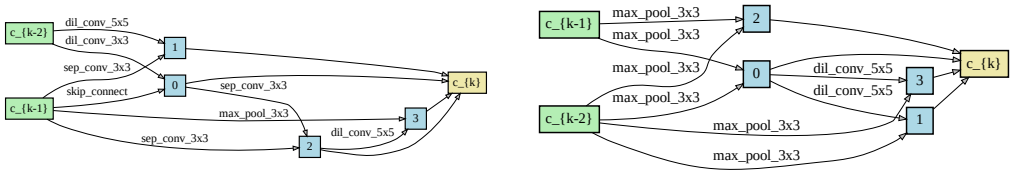
Figure 5: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on DARTS search space and CIFAR-10 dataset.



(a) Normal Cell

(b) Reduction Cell

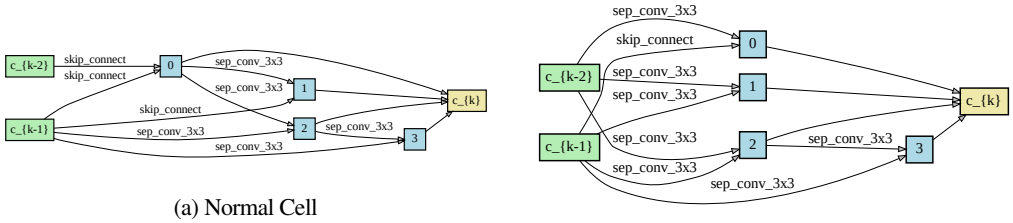
Figure 6: Best normal and reduction cells discovered by $\Lambda_{\pm}(\omega,\alpha)$ on DARTS search space and CIFAR-10 dataset.



(a) Normal Cell

(b) Reduction Cell

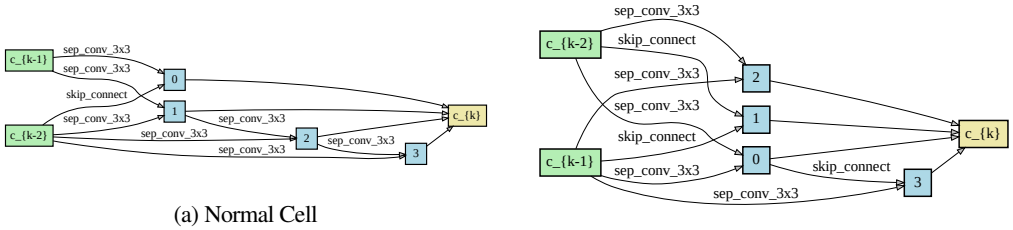
Figure 7: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S1 and CIFAR-10 dataset.



(a) Normal Cell

(b) Reduction Cell

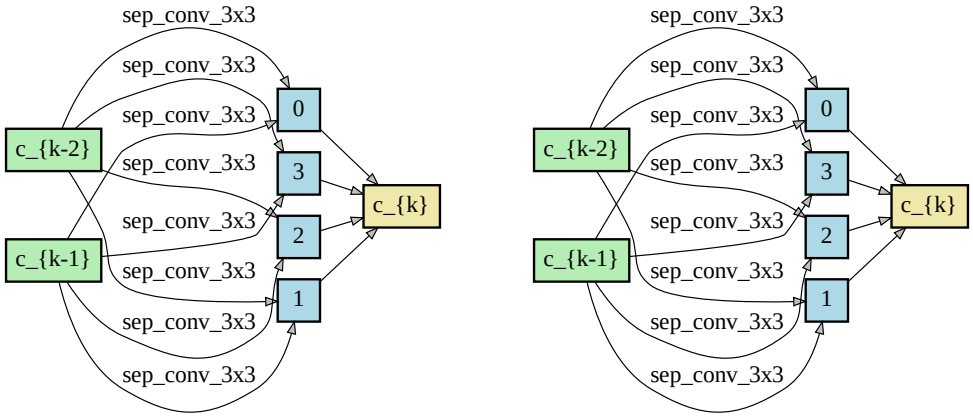
Figure 8: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S2 and CIFAR-10 dataset.



(a) Normal Cell

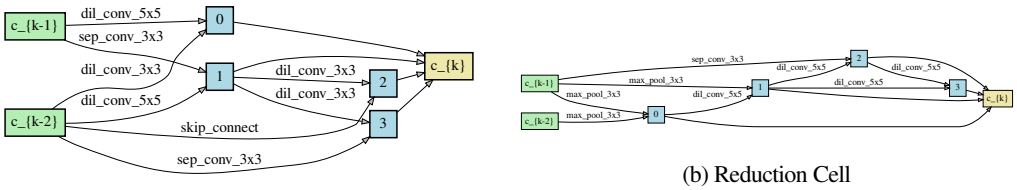
(b) Reduction Cell

Figure 9: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S3 and CIFAR-10 dataset.



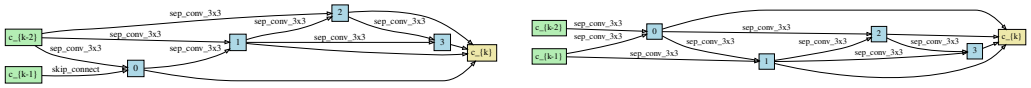
(a) Normal Cell (b) Reduction Cell

Figure 10: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S4 and CIFAR-10 dataset.



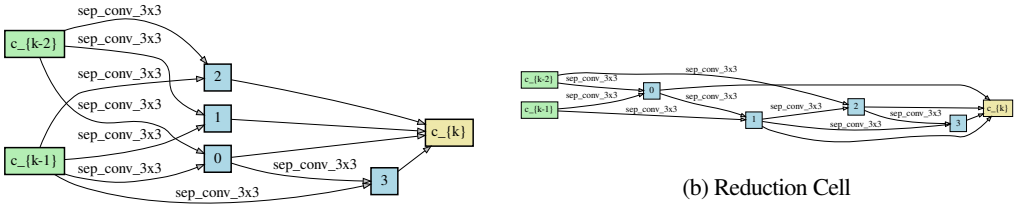
(a) Normal Cell (b) Reduction Cell

Figure 11: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S1 and CIFAR-100 dataset.



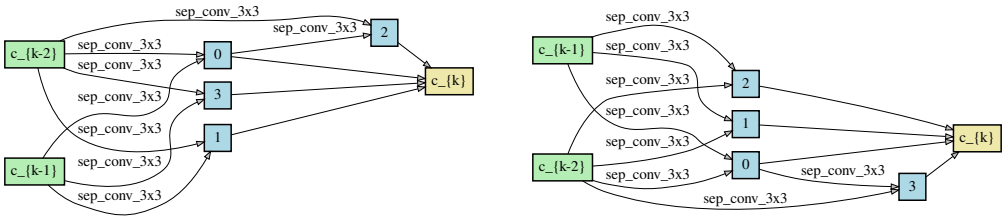
(a) Normal Cell (b) Reduction Cell

Figure 12: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S2 and CIFAR-100 dataset.



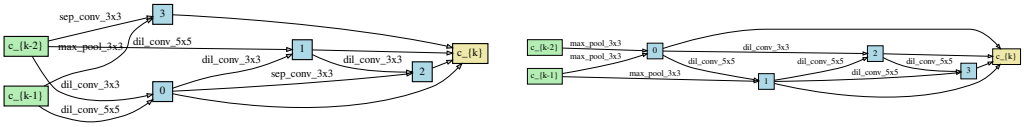
(a) Normal Cell (b) Reduction Cell

Figure 13: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S3 and CIFAR-100 dataset.



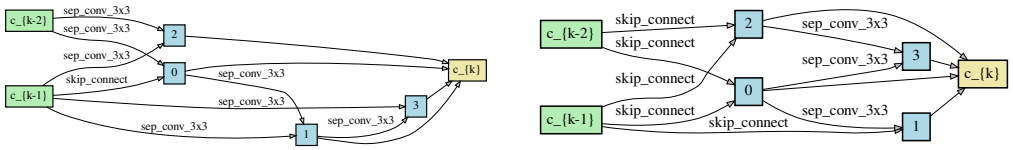
(a) Normal Cell (b) Reduction Cell

Figure 14: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S4 and CIFAR-100 dataset.



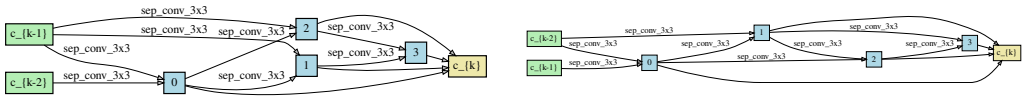
(a) Normal Cell (b) Reduction Cell

Figure 15: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S1 and SVHN dataset.



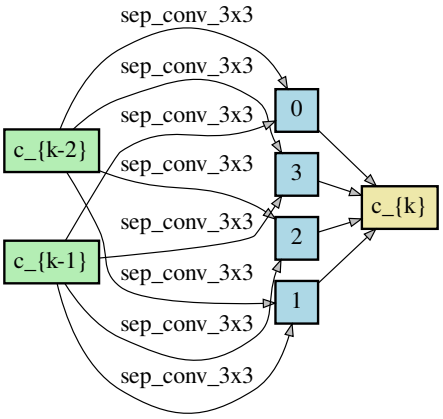
(a) Normal Cell (b) Reduction Cell

Figure 16: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S2 and SVHN dataset.

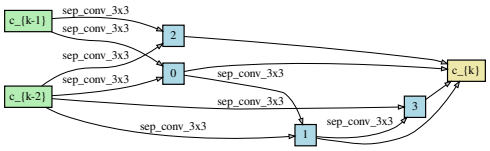


(a) Normal Cell (b) Reduction Cell

Figure 17: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S3 and SVHN dataset.



(a) Normal Cell



(b) Reduction Cell

Figure 18: Best normal and reduction cells discovered by $\Lambda(\omega,\alpha)$ on S4 and SVHN dataset.