STEALTHY SHIELD DEFENSE: A CONDITIONAL MUTUAL INFORMATION-BASED APPROACH AGAINST BLACK-BOX MODEL INVERSION ATTACKS

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ABSTRACT

Model inversion attacks (MIAs) aim to reconstruct the private training data by accessing a public model, raising concerns about privacy leakage. Black-box MIAs, where attackers can only query the model and obtain outputs, are closer to real-world scenarios. The latest black-box attacks have outperformed the state-of-the-art white-box attacks, and existing defenses cannot resist them effectively. To fill this gap, we propose Stealthy Shield Defense (SSD), a post-processing algorithm against black-box MIAs. Our idea is to modify the model's outputs to minimize the conditional mutual information (CMI). We mathematically prove that CMI is a special case of information bottlenecks (IB), and thus inherits the advantages of IB—making predictions less dependent on inputs and more dependent on ground truths. This theoretically guarantees our effectiveness, both in resisting MIAs and preserving utility. For minimizing CMI, we formulate a convex optimization problem and solve it via the water-filling method. Adaptive rate-distortion is introduced to reduce the modification to the outputs, and the water-filling is implemented on GPUs to address computational cost. Without the need to retrain the model, our algorithm is plug-and-play and easy to deploy. Experimental results indicate that SSD outperforms existing defenses, in terms of MIA resistance and model's utility, across various attack algorithms, training datasets, and model architectures. Our code is available at `https://github.com/ZhuangQu/Stealthy-Shield-Defense`.

1 INTRODUCTION

The rapid advancement of deep neural networks (DNNs) has driven their widespread deployment in multiple mission-critical domains, such as computer vision (He et al., 2016), natural language processing (Devlin et al., 2019) and dataset distillation (Zhong et al., 2024b;a). However, their growing integration with sensitive data has raised significant concerns about privacy vulnerabilities. Recent studies (Fang et al., 2024b;a; 2025) have explored various attack methods to probe these privacy vulnerabilities, such as gradient inversion (Fang et al., 2023; Yu et al., 2024b) and membership inference (Hu et al., 2022). Among the emergent privacy threats, model inversion attacks (MIAs) (Qiu et al., 2024b) aim to reconstruct private training data by accessing a machine learning model, posing significant risks of privacy breaches. For instance, consider a face recognition access control system with a publicly accessible interface. Through carefully crafted malicious queries, attackers might employ MIA to infer the sensitive facial images stored in the system database, along with the associated user identities, leading to privacy leakage.

MIAs are divided into *white-box* and *black-box* (Fang et al., 2024c). White-box attackers know the details of the model, whereas black-box attackers can only query the model and obtain outputs. Black-box MIAs become more threatening than white-box because: **(1) Black-box scenarios are more common.** As models grow larger nowadays, they are mostly stored on servers and can only be accessed online, which are typical black-box scenarios. **(2) Black-box attacks are more powerful.** The latest soft-label attack RLBMI (Han et al., 2023) and hard-label attack LOKT (Nguyen et al.,

2024) have outperformed the state-of-the-art white-box attacks. **(3) Existing defenses cannot resist black-box attacks effectively.** Existing defenses focus on modifying the weights and structure of the model, but black-box attackers only exploit the outputs, and are thus less susceptible.

To address these concerns, we propose Stealthy Shield Defense (SSD), a post-processing algorithm against black-box MIAs. The idea of SSD is to modify the model's outputs to minimize the conditional mutual information (CMI), as shown in Figure 1. CMI quantifies the dependence between inputs and predictions when the ground truth labels are given. In Theorem 1, we prove that CMI is a special case of information bottlenecks (IB), and thus inherits the advantages of IB—making predictions less dependent on inputs and more dependent on ground truths. Under this theoretical guarantee, SSD achieves a better trade-off between MIA resistance and model's utility. Without the need to retrain the model, SSD is plug-and-play and easy to deploy.

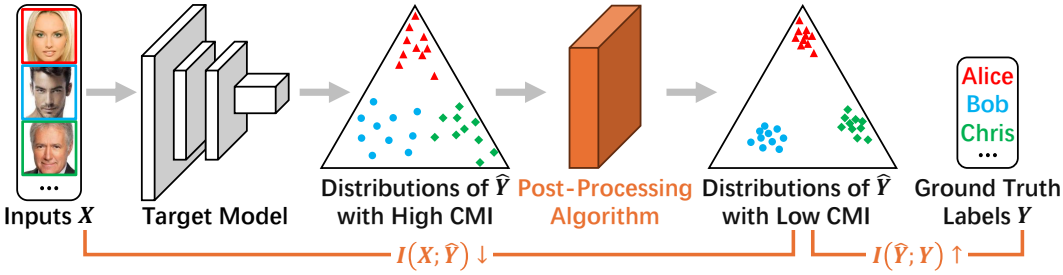

Figure 1: An overview of Stealthy Shield Defense (SSD). The probability simplex is a triangle when the number of classes is three. CMI is defined as $\mathcal{I}(X;\hat{Y}|Y)$. According to our Theorem 1, minimizing CMI makes the mutual information $\mathcal{I}(X;\hat{Y})$ decreased and $\mathcal{I}(\hat{Y};Y)$ increased. As shown by Yang et al. (2024), minimizing CMI makes the outputs more concentrated class-wisely.

The contributions of this paper are:

- We introduce CMI into model inversion defense and prove that CMI is a special case of IB, theoretically guaranteeing the effectiveness of CMI.
- We propose a post-processing algorithm to minimize CMI without retraining models. In our algorithm, temperature is introduced to calibrate the probabilities and adaptive rate-distortion is introduced to constrain the modification to the outputs. We speed up our algorithm by GPU-accelerated water-filling method as well.
- Our experiments demonstrate that SSD outperforms all competitors, in terms of MIA-resistance and model's utility, exhibiting good generalizability across various attack algorithms, training datasets, and model architectures.

2 Related Work

2.1 Model Inversion Attacks and Defenses

Model inversion attacks (MIAs) are a serious privacy threat to released models in the inference time (Fang et al., 2024c). MIAs can be divided into white-box attacks (Zhang et al., 2020; Chen et al., 2021; Yuan et al., 2023; Struppek et al., 2022; Qiu et al., 2024a) and black-box attacks (Kahla et al., 2022; An et al., 2022; Ye et al., 2023; Nguyen et al., 2024; Han et al., 2023). In this paper, we only focus on the black-box attacks, where attackers can only access the output of the victim model. In this scenario, BREP (Kahla et al., 2022) utilizes zero-order optimization to urge the latent vector to move away gradually from the decision boundary. Mirror (An et al., 2022) and C2F (Ye et al., 2023) explore the genetic algorithm. LOKT (Nguyen et al., 2024) train multiple surrogate models and apply white-box attacks to them.

In order to reduce the threat of model inversion attacks, a variety of defense algorithms have been proposed (Wang et al., 2021; Peng et al., 2022; Struppek et al., 2024; Ho et al., 2024; Yu et al.,

2024a). They mainly focus on the white-box scenario. However, the defense research in the black-box scenarios is limited. Moreover, these researches typically defense by modifying either training losses (Wang et al., 2021; Peng et al., 2022; Struppek et al., 2024) or model architectures (Yu et al., 2024a), which have a significant impact on model performance. To overcome the limitations, we propose a plug-and-play post-processing method.

2.2 Information Bottleneck and Conditional Mutual Information

Tishby et al. (1999) proposed the information bottleneck (IB) principle: a good machine learning model should compress the redundant information in inputs while preserving the useful information for tasks. They later highlighted that information is compressed layer-by-layer in DNNs (Tishby & Zaslavsky, 2015; Shwartz-Ziv & Tishby, 2017). Alemi et al. (2017) proposed variational information bottleneck (VIB) to estimate the bounds of IB, and Wang et al. (2021) applied VIB in their model inversion defense.

Yang et al. (2024) proposed to use conditional mutual information (CMI) as a performance metric for DNNs, providing the calculation formula and geometric interpretation of CMI. By minimizing CMI, they improved classifiers (Yang et al., 2025) and addressed class imbalance (Hamidi et al., 2024). By maximizing CMI, they improved knowledge distillation (Ye et al., 2024) and addressed nasty teachers (Yang & Ye, 2024).

In this paper, we theoretically prove that CMI is a special case of IB and thus inherits the advantages of IB. Furthermore, we propose a novel model inversion defense based on CMI.

3 Preliminary

3.1 Notation

Let $f\colon \mathbb{X} \to \mathbb{Y}$ be a neural classifier, $X \in \mathbb{X}$ be the input to f, $Y \in \mathbb{Y}$ be the ground truth label, $\hat{Y} \in \mathbb{Y}$ be the label predicted by f, and $Z \in \mathbb{Z}$ be the intermediate feature in f. Note that $Y \to X \to Z \to \hat{Y}$ is a Markov chain. Let $\mathcal{P}$ be the probability function and, for brevity, let $\mathcal{P}(x) \coloneqq \mathcal{P}\{X = x\}$, $\mathcal{P}(y) \coloneqq \mathcal{P}\{Y = y\}$, $\mathcal{P}(x, \hat{y}|y) \coloneqq \mathcal{P}\{X = x, \hat{Y} = \hat{y} \mid Y = y\}$, etc.

Let $\Delta^{\mathbb{Y}}$ be the probability simplex with $|\mathbb{Y}|$ vertices. Let $\boldsymbol{f}(x) \in \Delta^{\mathbb{Y}}$ be the output from the softmax layer of f when x is input to f, and $f_{\hat{y}}(x)$ be the $\hat{y}$-th component of $\boldsymbol{f}(x)$, $\hat{y} \in \mathbb{Y}$. Note that $f(x) = \arg\max_{\hat{y} \in \mathbb{Y}} f_{\hat{y}}(x)$.

3.2 Model Inversion Attacks

Let $D \subseteq \mathbb{X} \times \mathbb{Y}$ be the dataset learned by f. MIAs aim to reconstruct $\hat{D}$ as close to D as possible. According to the access to f, MIAs are categorized as:

Hard-label: Attackers can query any $x \in \mathbb{X}$ and obtain $f(x) \in \mathbb{Y}$.

Soft-label: Attackers can query any $x \in \mathbb{X}$ and obtain $\boldsymbol{f}(x) \in \Delta^{\mathbb{Y}}$.

White-box: Attackers know the details of f.

Hard-label and soft-label, collectively called *black-box*,[1] are what we aim to defend against.

3.3 Defense via Mutual Information

Wang et al. (2021) proposed to resist MIAs via the mutual information, which is defined as

$$\mathcal{I}(X; \hat{Y}) \coloneqq \sum_{x \in \mathbb{X}} \sum_{\hat{y} \in \mathbb{Y}} \mathcal{P}(x, \hat{y}) \log \frac{\mathcal{P}(x, \hat{y})}{\mathcal{P}(x)\mathcal{P}(\hat{y})}. \tag{1}$$

[1] Some literature refer to *hard-label* as *label-only*, and *soft-label* as *black-box*.

$\mathcal{I}(X;\hat{Y})$ quantifies the dependence between X and $\hat{Y}$. They minimized it to prevent attackers from obtaining the information about D, but they hurt the model's utility. Especially, $\mathcal{I}(X;\hat{Y}) = 0$ iff X and $\hat{Y}$ are independent, in which case f is immune to any attack but useless at all.

As an alternative, they introduced information bottlenecks (IB), which is defined as

$$\mathcal{I}(X;Z) - \lambda \cdot \mathcal{I}(Z;Y), \tag{2}$$

where $\lambda > 0$. They used (2) as a regularizer to train f, minimizing $\mathcal{I}(X;Z)$ to resist MIAs while maximizing $\mathcal{I}(Y;Z)$ to preserve model's utility. They achieved a better trade-off by adjusting λ.

4 METHODOLOGY

4.1 DEFENSE VIA CONDITIONAL MUTUAL INFORMATION

We aim to resist black-box MIAs, so we still focus on $\hat{Y}$ rather than Z. Furthermore, we observe that all MIA algorithms target one fixed label during attacking. Formally, given $y \in \mathbb{Y}$, let

$$D^y \coloneqq \{x \in \mathbb{X} \mid (x,y) \in D\}$$

be the training subset whose ground truth label is y, and attackers aim to reconstruct $\hat{D}^y$ as close to D^y as possible. Based on their behavior, we propose to resist MIAs via

$$\mathcal{I}(X;\hat{Y}|Y=y) \coloneqq \sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}} \mathcal{P}(x,\hat{y}|y) \log \frac{\mathcal{P}(x,\hat{y}|y)}{\mathcal{P}(x|y)\mathcal{P}(\hat{y}|y)}. \tag{3}$$

$\mathcal{I}(X;\hat{Y}|Y=y)$ quantifies the dependence between X and $\hat{Y}$ when $Y=y$. We minimize it to prevent attackers from obtaining the information about D^y.

Minimizing (3) on each $y \in \mathbb{Y}$ is equivalent to minimizing the conditional mutual information (CMI), which is defined as

$$\mathcal{I}(X;\hat{Y}|Y) \coloneqq \sum_{y\in\mathbb{Y}} \mathcal{P}(y) \cdot \mathcal{I}(X;\hat{Y}|Y=y). \tag{4}$$

Theorem 1. *CMI is a special case of information bottlenecks (IB) when $Z = \hat{Y}$ and $\lambda = 1$, i.e.*

$$\mathcal{I}(X;\hat{Y}|Y) = \mathcal{I}(X;\hat{Y}) - \mathcal{I}(\hat{Y};Y).$$

Our proof is provided in Appendix A. Theorem 1 proves that CMI inherits the benefits of IB, i.e. minimizing CMI has effects in two aspects:

- Minimizing $\mathcal{I}(X;\hat{Y})$ to compress the redundant information in inputs, and decrease the dependence between inputs and predictions. This helps to resist MIAs (Wang et al., 2021).
- Maximizing $\mathcal{I}(\hat{Y};Y)$ to preserve the useful information for tasks, and increase the dependence between predictions and ground-truths. This helps to improve model's performance.

The $\mathcal{I}(X;Z)$ in (2) is challenging to calculate because the input space $\mathbb{X}$ and feature space $\mathbb{Z}$ are both high-dimensional. Prior work can only estimate variational bounds for IB (Tishby et al., 1999; Tishby & Zaslavsky, 2015; Alemi et al., 2017; Shwartz-Ziv & Tishby, 2017). Fortunately, as a special case of IB, CMI can be calculated and minimized directly, as described in the next section.

4.2 MINIMIZE CMI BY POST-PROCESSING

Previous work used CMI as a loss function and minimized it during training models (Yang et al., 2024; Hamidi et al., 2024; Yang et al., 2025). Unlike them, we propose a post-processing algorithm to minimize CMI. Without the need to retrain the model, our algorithm is plug-and-play and easy to deploy.

CMI can be calculated as follows:

$$\begin{aligned}\mathcal{I}(X;\hat{Y}|Y) &= \sum_{y\in\mathbb{Y}}\mathcal{P}(y)\sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}}\mathcal{P}(x,\hat{y}|y)\log\frac{\mathcal{P}(x,\hat{y}|y)}{\mathcal{P}(x|y)\mathcal{P}(\hat{y}|y)}, \qquad \text{by definitions (3-4)},\\ &= \sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}}\sum_{y\in\mathbb{Y}}\mathcal{P}(x,\hat{y},y)\log\frac{\mathcal{P}(\hat{y}|x,y)}{\mathcal{P}(\hat{y}|y)},\\ &= \sum_{x\in\mathbb{X}}\mathcal{P}(x)\sum_{y\in\mathbb{Y}}\mathcal{P}(y|x)\sum_{\hat{y}\in\mathbb{Y}}\mathcal{P}(\hat{y}|x,y)\log\frac{\mathcal{P}(\hat{y}|x,y)}{\mathcal{P}(\hat{y}|y)},\\ &= \sum_{x\in\mathbb{X}}\mathcal{P}(x)\sum_{y\in\mathbb{Y}}\mathcal{P}(y|x)\sum_{\hat{y}\in\mathbb{Y}}\mathcal{P}(\hat{y}|x)\log\frac{\mathcal{P}(\hat{y}|x)}{\mathcal{P}(\hat{y}|y)}, \qquad \text{by Markov } Y\to X\to\hat{Y}.\end{aligned}$$

Based on the above mathematical transformation, minimizing $\mathcal{I}(X;\hat{Y}|Y)$ is equivalent to minimizing $\sum_{y\in\mathbb{Y}}\mathcal{P}(y|x)\sum_{\hat{y}\in\mathbb{Y}}\mathcal{P}(\hat{y}|x)\log\frac{\mathcal{P}(\hat{y}|x)}{\mathcal{P}(\hat{y}|y)}$ for each x input to f. However, this objective function is too complex to optimize. For simplicity, we sample $y\in\mathbb{Y}$ with the probability of $\mathcal{P}(y|x)$ and minimize $\sum_{\hat{y}\in\mathbb{Y}}\mathcal{P}(\hat{y}|x)\log\frac{\mathcal{P}(\hat{y}|x)}{\mathcal{P}(\hat{y}|y)}$ instead, which is equivalent to the original objective in terms of mathematical expectation. Next, we find a way to calculate $\mathcal{P}(\hat{y}|x)$ and $\mathcal{P}(\hat{y}|y)$.

We consider $\mathcal{P}(\hat{y}|x)=f_{\hat{y}}(x)$ according to the design of neural classifiers. Note that

$$\begin{aligned}\mathcal{P}(\hat{y}|y) &= \sum_{x\in\mathbb{X}}\mathcal{P}(x,\hat{y}|y)=\sum_{x\in\mathbb{X}}\mathcal{P}(x|y)\mathcal{P}(\hat{y}|x,y)=\sum_{x\in\mathbb{X}}\mathcal{P}(x|y)\mathcal{P}(\hat{y}|x)=\sum_{x\in\mathbb{X}}\mathcal{P}(x|y)f_{\hat{y}}(x),\\ &= \mathbb{E}_{X|Y=y}[f_{\hat{y}}(X)], \qquad \hat{y},y\in\mathbb{Y}.\end{aligned}$$

By expressing $\mathcal{P}(\hat{y}|y)$ as a conditional mathematical expectation, we can estimate it by samples. Note that the samples in D^y are i.i.d. with $X|Y=y$, so we let

$$\mathcal{P}(\hat{y}|y)\approx\operatorname*{mean}_{x'\in D^y}f_{\hat{y}}(x'), \qquad \hat{y},y\in\mathbb{Y}.$$

Let $\boldsymbol{q}^y\coloneqq\operatorname*{\mathbf{mean}}_{x'\in D^y}\boldsymbol{f}(x')$ and $q^y_{\hat{y}}$ be the $\hat{y}$-th component of $\boldsymbol{q}^y$, and then we have

$$\sum_{\hat{y}\in\mathbb{Y}}\mathcal{P}(\hat{y}|x)\log\frac{\mathcal{P}(\hat{y}|x)}{\mathcal{P}(\hat{y}|y)}\approx\sum_{\hat{y}\in\mathbb{Y}}f_{\hat{y}}(x)\log\frac{f_{\hat{y}}(x)}{q^y_{\hat{y}}}=\mathrm{KL}(\boldsymbol{f}(x)||\boldsymbol{q}^y), \tag{5}$$

where KL is the Kullback-Leibler divergence, a binary convex function.

To determine the sampling probability $\mathcal{P}(y|x)$, a simple idea is to consider

$$\mathcal{P}(y|x)=\mathcal{P}(\hat{y}|x)=f_{\hat{y}}(x)\text{ for }y=\hat{y}\in\mathbb{Y}. \tag{6}$$

But Guo et al. (2017) have demonstrated that (6) is inaccurate for modern neural networks. Inspired by their work, we introduce temperature mechanism to calibrate it.

When x is inputted to f, we minimize CMI by modifying the prediction $\boldsymbol{f}(x)$. Let $\boldsymbol{p}\in\Delta^{\mathbb{Y}}$ be the modified prediction, and our objective function is $\mathrm{KL}(\boldsymbol{p}||\boldsymbol{q}^y)$ based on the above derivation. To preserve the model's utility, we constrain $\|\boldsymbol{p}-\boldsymbol{f}(x)\|_1\le\varepsilon$ where $\varepsilon>0$ is the distortion bound.

In rate-distortion theory (Shannon, 1959), minimizing mutual information under bounded distortion constraint is for signal compression. If a signal has less information, it is easier to compress, and a stricter distortion bound can be applied. Regarding $\boldsymbol{f}(x)$ as a signal and $\boldsymbol{p}$ as the compressed signal, we can use normalized entropy to quantify the information in $\boldsymbol{f}(x)$, which is defined as

$$\bar{\mathcal{H}}(x)\coloneqq\frac{-1}{\log|\mathbb{Y}|}\sum_{\hat{y}\in\mathbb{Y}}f_{\hat{y}}(x)\log f_{\hat{y}}(x). \tag{7}$$

Smaller $\bar{\mathcal{H}}(x)$ means less information in $\boldsymbol{f}(x)$, and smaller modification it need. Thus we propose the adaptive constraint $\|\boldsymbol{p}-\boldsymbol{f}(x)\|_1\le\varepsilon\cdot\bar{\mathcal{H}}(x)$ to further control the distortion. Note that the old

Algorithm 1: Our post-processing to minimize CMI.

Input: original output $\boldsymbol{f}(x)$, temperature T, distortion bound ε, validation set D.
Output: modified output $\boldsymbol{p}$.
$y \leftarrow$ Sample in $\mathbb{Y}$ with the probability of $\mathbf{softmax}(\frac{\boldsymbol{f}(x)}{T})$;
$\boldsymbol{q}^y \leftarrow \underset{x' \in D^y}{\mathbf{mean}}\, \boldsymbol{f}(x')$;
$\bar{\mathcal{H}}(x) \leftarrow \frac{-1}{\log|\mathbb{Y}|} \sum_{\hat{y} \in \mathbb{Y}} f_{\hat{y}}(x) \log f_{\hat{y}}(x)$;
Solve the convex optimization problem and return the optimal $\boldsymbol{p}$:

$$\begin{aligned} &\min \mathrm{KL}(\boldsymbol{p} || \boldsymbol{q}^y), \\ &\text{s.t. } \|\boldsymbol{p} - \boldsymbol{f}(x)\|_1 \leq \varepsilon \cdot \bar{\mathcal{H}}(x), \\ &\quad \boldsymbol{p} \in \Delta^{\mathbb{Y}}. \end{aligned} \tag{8}$$

constraint $\|\boldsymbol{p} - \boldsymbol{f}(x)\|_1 \leq \varepsilon$ still holds due to the property of $0 \leq \bar{\mathcal{H}}(x) \leq 1$. This method is called adaptive rate distortion.

Our defense is summarized as Algorithm 1:

Note that the $\tilde{\boldsymbol{q}}^y$ can be calculated and stored in advance, making our algorithm not require f and D. (8) is a typical convex optimization problem that can be solved by any convex optimizer. Furthermore, we provide an efficient algorithm in Appendix C and evaluate its time cost in Appendix D.

5 EXPERIMENT

5.1 EXPERIMENT SETTINGS

Datasets. Following previous researches of model inversion attacks, we use the FaceScrub (Ng & Winkler, 2014) and CelebFaces Attributes (CelebA) (Liu et al., 2015) as private datasets. FaceScrub consists of 530 identities. CelebA contains 10177 identities and we only take 1000 identities with the most images (Kahla et al., 2022). To adapt to the classifiers, all images in the various datasets are cropped and resized to 64×64 pixels in our experiment. We use FFHQ as public dataset to train GANs. The GANs of each attack are consistent with their original implementations.

Models. We train a variety of classifiers using the private datasets mentioned above, measuring robust model performance under various conditions. For target models, we employ VGG-16 (Simonyan & Zisserman, 2014) and IR-152 (He et al., 2016), both of which are trained with different defense methods. We select FaceNet (Cheng et al., 2017) as the evaluation model.

Model inversion attacks. In our experiments, we foucus on four state-of-the-art (SOTA) black-box model inversion attacks, including BREP (Kahla et al., 2022), Mirror (An et al., 2022), C2FMI (Ye et al., 2023) and LOKT (Nguyen et al., 2024). We attack the first 100 classes in the private dataset, reconstructing 5 images for each class.

Metrics. Following previous works (Struppek et al., 2024), we utilize multiple metrics to comprehensively evaluate the performance of our defense methods. To measure the model robustness and utility of each method, we consider the following metrics:

- **Attack Accuracy.** The metric is used to imitate a human to determine whether reconstructed images correspond to the target identity or not. Specifically, we employ an evaluation model trained on the same dataset as the target model to re-classify the reconstructed images. We then compute the top-1 and top-5 classification accuracies, denoted as $Acc@1$ and $Acc@5$, respectively.
- **Feature Distance.** The feature is extracted from the second-to-last layer of the model. This distance metric measures the average l_2 distance between the features of reconstructed

images and the nearest private images. Consistent with previous research, we use both the evaluation model and a pre-trained FaceNet model Schroff et al. (2015) to generate the features. The corresponding feature distances are denoted as σ_{eval} and σ_{face}. A lower feature distance indicates a closer semantic similarity between the reconstructed images and private samples.

- **Test Accuracy.** The top-1 classification accuracy on the private test set. This metric is used to evaluate the utility of the target model with defense.
- **Predictive Bias.** This metric is used to quantify the modification to the predicted probability vectors by defense methods. For an identical input, we take the L_1 norm of the difference between the outputs with and without defense. Avg L_1 is the average norm over all private test samples, and Max L_1 is the largest one. Lower values of both suggest that the defense method causes less harm to the predicted probability vectors.

5.2 Comparison with Previous State-of-the-art Defenses

In this section, we evaluate the robustness of our defense method by comparing it against an undefended model and prior state-of-the-art (SOTA) defense strategies, including MID (Wang et al., 2021), BiDO (Peng et al., 2022), LS (Struppek et al., 2024) and TL (Ho et al., 2024). We adhere to the official configurations for each defense method, and the corresponding hyperparameters are detailed in Appendix B.

We evaluate the model's robustness under various types of black-box model inversion attacks, including both soft-label and hard-label attacks. We conduct experiments on different target models and private datasets to demonstrate that our approach performs effectively across diverse scenarios.

For soft-label attacks, we compare our method with previous defense strategies under the Mirror and C2FMI attacks. The attack results are listed in Table 1. We can observe that our method achieves significant improvements over existing defense strategies, especially when the attack has a strong performace. Specifically, under the Mirror attack against IR-152 trained on the FaceScrub dataset, our method reduces the attack accuracy from 52.3% to 19.4%, achieving a 3.6% greater reduction compared to the previous SOTA method TL. For C2FMI attacks against VGG16 models trained on the FaceScrub dataset, our method reduces the attack accuracy to approximately $1/9$ of that without defense, which is only a quarter of the accuracy achieved under the TL defense.

Table 1: Experiment results against soft-label attacks.

Model Dataset	Defense	**Mirror**				**C2FMI**			
		$\downarrow Acc@1$	$\downarrow Acc@5$	$\uparrow \delta_{eval}$	$\uparrow \delta_{face}$	$\downarrow Acc@1$	$\downarrow Acc@5$	$\uparrow \delta_{eval}$	$\uparrow \delta_{face}$
IR-152 CelebA	**None**	10.0%	18.8%	2526	1.31	3.6%	8.0%	2521	1.36
	MID	9.0%	17.6%	2448	1.23	0.2%	0.4%	2382	1.56
	BiDO	4.8%	11.4%	**2758**	1.17	0.8%	3.8%	**2598**	1.31
	LS	3.2%	7.8%	2602	1.33	1.4%	4.2%	2536	1.39
	TL	6.6%	14.4%	2613	1.27	2.6%	7.0%	2528	1.37
	SSD	**1.2%**	**3.0%**	2527	**1.56**	**0%**	**0.4%**	2377	**1.67**
IR-152 FaceScrub	**None**	52.4%	74.6%	1893	0.79	27.0%	49.8%	1952	0.98
	MID	43.6%	63.4%	2067	0.86	3.0%	9.6%	**2754**	1.44
	BiDO	27.6%	53.0%	2132	0.99	14.2%	24.4%	2242	1.20
	LS	33.4%	56.6%	2153	0.88	21.8%	46.8%	2022	1.02
	TL	23.0%	47.2%	2155	0.95	6.8%	16.8%	2191	1.23
	SSD	**19.4%**	**28.2%**	**2415**	**1.31**	**2.0%**	**6.4%**	2517	**1.49**
VGG-16 FaceScrub	**None**	8.0%	15.0%	2577	0.78	23.8%	37.0%	2315	0.93
	MID	6.4%	12.2%	2627	0.79	18.4%	31.8%	2239	0.93
	BiDO	11.4%	21.0%	2530	0.79	10.6%	19.2%	2552	0.94
	LS	10.2%	18.4%	2526	0.75	17.0%	29.2%	2424	0.95
	TL	6.8%	12.0%	2624	**0.88**	10.4%	17.6%	2602	1.03
	SSD	**5.6%**	**10.6%**	**2665**	0.80	**8.8%**	**15.2%**	**2681**	**1.07**

In hard-label scenarios with BREP and LOKT attacks. We provided a quantitive results in Table 2. Note that LOKT is the SOTA black-box attack method. It demonstrates very high attack performance

across various kinds of settings. While previous defenses only showed limited defensive capabilities, our approach almost completely defeats this attack. Especially in the attack against IR-152 with FaceScrub dataset, without any defense, LOKT showed an attack accuracy of up to 82.9%. However, our defense method reduce it to only 1.7%, making it almost impossible to launch a successful attack. Moreover, our defense largely enhance the feature distance σ_{face} from 0.66 to 1.53, which indicate that our defense method make the attack failed to capture the privacy characteristics.

Table 2: Experiment results against hard-label attacks.

Model Dataset	Defense	BREP				LOKT			
		$\downarrow Acc@1$	$\downarrow Acc@5$	$\uparrow \delta_{eval}$	$\uparrow \delta_{face}$	$\downarrow Acc@1$	$\downarrow Acc@5$	$\uparrow \delta_{eval}$	$\uparrow \delta_{face}$
IR-152 CelebA	**None**	7.2%	24.4%	1654	0.95	51.6%	74.4%	1469	0.85
	MID	12.6%	28.8%	1973	1.28	29.8%	51.0%	1713	1.04
	BiDO	13.0%	30.6%	1670	1.03	48.4%	66.8%	1551	0.95
	LS	15.6%	40.0%	1584	0.97	52.0%	73.6%	1489	0.88
	TL	10.2%	27.2%	1643	1.05	56.4%	74.6%	1510	0.92
	SSD	**0.4%**	**1.6%**	**2362**	**1.61**	**0.2%**	**1.0%**	**2321**	**1.54**
IR-152 FaceScrub	**None**	32.8%	56.6%	2161	1.00	83.0%	93.2%	1488	0.66
	MID	34.0%	51.0%	2178	1.06	54.0%	74.4%	1856	0.82
	BiDO	24.2%	39.4%	2235	1.07	59.8%	77.6%	1694	0.77
	LS	22.8%	45.8%	2384	1.07	60.0%	77.6%	1748	0.74
	TL	14.2%	27.2%	2353	1.15	62.6%	78.2%	1682	0.73
	SSD	**3.4%**	**7.0%**	**2622**	**1.51**	**1.8%**	**4.4%**	**2694**	**1.53**
VGG-16 FaceScrub	**None**	33.6%	56.6%	2327	0.94	93.8%	98.0%	1359	0.57
	MID	37.4%	58.2%	2249	0.90	82.4%	92.8%	1526	0.60
	BiDO	30.4%	51.8%	2349	0.96	78.8%	87.4%	1567	0.63
	LS	29.6%	49.0%	2402	0.94	78.2%	88.6%	1573	0.65
	TL	29.0%	47.8%	2381	0.98	58.2%	74.0%	1771	0.71
	SSD	**9.8%**	**15.0%**	**2586**	**1.45**	**12.6%**	**21.4%**	**2370**	**1.18**

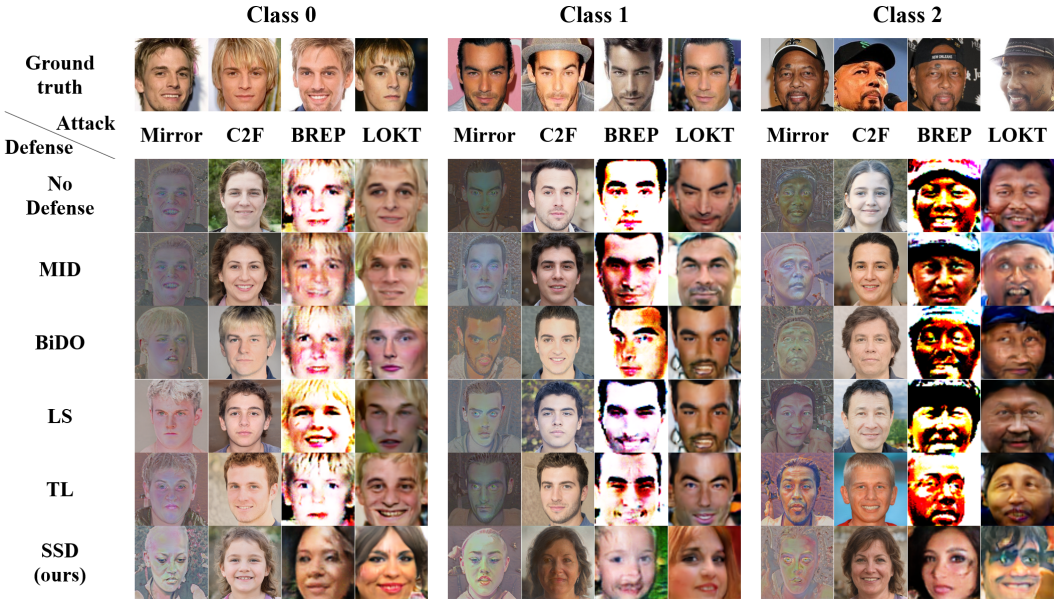

Figure 2: Visual comparison of reconstructed images using various black-box attack methods against an IR-152 model trained on CelebA, evaluated under different defense strategies. The top row displays the ground truth images of the target class from the private train dataset for reference.

Visualization results of the reconstructed images with different defense methods under different black-box attacks are shown in Fig. 2. Compared to previous approaches, our defense strategy

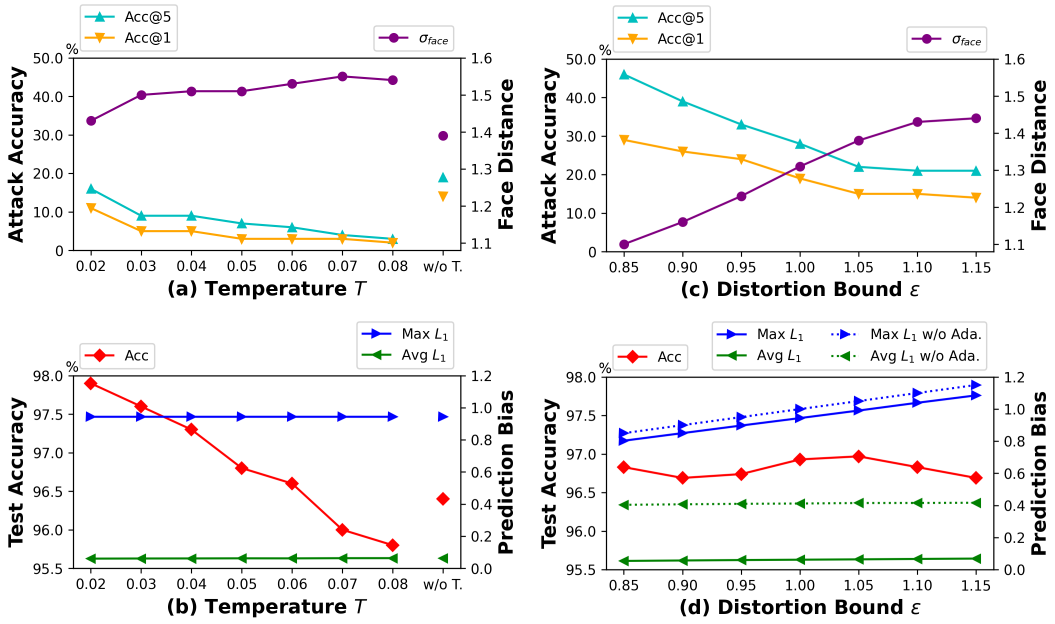

Figure 3: Ablation Study on temperature T and distortion bound ε.

produces reconstructed images that deviate more significantly from the private images, demonstrating its effectiveness in increasing the challenge for attackers to extract sensitive visual features and thereby enhancing privacy protection.

Table 3: Evaluation results on model's utility.

Defense	IR-152 & CelebA			IR-152 & FaceScrub			VGG-16 & FaceScrub		
	↑Acc	↓Avg L_1	↓Max L_1	↑Acc	↓Avg L_1	↓Max L_1	↑Acc	↓Avg L_1	↓Max L_1
None	94.2%	0	0	98.6%	0	0	97.9%	0	0
MID	88.9%	0.44	1.93	96.5%	0.32	1.96	95.1%	0.36	1.78
BiDO	88.2%	0.37	1.96	94.0%	0.58	1.95	94.3%	0.27	1.90
LS	90.1%	0.37	1.99	94.9%	0.18	1.96	94.9%	0.19	1.88
TL	89.1%	0.35	1.84	95.3%	0.33	1.97	94.5%	0.15	1.96
SSD	**90.3%**	**0.15**	**0.95**	**96.7%**	**0.06**	**0.94**	**96.3%**	**0.05**	**0.74**

The evaluation results for the target model's utility are presented in Table 3. The results indicate that our defense holds the best utility, outperforming all competitors across different metrics, training datasets and model structures. Thanks to our bounded distortion constraint, our Max $L_1 \leq \varepsilon$ always holds strictly, where the competitors' are close to the maximum of 2. In particular, thanks to our adaptive rate-distortion, our Avg L_1 is only 1/5 to 1/2 of the competitors'.

5.3 Ablation Studies

In this section, we conduct ablation experiments to explore the effects of the temperature and distortion bound in our defense. The target model is IR-152 trained on FaceScrub.

Figure 3 shows the experimental results on temperature, where the attack accuracy is measured on BREP. It can be seen that as the temperature T rises, our MIA robustness becomes stronger. This is because the y in Algorithm 1 is closer to uniform distribution, which makes it easier to return misleading labels to hard-label attackers. However, high temperature impairs the model's utility. In particular, the "w/o T." in Figure (a)(b) represents the case without temperature mechanism. In

that case, neither MIA robustness nor model's utility is ideal, which demonstrates the necessity of introducing a temperature mechanism.

For the distortion bound, the experiment results are displayed in Figure (c)(d). The attack accuracy is measured on Mirror. As the distortion bound goes up, our defense can make more alterations to the output, resulting in better MIA robustness. It can be seen that relaxing the distortion bound mainly affects the maximum distortion Max L_1, while having almost no effect on the average distortion Avg L_1. Especially, without the adaptive mechanism, our Avg L_1 would become as high as that of other defenses. This demonstrates the necessity of introducing the adaptive mechanism.

6 Conclusion

In contrast to previous researches on model inversion defense with focus on white-box attacks, we conduct a specific study on black-box attacks. Specifically, we investigate the impact of conditional mutual information (CMI) and develop a CMI-based defense strategy. We conduct our defense in the post-processing stage, instead of re-training the model. Our method modify the model output by reducing the dependence between model inputs and outputs. To further reduce the modifications to outputs, we introduce an adaptive rate-distortion framework and optimize it by water-filling method. Experimental results demonstrate that our defense method achieves state-of-the-art (SOTA) performance against black-box attacks. We hope that our findings will help shift attention towards robust defense mechanisms in black-box settings and inspire further research in this area.

7 Acknowledgement

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A PROOF OF THEOREM 1

$$
\begin{aligned}
&\mathcal{I}(X;\hat{Y}|Y)\\
=&\sum_{y\in\mathbb{Y}}\mathcal{P}(y)\sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}}\mathcal{P}(x,\hat{y}|y)\log\frac{\mathcal{P}(x,\hat{y}|y)}{\mathcal{P}(x|y)\mathcal{P}(\hat{y}|y)}, && \text{by definitions (3-4),}\\
=&\sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}}\sum_{y\in\mathbb{Y}}\mathcal{P}(x,\hat{y},y)\log\frac{\mathcal{P}(\hat{y}|x,y)}{\mathcal{P}(\hat{y}|y)},\\
=&\sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}}\sum_{y\in\mathbb{Y}}\mathcal{P}(x,\hat{y},y)\log\frac{\mathcal{P}(\hat{y}|x)}{\mathcal{P}(\hat{y}|y)}, && \text{by Markov chain } Y\to X\to\hat{Y},\\
=&\sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}}\sum_{y\in\mathbb{Y}}\mathcal{P}(x,\hat{y},y)\log\left(\frac{\mathcal{P}(x,\hat{y})}{\mathcal{P}(x)}\Big/\frac{\mathcal{P}(\hat{y},y)}{\mathcal{P}(y)}\right),\\
=&\sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}}\sum_{y\in\mathbb{Y}}\mathcal{P}(x,\hat{y},y)\log\left(\frac{\mathcal{P}(x,\hat{y})}{\mathcal{P}(x)\mathcal{P}(\hat{y})}\Big/\frac{\mathcal{P}(\hat{y},y)}{\mathcal{P}(\hat{y})\mathcal{P}(y)}\right),\\
=&\sum_{x\in\mathbb{X}}\sum_{\hat{y}\in\mathbb{Y}}\mathcal{P}(x,\hat{y})\log\frac{\mathcal{P}(x,\hat{y})}{\mathcal{P}(x)\mathcal{P}(\hat{y})}-\sum_{\hat{y}\in\mathbb{Y}}\sum_{y\in\mathbb{Y}}\mathcal{P}(\hat{y},y)\log\frac{\mathcal{P}(\hat{y},y)}{\mathcal{P}(\hat{y})\mathcal{P}(y)},\\
=&\mathcal{I}(X;\hat{Y})-\mathcal{I}(\hat{Y};Y), && \text{by definition (1).}
\end{aligned}
$$

B SETTINGS FOR DEFENSES

Table 4: The parameter settings for all defenses.

Defense	IR-152 & CelebA	IR-152 & FaceScrub	VGG-16 & FaceScrub
MID	$\beta=0.005$	$\beta=0.01$	$\beta=0.02$
BiDO	$\lambda_x=0.001$, $\lambda_y=0.01$	$\lambda_x=0.002$, $\lambda_y=0.02$	$\lambda_x=0.002$, $\lambda_y=0.02$
LS	$\alpha=-0.05$	$\alpha=-0.1$	$\alpha=-0.1$
TL	Freeze the first 50% of the layers.		
SSD	$T=0.03$, $\varepsilon=1$	$T=0.05$, $\varepsilon=1$	$T=0.3$, $\varepsilon=1$

C OUR WATER-FILLING ALGORITHM TO OPTIMIZE (8)

For brevity, let $\boldsymbol{q}\coloneqq\tilde{\boldsymbol{q}}^y$, $\boldsymbol{f}\coloneqq\boldsymbol{f}(x)$, and $\varepsilon\coloneqq\varepsilon\cdot\bar{\mathcal{H}}(x)$. The problem (8) is restated as

$$
\begin{aligned}
&\min \mathrm{KL}(\boldsymbol{p}||\boldsymbol{q}),\\
&\text{s.t. } \|\boldsymbol{p}-\boldsymbol{f}\|_1\le\varepsilon,\\
&\boldsymbol{p}\in\Delta^{\mathbb{Y}}.
\end{aligned}
\tag{9}
$$

Note that Kullback-Leibler divergence is a metric. $\mathrm{KL}(\boldsymbol{p}||\boldsymbol{q})\ge 0$ always holds and $\mathrm{KL}(\boldsymbol{p}||\boldsymbol{q})=0$ iff $\boldsymbol{p}=\boldsymbol{q}$. Trivially, when $\|\boldsymbol{q}-\boldsymbol{f}\|_1\le\varepsilon$, the optimal solution is $\boldsymbol{p}=\boldsymbol{q}$.

When $\|\boldsymbol{q}-\boldsymbol{f}\|_1>\varepsilon$, the optimal $\boldsymbol{p}$ must be between $\boldsymbol{f}$ and $\boldsymbol{q}$ due to the properties of KL, i.e.

$$
\text{Either } f_i\le p_i\le q_i \text{ or } f_i\ge p_i\ge q_i, \text{ for each } i\in\mathbb{Y}. \tag{10}
$$

Furthermore, due to $\boldsymbol{f},\boldsymbol{p}\in\Delta^{\mathbb{Y}}$, there must be

$$
\sum_{i\in\mathbb{Y}:f_i<q_i}p_i-f_i=\sum_{i\in\mathbb{Y}:f_i>q_i}f_i-p_i=\frac{\varepsilon}{2}. \tag{11}
$$

In the following we consider the case $f_i < q_i$ (another is symmetric). Assuming that $f_i < q_i$ iff $i \in \{1, 2, \ldots, n\}$, a semi-problem of (9) is

$$\begin{aligned} \min \ & \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i}, \\ \text{s.t.} \ & \sum_{i=1}^{n} p_i - f_i = \frac{\varepsilon}{2}, \\ & p_i \geq f_i, \qquad i = 1, 2, \ldots, n. \end{aligned} \tag{12}$$

Introducing Lagrange multipliers $\boldsymbol{\lambda} \in \mathbb{R}^n_{\geq 0}$ and $v \in \mathbb{R}$, the KKT conditions are

$$(p_i - f_i)\lambda_i = 0, \tag{13}$$

$$1 + \log \frac{p_i}{q_i} - v - \lambda_i = 0, \tag{14}$$

where $i = 1, 2, \ldots, n$. Eliminating $\lambda_i \geq 0$ yields

$$(p_i - f_i)\left(1 + \log \frac{p_i}{q_i} - v\right) = 0, \tag{15}$$

$$1 + \log \frac{p_i}{q_i} \geq v. \tag{16}$$

When $v > 1 + \log \frac{f_i}{q_i}$, (16) implies $p_i > f_i$, and (15) implies $p_i = q_i \exp(v - 1)$.

When $v \leq 1 + \log \frac{f_i}{q_i}$, $p_i > f_i$ implies $\left(1 + \log \frac{p_i}{q_i} - v\right) > 0$ that against (15), so $p_i = f_i$.

In summary, the optimal solution is

$$p_i = \begin{cases} q_i \exp(v - 1) & v > 1 + \log \dfrac{f_i}{q_i} \\ f_i & \text{other} \end{cases}, \qquad i = 1, 2, \ldots, n, \tag{17}$$

where v is determined by the constraint $\sum_{i=1}^{n} p_i - f_i = \frac{\varepsilon}{2}$.

Let $w \coloneqq \exp(v - 1) \in \mathbb{R}_{>0}$ and (17) is simplified to

$$p_i = \max(f_i, wq_i), \qquad i = 1, 2, \ldots, n. \tag{18}$$

We propose Algorithm 2 to calculate (18) efficiently. Our algorithm is known as "water-filling", because w is like a rising water level and $\frac{\varepsilon}{2}$ is like the maximum volume of water. Its time complexity is $O(n \log n)$ due to the sorting at the beginning.

Algorithm 2: Water-filling on CPU.

Input: f_i, q_i for $i = 1, 2, \ldots, n$.
Output: p_i for $i = 1, 2, \ldots, n$.
Reindex f_i, q_i so that $\frac{f_1}{q_1} \leq \frac{f_2}{q_2} \leq \ldots \leq \frac{f_n}{q_n}$;
$i \leftarrow 1$;
$f_{\text{sum}} \leftarrow 0$;
$q_{\text{sum}} \leftarrow 0$;
while $q_{sum}\frac{f_i}{q_i} - f_{sum} < \frac{\varepsilon}{2}$ **do**
  $i \leftarrow i + 1$;
  $f_{\text{sum}} \leftarrow f_{\text{sum}} + f_i$;
  $q_{\text{sum}} \leftarrow q_{\text{sum}} + q_i$;
end
$w \leftarrow \dfrac{f_{\text{sum}} + \frac{\varepsilon}{2}}{q_{\text{sum}}}$;
Reindex f_i, q_i back to the original;
return $\max(f_i, wq_i)$ for $i = 1, 2, \ldots, n$;

Algorithm 3: Water-filling on GPU.

Input: PyTorch tensors $\boldsymbol{f}, \boldsymbol{q}$ of size n.
Output: PyTorch tensor $\boldsymbol{p}$ of size n.
Reindex $\boldsymbol{f}, \boldsymbol{q}$ by torch.sort($\frac{\boldsymbol{f}}{\boldsymbol{q}}$);

$\boldsymbol{f}_{\text{sum}} \leftarrow \boldsymbol{f}$.cumsum();
$\boldsymbol{q}_{\text{sum}} \leftarrow \boldsymbol{q}$.cumsum();
mask $\leftarrow \boldsymbol{q}_{\text{sum}}\frac{\boldsymbol{f}}{\boldsymbol{q}} - \boldsymbol{f}_{\text{sum}} < \frac{\varepsilon}{2}$;
$i \leftarrow$ **mask**.argmax();

$w \leftarrow \dfrac{\boldsymbol{f}_{\text{sum}}[i] + \frac{\varepsilon}{2}}{\boldsymbol{q}_{\text{sum}}[i]}$;
Reindex $\boldsymbol{f}, \boldsymbol{q}$ back to the original;
return torch.max($\boldsymbol{f}, w\boldsymbol{q}$);

To further speed up, we also propose Algorithm 3, a GPU-based water-filling. Specifically, we manage to eliminate the loop and branch in Algorithm 2, making it completely sequential and suitable for GPUs. By utilizing the operators of PyTorch tensors, we fully leverage the parallelism capabilities of GPUs.

D EXPERIMENTS ON COMPUTATIONAL COST

We quantitatively demonstrate the efficiency of our post-processing Algorithm 1 by experiments. The target models, training sets, and defense settings are consistent with Table 4. We take a batch with 512 test samples and let the model infer 100 times on it. We record the time cost by torch.profiler, an official tool provided by PyTorch. We exclude the time for I/O (i.e. the time from disk to memory, and from CPU to GPU), and only include the time for forward propagation on GPU. Our experiment is conducted on one NVIDIA GeForce RTX 3090. The results are in Table 5.

Table 5: The time cost of our post-processing algorithm.

	IR-152 & CelebA	IR-152 & FaceScrub	VGG-16 & FaceScrub
Time without defense	18.63 s	17.70 s	5.65 s
Time with our defense	19.22 s	18.16 s	6.07 s
Percent of increased time	3.1%	2.5%	7.4%

It can be seen that we only increase the time by 2.5% to 7.4%. The higher percent on VGG is due to the shallower model structure. In absolute terms, modifying 512 predictions for 100 times only needs 0.5 seconds. If we take the I/O time into account, the percents will be small enough to be ignored.

We further investigate the relationship between $|\mathbb{Y}|$ and the time cost of our Algorithm 3. We generate $\boldsymbol{s} \in \mathbb{R}^{|\mathbb{Y}|} \sim N(\mathbf{0}, \boldsymbol{I})$ and let $\boldsymbol{r} \leftarrow \text{softmax}(10\boldsymbol{s})$. It is observed that the $\boldsymbol{r}$ generated in this way is close to the real probability distributions. We use these $\boldsymbol{r}$ to simulate the real $\boldsymbol{f}(x)$ and $\boldsymbol{q}^y$, and let our GPU-based water-filling to find the optimal solution $\boldsymbol{p}$. We take a batch with 256 pairs $(\boldsymbol{f}(x), \boldsymbol{q}^y)$ and solve in parallel. The time costs are shown in Table 6.

Table 6: The relationship between $|\mathbb{Y}|$ and the time cost of our GPU-based water-filling.

$\lvert\mathbb{Y}\rvert$	10^1	10^2	10^3	10^4	10^5	10^6
Time	131 ms	132 ms	143 ms	163 ms	249 ms	1301 ms

It shows that even when $|\mathbb{Y}|$ reaches a million, solving 256 convex optimization problems only takes 1.3 seconds. We believe that at this point, our post-processing will not be the performance bottleneck, but the slow inferring and massive parameters of the target model will be.

E EXPERIMENTS UNDER RLB ATTACK

We evaluate the all defenses' MIA robustness against RLB (Han et al., 2023), a SOTA soft-label attack method. All settings are consistent with Tables 1-4, where the target model is IR-152 and the private dataset is CelebA. The first 10 classes of CelebA are attacked and each class reconstructed 5 images. The results are shown in Table 7.

Table 7: The MIA robustness of all defense under RLB attack.

	$\downarrow Acc@1$	$\downarrow Acc@5$	$\uparrow \delta_{eval}$	$\uparrow \delta_{face}$
No Defense	32%	64%	2006	0.77
MID	30%	48%	2088	0.84
BiDO	16%	28%	2254	0.94
LS	12%	34%	2204	0.85
TL	22%	34%	2107	0.82
SSD (ours)	**8%**	**12%**	**2480**	**1.26**

It can be seen that our defense has the best MIA robustness against RLB. The models' utility and defenses' settings are consistent with the Tables 3-4, which shows that we also preserve the best model's utility.

F EXPERIMENTS ON HIGH RESOLUTION

To adapt to high resolution, we choose Mirror as the attacker. The prior distribution is StyleGAN2 trained on FFHQ with a resolution of 1024×1024. The generated images are center-cropped to 800×800, resized to 224×224, and inputted to the target model. The target model is ResNet-152 and the evaluation model is Inception-v3. The first 10 classes of FaceScrub are attacked and each class reconstructed 5 images. The attack results are shown in Table 8 and the models' utility are shown in Table 9. Although models are more vulnerable on high resolution, our defense still achieves the best MIA robustness, with a good utility.

Table 8: The MIA robustness of all defenses under Mirror attack on high resolution.

	$\downarrow Acc@1$	$\downarrow Acc@5$	$\uparrow \delta_{eval}$	$\uparrow \delta_{face}$
No Defense	70%	94%	195	0.84
MID	62%	90%	183	0.76
BiDO	66%	86%	194	0.90
LS	48%	82%	202	0.87
TL	58%	92%	191	0.80
SSD (ours)	**42%**	**66%**	**211**	**1.13**

Table 9: The target models' utility and defenses' settings on high resolution.

	$\uparrow$ Acc	$\downarrow$ Avg L_1	$\downarrow$ Max L_1	Settings
No Defense	98.5%	0	0	–
MID	96.7%	0.30	**1.97**	$\beta = 0.005$
BiDO	96.3%	0.09	1.99	$\lambda_x = 0.15, \lambda_y = 1.5$
LS	96.5%	0.11	1.99	$\alpha = -0.01$
TL	96.7%	0.19	1.99	First 70% layers
SSD (ours)	**96.9%**	**0.07**	1.98	$T = 1, \varepsilon = 20$

G DISCUSSION ON ADAPTIVE ATTACKS

In this section we discuss adaptive attacks, where attackers are aware of our defense and take targeted actions.

Firstly, we believe that launching adaptive attacks in black-box scenarios is unrealistic, because attackers don't know the target model, and naturally don't know its defense strategy. If they were

to guess the defense strategy based on the model's behavior, they would need to consume a large number of queries.

Step back and consider, if attackers know our defense, their best strategy is:

1. Query the same x repeatedly and count the frequency of different outputs.
2. Estimate our sampling probability $\mathcal{P}(y|x)$ by the frequency they count.
3. Infer our true prediction $\mathcal{P}(\hat{y}|x)$ by the $\mathcal{P}(y|x)$ they estimate and the temperature T (assuming they know).

If an online server detects such pattern of queries, it can block them. Step back and consider again, we propose a memory-free and low-cost improvement to block such adaptive attacks:

Design a hash function $h : \mathbb{X} \to \mathbb{N}$, where $\mathbb{X}$ is the input space and $\mathbb{N}$ is the set of integers. When users/attackers query x, we take $h(x)$ as the random seed for sampling, ensuring same-input-same-output. However, attackers can add subtle perturbations to x, therefore our h needs to be robust. For example, it can be

$$h(x) \coloneqq \sum_{i=1}^{m} \lfloor k \cdot z_i(x) \rfloor, \tag{19}$$

where $\boldsymbol{z}(x) \in \mathbb{R}^m$ is the penultimate layer feature in target model, and k is the sensitivity coefficient. Note that $\boldsymbol{z}(x)$ are commonly used to evaluate the similarity between two images, i.e., the closer the two $\boldsymbol{z}(x)$ are, the more similar the two x look. The larger k is, the more numerically sensitive h is, and the more random our defense is.

How to evaluate and improve h is a new and interesting topic, worth studying deeply in the future.