Expressiveness and Learnability: A Unifying View for Evaluating Self-Supervised Learning

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Abstract

We propose a unifying view to analyze the representation quality of self-supervised learning (SSL) models without access to supervised labels, while being agnostic to the architecture, learning algorithm or data manipulation used during training. We argue that representations can be evaluated through the lens of *expressiveness* and *learnability*. We propose to use the Intrinsic Dimension (ID) to assess expressiveness and introduce Cluster Learnability (CL) to assess learnability. CL is measured as the learning speed of a KNN classifier trained to predict labels obtained by clustering the representations with K-means. We thus combine CL and ID into a single predictor – CLID. Through a large-scale empirical study with a diverse family of SSL algorithms, we find that CLID better correlates with in-distribution model performance than other competing recent evaluation schemes. We also benchmark CLID on out-of-domain generalization, where CLID serves as a predictor of the transfer performance of SSL models on several classification tasks, yielding improvements with respect to the competing baselines.

1 Introduction

Despite impressive recent progress in self-supervised learning (SSL) (Chen et al., 2020a; Caron et al., 2021; Grill et al., 2020; Caron et al., 2020; 2018; He et al., 2020; Chen & He, 2021), the problem of properly evaluating the quality of the learned representations *without using labelled data* has not been fully explored. This is an important problem because solving it could give us a practical tool to choose which SSL model to use when downstream task labels are not accessible or when fine-tuning every SSL model to choose the best one is costly. Additionally, it can also shed light on how these methods function.

In this paper, we approach it by drawing an analogy between unsupervised learning and the evolution of human language. It has been suggested in the language evolution literature (see e.g., Smith et al., 2013) that linguistic structure results from competing pressures for expressiveness (to discriminate objects and concepts of the world) and learnability (to be transmitted across generations). Here, we take the view that a similar guiding principle may be applied to representations of natural images. Just as a language associates objects and scenes with utterances, SSL algorithms do it with continuous vectors to images. We hypothesize that *a good representation should not only cover the diverse visual concepts in the datasets, but also compress them in a manner that could be efficiently learned*. Our goal is to test this hypothesis by (*i*) proposing tractable metrics to assess *expressiveness* and *learnability*, (*ii*) designing performance predictors based on these, and (*iii*) testing these predictors through a large-scale empirical study of a diverse class of SSL methods.

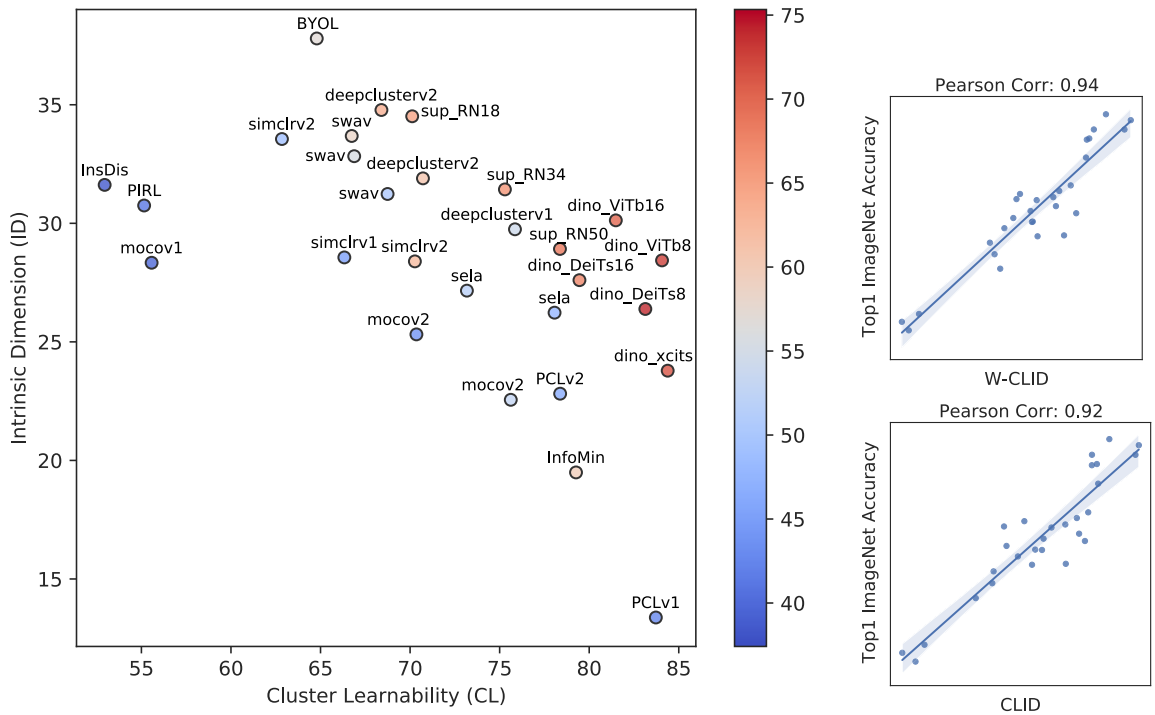

Figure 1: We propose to use Cluster Learnability (CL) to measure learnability and Intrinsic Dimension (ID) for expressiveness. **Left**: Each circle is a SSL pre-trained checkpoint, and the color is used to show its KNN Top1 ImageNet accuracy. Red indicates high accuracy, while blue indicates low accuracy. Good self-supervised learning representations are learnable and expressive, distributed over the upper-right portion of the graph. Our predictors (**Right Column**) are highly correlated with ImageNet performance, even without access to gold labels and by staying agnostic to the model's architecture or training algorithm.

The concepts of learnability and expressiveness, while not being explicitly mentioned, can be found in existing literature. For example, many SSL strategies emphasize maximizing the mutual information between inputs and representations (Vincent et al., 2008; Higgins et al., 2017; Oord et al., 2018; Bachman et al., 2019b). These can be viewed as attempts to increase expressiveness, since high mutual information leads to correspondence between inputs and representations, and thus the visual concepts among the inputs are mapped to the representation space. Recent works also pay attention to the emerging properties of learnability in the cluster structure from SSL models through human studies (Laina et al., 2020). The emphasis of learnability can be also found in the recent attempt to design a compression regularizor called Conditional Entropy Bottleneck (Lee et al., 2021). Finally, the intuition of a trade-off between learnability and expressiveness underpins other representation analysis frameworks, such as alignment and uniformity (Wang & Isola, 2020) or Maximal Coding Rate Reduction (Yu et al., 2020), which are further explained in Section 2. In this paper, we aim at turning this intuition into a practical self-supervised evaluation tool, especially on predicting performances in out-of-distribution transfer tasks.

Concretely, we propose to quantify expressiveness via an estimator of the intrinsic dimension (ID) of the data representations (Ansuini et al., 2019). To assess learnability, we take inspiration from Laina et al. (2020) and propose a novel method called Cluster Learnability (CL). We compute it by measuring the learning speed of a KNN learner trained to predict labels induced by clustering held-out representations with K-Means. We show that a combination of CL and ID, dubbed CLID, is correlated with the model performance across different architectures (see Figure 1), better than with existing evaluation scheme baselines (Wang & Isola, 2020; Reed et al., 2021; Yu et al., 2020). To further demonstrate the usefulness of our framework, we conduct out-of-domain generalization prediction experiments on seven downstream transfer tasks. We find that the proposed CLID predictor outperforms baseline concurrent methods at predicting transfer performance.

Our contributions are summarized below:

- We propose a unifying evaluation framework for self-supervised learning relying on expressiveness and learnability, yielding new insights into existing techniques in the literature.
- We propose tractable and generic metrics to estimate *(i)* expressiveness via the Intrinsic Dimension (ID) and *(ii)* learnability via our novel Cluster Learnability (CL) metric.
- We show that CLID predictors are well correlated with the ImageNet accuracies, and are robust with respect to the hyperparameters.
- We show that CLID predictors predict the transfer task performance of pretrained SSL models, especially when used jointly with in-domain accuracies.

2 Learnability and Expressiveness

In this section, we briefly discuss existing evaluation schemes of self-supervised learning methods, which will be used as baselines in our experiments. We then describe our own evaluation scheme, based on specific metrics to quantify learnability and expressiveness.

Notation We consider the following setting: we assume we have a dataset $\{x_i\}_{i=1}^N$ represented in $\mathcal{X} \subset \mathbb{R}^d$, where $x_i \sim \mathcal{P}$ are sampled i.i.d. from some distribution $\mathcal{P}$ over $\mathcal{X}$. We also consider *representation maps* $\mathcal{F} : \mathcal{X} \to \mathbb{R}^m$, typically pretrained neural networks, which represent any input as an m-dimensional vector. Any such map forms a representation dataset $Z = \{z_i\}_{i=1}^N$ where $z_i = \mathcal{F}(x_i)$.

2.1 Existing SSL Evaluation

Alignment and Uniformity (Wang & Isola, 2020) By decomposing contrastive learning loss, Wang & Isola (2020) proposes to understand SSL with alignment and uniformity. Formally, they introduce two metrics,

$$\begin{aligned} \mathcal{L}_{align} &:= \mathbb{E}_{x\sim\mathcal{P},x'\sim\mathcal{P}_{\text{aug}}(\cdot|x)} \|\mathcal{F}(x) - \mathcal{F}(x')\|_2^\alpha, \quad \alpha > 0 \\ \mathcal{L}_{unif} &:= \log \mathbb{E}_{x_1,x_2\sim\mathcal{P}} \left[e^{-t\|\hat{\mathcal{F}}(x_1)-\hat{\mathcal{F}}(x_2)\|_2^2)} \right], \quad t > 0 \end{aligned} \tag{1}$$

where $\mathcal{P}_{\text{aug}}$ is the conditional distribution defined by the data augmentation procedure, $\hat{\mathcal{F}}(x) := \mathcal{F}(x)/\|\mathcal{F}(x)\|$ are the normalized features and t, α are tunable parameters. These two metrics respectively correspond to the intuition of learnability and expressiveness. By making the feature of positive pairs to be similar, $\mathcal{L}_{align}$ would induce a learnable representations since one can generalize from one positive example to another. The other component $\mathcal{L}_{unif}$ ensures the features (normalized) distributed over a hyper-sphere, so that the representations can cover more concepts and achieve high expressiveness.

Maximal Coding Rate Reduction (MCR2) (Yu et al., 2020) MCR2 measures the reduction between the coding rate of a representation before and after a category structure Π is observed,

$$\Delta R := R(Z, \epsilon) - R^c(Z, \epsilon|\Pi) \tag{2}$$

where Π is a given dataset partition. More details on the definition of coding rate R and R^c can be found in the paper. The goal of this approach is to learn representations that is maximally diverse (large $R(Z, \epsilon)$) as well as between-class discriminative (small $R^c(Z, \epsilon|\Pi)$). Since the second term can be also interpreted as the coding rate of the representations under the category structure R^c, then MCR2 corresponds to the intuition of learning a diverse and yet compressible (w.r.t. the partition Π) representations.

Mutual Information (MI) and Pretext Tasks Maximizing mutual information between inputs and representations (Oord et al., 2018; Bachman et al., 2019a) can be viewed as increasing the expressiveness of the representations. High mutual information implies the highly correspondence between inputs and representation, suggesting each concept in the inputs is covered in the representation space. However, as pointed out by Tschannen et al. (2020), MI is not a good predictor of the model performance. Arguably, this is because MI alone does not cover the measurement of the learnability of the representations. Similar arguments can be applied to pretext tasks evaluation scheme like rotation prediction and solving jigsaw puzzles (Reed et al., 2021).

2.2 Our Proposal

Here we describe our evaluation scheme under the view of learnability and expressiveness. Our proposal can be efficiently computed on the validation set of the dataset[1].

Intrinsic Dimension (ID) We propose to use the notion of intrinsic dimensionality (ID) of the data in the representation space (Pettis et al., 1979a) to measure expressiveness. Our intuition is that, as more and more fine-grained categories emerging in the representation space, we expect the manifold complexity to go up, which can be characterized as the number of parameters needed to describe the representation manifold without losing information.

Inferring the intrinsic dimension of a highly nonlinear manifold is a challenging problem (e.g., Levina & Bickel, 2005). In this work, we leverage the nearest neighbor-based method of Facco et al. (2017) to estimate ID[2]. This estimator (TwoNN) is shown to be reliable with respect to representation dimensions and scalable to real-world datasets with deep neural networks (Ansuini et al., 2019). The concept of intrinsic dimension is also connected to the entropy estimation, which has been widely used to quantify the amount of information contained (See Appendix B for a detailed discussion).

Formally, given $z_i \in Z$ and an integer $k \geq 1$, we denote by $r_{ik} = D(z_i, NN(z_i, k))$ the distance[3] of z_i to its k-th nearest neighbor $NN(z_i, k)$. We also introduce the ratio of distances $\mu_i = r_{i2}/r_{i1}$ for each data point z_i, $1 \leq i \leq N$. Assuming that the points are sampled on a manifold with intrinsic dimension d, it can be shown that, under the assumption of local uniformity[4], μ_i follows a Pareto distribution with parameter $d+1$ on $[1, \infty)$, i.e., $\mu_i \sim P(\mu|d) := d\mu^{-(d+1)}$. While d can be computed by maximizing the likelihood, we follow a much simpler method proposed by Facco et al. (2017) basd on the cumulative distribution $F(\mu) = 1 - \mu^{-d}$ associated with $P(\mu|d)$. The idea is to estimate d with a linear regression on the empirical cumulate of the distribution. Specifically, assuming we sort μ_i ascendingly, that is $\mu_1 \leq \mu_2 \leq ... \leq \mu_N$, we estimate the cumulative distribution as $F_i^{emp} = i/N$ and fit a straight line on the datasets $\{(\log \mu_i, -\log(1 - F_i^{emp}))\}_{i=1}^N$ in the two dimensional plane. The slope is the estimated ID.

Cluster Learnability (CL) We take a similar stand as MCR2, which is that learnability and compression happen when there is a category structure. In order to measure the learnability of the emerging visual concepts in the representation space, we take inspiration from Laina et al. (2020). The authors show that the classes obtained by clustering the representations of pretrained self-supervised models can be easily learned by humans. We propose a *cluster learnability* (CL) metric as a way to automate such procedure.

Let $\{z_i, \tilde{y}_i\}_{i=1}^N$ be the labelled dataset obtained by clustering the representation (e.g., with K-means). We define the learnability of the representation as the learning speed of a KNN classifier with this labelled dataset. To formalize this, we use a prequential approach (Dawid, 1984). Let $\hat{y}_i = KNN(z_i|\{z_{k<i}, \tilde{y}_{k<i}\})$ denote the prediction of the i-th label after seeing the pairs $(z_k, \tilde{y}_k)$ for $k < i$. Learnability is defined as the

[1] We also experiment with computing on the train set but there is no difference in the results.

[2] We experiment with the ID estimator based on the maximum likelihood estimation (Levina & Bickel, 2004; Ma et al., 2018), without noticing a difference in performance.

[3] While we generally consider nearest neighboors w.r.t Euclidean distance, in practice we will also use the cosine distance function, $D(z_1, z_2) = 2 - 2\cos(z_1, z_2)$. Both produce similar results in our experiments.

[4] While the original derivation (Facco et al., 2017) assumes global uniformity, but post-hoc analysis shows that it only needs local uniformity up to the second neighbor.

online learning accuracy of our KNN classifier:

$$CL = \frac{1}{N}\sum_{i=1}^{N}[\hat{y}_i == \tilde{y}_i] \tag{3}$$

In order to scale to large datasets, we choose to chunk our dataset and average the chunk accuracies. This metric can be connected to the prequential codelength of the cluster assignment dataset where the conditional model is a formed by a KNN classifier. We elaborate on this connetion in Appendix C.

CLID Predictor We propose two versions of the CLID predictor as follows:

$$\begin{aligned} \text{CLID}: &\quad CL + ID \\ \text{W-CLID}: &\quad \mathbf{w}^T[CL; ID; 1] \end{aligned}$$

where $\mathbf{w}$ is a weight that can be learned by linear regression if in-domain performance of the models considered is available.

3 Empirical Study

3.1 Setup

We select in total 28 self-supervised learning checkpoints trained on ImageNet over different algorithms, architecture, and training epochs. A complete list can be found in Table 4 in the appendix. We use the KNN evaluation on the validation data using the ground-truth labels to measure the performance of the model, which has been shown to be well correlated with the linear evaluation but computationally less expensive (Caron et al., 2021).

For the computation of cluster learnability, we choose the square root of the dataset size as the number of clusters in Kmeans. We report results with 1 neighbor for our KNN learner. We also show that our results are robust to other choices of hyper-parameters. We normalize the features and use cosine distance for the K-means clustering and KNN learner[5]. Other configurations of cluster numbers and neighbor numbers are also explored in section 3.4. All our experiments are computed on a single V100 GPU.

3.2 Baselines

Wang & Isola (2020) also proposes to predict the ImageNet performance of pre-trained SSL checkpoints as an evaluation scheme. Therefore, in our experiments, we follow the official implementation[6] with $\alpha = 2$ and $t = 2$ as default values for the tunable parameters in Eqn 1. We additionally define $-\mathcal{L}_{contrast} = -\mathcal{L}_{align} - \mathcal{L}_{unif}$ to compute the predictor for Wang & Isola (2020)'s method, which reduces to the negative contrastive loss. We add a negative sign, since we requre a predictor to be in proportional to the model performance.

The original MCR2 requires the partition Π to be pre-specified. In order to adapt it into an unsupervised evaluation scheme, we use a K-means clustering as the dataset partition. We follow the default settings[7] where $\epsilon^2 = 0.5$ from Eqn 2, and we normalize the features. We also experiment with using the ground-truth label as the dataset partition, but we found no improvements. For MCR2, as a predictor, we simply use ΔR as is defined above.

For the Mutual Information baseline, we use the MINE estimator (Belghazi et al., 2018) with a fixed student ResNet18 netowrk (He et al., 2016) to estimate the MI $I(X, Z)$. We use a batch size 128, learning rate 0.0005 and weight decay 0.001. The network is trained for 50000 steps on the training images, and we report the MINE estimation on the validation data. We find that training longer is computational intensive, and the results are similar.

[5]We find similar results when using L2 distance.
[6]See `https://github.com/SsnL/moco_align_uniform`
[7]See `https://github.com/ryanchankh/mcr2`

Table 1: Correlation results between ImageNet performances and different predictors. We compute both Pearson ρ and Kendall τ. Our CLID predictors achieve the highest correlation.

Predictors	Pearson ρ	Kendall τ
CL	0.74	0.44
ID	0.12	0.09
CLID	0.92	**0.75**
W-CLID	**0.94**	0.73
$-\mathcal{L}_{align}$	0.42	0.26
$-\mathcal{L}_{unif}$	-0.05	0.03
$\mathcal{L}_{contrast}$	0.37	0.24
$R(Z)$	-0.62	-0.33
$-R^c(Z, \Pi)$	-0.58	-0.34
ΔR	-0.62	-0.33
MI	0.13	0.08
Pretext	0.61	0.27

For the pretext task, we experiment with rotation prediction by constructing a 4-way classification. We randomly rotate the training images by $0, 90, 180, 270$ degrees, train a KNN classifier on the training images to predict a 4-way classification, and then report the rotation prediction accuracy on the validation images. This is shown to be an effective evaluation in both self-supervised learning (Reed et al., 2021) and architecture search (Liu et al., 2020).

3.3 Is CLID scheme correlated with ImageNet performance?

We first focus on whether the proposed evaluation scheme is useful for in-domain (ImageNet) generalization. We perform both qualitative and quantitative examination and the results are in Figure 1. On the left, we find that the self-supervised learning checkpoints with higher ImageNet accuracies tend to be more learnable as well as expressive (e.g., in the upper-right corner of the graph). In the meantime, we find that methods favouring only one quantity like PCLv1 and PIRL also have poor ImageNet accuracies. In Figure 1 middle and right, we compute the correlation of our CLID and W-CLID predictors with respect to the ImageNet accuracy of the model considered and show that we achieve a Pearson ρ of 0.92 for CLID and 0.94 for W-CLID.

In Table 1, we compare our results with respect the baselines. The regression plots for the baselines can be found in Figure 3. Our proposed predictors achieve the highest correlation both in terms of Pearson ρ and Kendall τ coefficient. We find that $\mathcal{L}_{contrast}$ achieves a relatively low correlation $\rho = 0.37$, which apparently contradicts the observation made in Wang & Isola (2020). We hypothesize that this is due to the fact that their analysis is restricted to SSL checkpoints trained with contrastive learning, and therefore $\mathcal{L}_{contrast}$ might not be general enough to characterize representations obtained by alternative approaches. Additionally, the results in Wang & Isola (2020) are computed on the representations used to compute the noise-contrastive objective, which are usually transformed by a final projection layer that is not always present in other SSL methods. This limits its generality. Interestingly, we find that MCR2 (ΔR) gives negative correlation which warrants further investigation. While the MI baseline only achieves $\rho = 0.13$, the Pretext baseline is still surprisingly good with a $\rho = 0.61$, suggesting that the rotation prediction task remains a simple but effective baseline.

3.4 Is CLID sensitive to its hyper-parameters?

In this section, we check the sensitivity of CLID to its hyperparameters. Since ID does not have any hyper-parameters, we mainly examine the following hyper-parameters for CL: the number of clusters and the number of neighbors used in the KNN Learner.

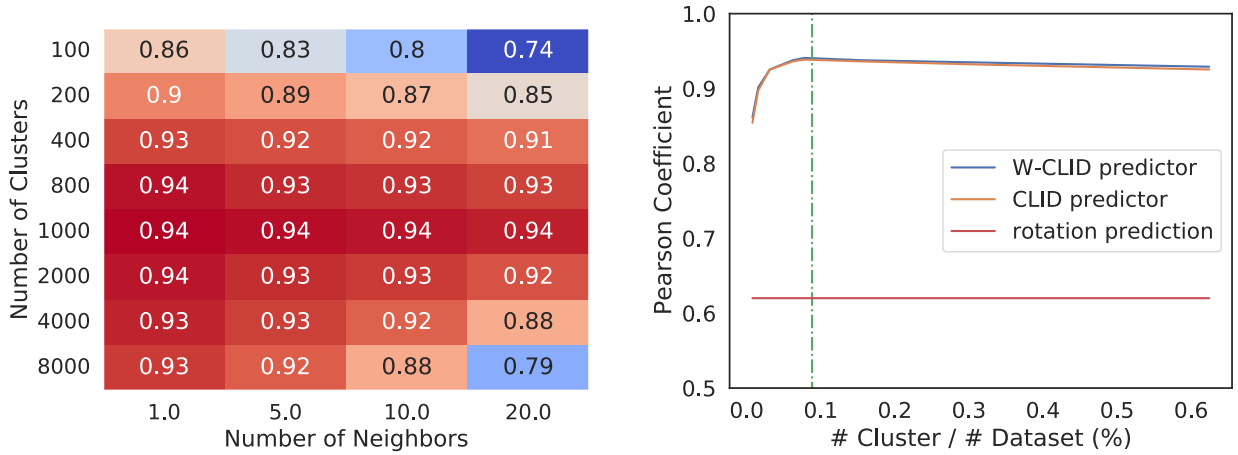

Figure 2: Robustness Analysis for ImageNet. **Left**: The heat map of Pearson Coefficient between the Top-1 accuracy and W-CLID predictor. **Right**: Pearson coefficient vs. the ratio between the number of clusters and the dataset size when using one neighbor. The result is stable with a reasonably large number of clusters, e.g., the square-root of the dataset size **(Green Dashed Line)**.

Table 2: Kendall Ranking Coefficient results on Out-of-Domain Tasks when ImageNet labels are not available. The CLID predictor has the best predictive performance for transfer tasks. Full results in Table 5.

	CLID	W-CLID	$-\mathcal{L}_{contrast}$	ΔR	MI	Pretext
foods	**0.75**	0.72	0.17	-0.35	0.01	0.51
flowers	**0.81**	0.73	0.25	-0.35	0.14	0.46
pets	**0.71**	0.70	0.23	-0.25	-0.08	0.54
caltech101	0.56	0.52	0.11	-0.18	-0.05	**0.67**
stl10	0.56	**0.64**	0.02	-0.34	-0.21	0.41
aircraft	**0.67**	0.57	0.21	-0.27	0.16	0.57
cars	**0.81**	0.72	0.32	-0.28	0.16	0.48
Avg	**0.70**	0.66	0.19	-0.29	0.02	0.52

The results can be found in Figure 2. The resulting predictors can still produce a higher correlation than the rotation prediction for a wide range of the configurations we tried. We observe that decreasing the number of neighbors leads to a better predictor. Given our result, we recommend setting number of neighbors to be 1.

We observe that there is an optimal cluster number so that the choice of neighbor numbers can be more flexible. If the cluster number is too low, the results become worse. Recall that for different checkpoints, the clusters are produced with the same K-means algorithm, so the separability among clusters might be roughly controlled. As a result, we hypothesize that the number of clusters might control the underlying difficulty of the classification problem faced by the KNN learner. In the extreme case, we either have 1 cluster or as many clusters as data points. In the former, KNN accuracy would always be 100% and in the latter always be close to 0%[8]. As a result, neither of them would be suitable enough to distinguish the collected checkpoints. As a rule of thumb, we recommend using a reasonably large number of clusters, e.g., the square root of the dataset size.

Table 3: Kendall Ranking Coefficient results on Out-of-Domain Tasks when ImageNet labels are obtained. The results are computed with the *joint rank product* between the predictors and the ImageNet accuracy. While the ImageNet accuracy can predict the transfer accuracy reasonabley, the proposed CLID predictor could further enhance the result. Full results in Table 6. Blue cells indicates the joint ranking outperform using ImageNet accuracy alone.

	Imagenet	CLID	W-CLID	Source Ent	Target Ent	$L_{contrast}$	Pretext
foods	0.86	0.84	0.81	**0.88**	0.84	0.56	0.75
flowers	0.70	**0.79**	0.74	0.74	0.71	0.58	0.64
pets	0.75	0.77	**0.78**	0.66	0.72	0.55	0.76
caltech101	0.62	0.59	0.58	0.60	0.66	0.41	**0.77**
stl10	**0.76**	0.71	0.71	0.61	0.71	0.36	0.64
aircraft	0.60	0.66	0.61	**0.68**	0.64	0.50	0.62
cars	0.68	**0.79**	0.74	0.72	0.74	0.64	0.66
Avg	0.71	**0.74**	0.71	0.70	0.72	0.52	0.69

3.5 Can CLID be used to predict transfer performance?

As is pointed out by Vedantam et al. (2021), ImageNet accuracies are not completely correlated with other downstream classification tasks. As a result, we want to see if the proposed CLID predictor can help improve it. We collect 7 out-of-domain downstream classification tasks. For each domain, we compare the obtained ranking and compare it with the actual ranking on that domain. We report the Kendall τ correlation score, which is consistent with the existing benchmarks on out-of-domain generalization (Vedantam et al., 2021).

First, we investigate the scenario where the ImageNet labels are not accessible (Table 2). This is an extension of our previous scenario, and it is also a realistic assumption especially when the models are pretrained on web-scale data without clear annotations. We find that our CLID predictor is the best, even beating the W-CLID predictor. One reason is because the linear coefficients $\mathbf{w}$ are obtained from regressing in the ImageNet domain, which might increase overfitting and hurt transfer. We also find that $-\mathcal{L}_{contrast}$ is not a good predictor, and a simple pretext task like rotation prediction already has reasonable performance even in transfer settings.

In the second setting, we assume to have access to ImageNet labels (Table 3) and explore whether transfer performance prediction can be improved using this additional information. We find that the ImageNet accuracies have pretty high correlation with the downstream tasks, with an average Kendall coefficients of 0.71. This is consistent with Vedantam et al. (2021) as in-domain accuracy is a good predictor for out-of-domain performance. To integrate ImageNet performance information to each our competing predictors, we proceed as follows. Let r^i_{pred} be the ranking of i-th checkpoint from the predictor, and r^i_{img} be that from the ImageNet accuracy. We compute a joint ranking by computing the rank product, which is the geometric mean of these two ranks:

$$r^i_{joint} = \sqrt{r^i_{pred} r^i_{img}}$$

In addition to comparing with previously presented baselines, we also add the strongest baselines from (Vedantam et al., 2021): we train a linear classifier and measure the negative label entropy $-H(Y|X)$. Since the entropy can be measured on either source domain or target domain, we denote each variant as "Source Ent" and "Target Ent". The intuition behind is that if the model has more confidence in the prediction (lower entropy), it should generalize better.

We find that the proposed CLID predictor could further enhance the ImageNet accuracies predictions, with an average Kendall coefficients of 0.74 for the joint ranking. We find that "Target Ent." and "Source Ent." both have a reasonable performance, which is consistent with observation in Vedantam et al. (2021), but they underperform our predictor. Note that computing "Target Ent." and "Source Ent." requires training an extra linear layer, which increases their computational requirements.

[8]To be specific it's $1/N$, where N is the dataset size. It's close to 0% when N is large.

4 Related Work

Representation Evaluation Several recent works address the question of representation evaluation in self-supervised learning. Whitney et al. (2021) propose to use the learning dynamics of the downstream classifiers to measure the representation complexity. However their method still depends on extra human labels. Pretext tasks like jigsaw or rotation prediction are shown to be well correlated with the self-supervised evaluation (Reed et al., 2021; Deng & Zheng, 2021) and architecture search (Liu et al., 2020). However, they also rely on crafting ad-hoc data augmentations. Previous work also investigate the intuition of learnability and expressiveness, but with their own terms. Maximal Coding Rate Reduction (MCR2) Yu et al. (2020) is one of them; however, this approach requires the ground-truth labels, so it is not directly applicable as an SSL evaluation framework. Wang & Isola (2020) studies contrastive loss, by decomposing it into an alignment loss and a uniformity loss. However, their method requires access to the projection head output, which limits its applicability to general architectures. Furthermore, we find that by focusing only on contrastive loss, this evaluation scheme provides limited predictive power when applying to a more diverse family of SSL models (section 3.3). Ericsson et al. (2021) argues that it is important to look at the transfer performance of the SSL models in order to judge its quality. We agree with this statement and therefore also test our method for out-of-domain generalization settings (section 3.5). Closely related to our work, a very recent paper Garrido et al. (2022) proposes to evaluate joint-embedding self-supervised methods without access to supervised labels, by means of the effective rank of the learned embedding matrix.

Learnability, Ease-of-Transmission and Compression Learnability has been argued to be a hallmark of the human language in order to be effortlessly transmitted through generations (Kirby et al., 2014; Rafferty et al., 2011; Beckner et al., 2017; Zhou & Yurovsky, 2021; Kampen, 2004), and it is also true for visual concepts like color (Xu et al., 2010), categories (Griffiths et al., 2006), shapes (Portelance et al., 2021) etc. In deep learning, it has been explored in the context of emergent communication (Ren et al., 2020; Guo et al., 2019; Li & Bowling, 2019), language drift (Lu et al., 2020), and neural module networks (Vani et al., 2021), but it is less explored for vision representation learning, except for a human study on just two SSL methods (Laina et al., 2020). While Lee et al. (2021) also investigates a regularizor for compressive self-supervised learning, it is unclear whether such notion is useful as a representation evaluation framework. Learnability has a tight connection to compression (Chaitin, 2007) and prequential codelength (Dawid, 1984), which quantifies the compression levels with the online learning error. Existing work (Blier & Ollivier, 2018) has uses it to support the generalization ability of the learner (e.g., deep neural nets) on the dataset (e.g., labeled images). Here, we use this concept to quantify the learnability of the representation, in the sense that if the emerged Kmeans clustering is more learnable, then the same KNN learner could achieve a lower compression bound via prequential coding.

Manifold Intrinsic Dimension Intrinsic dimension can be thought of as the smallest number of variable needed to approximate the representation manifold. Applying local neighborhood information to estimate the intrinsic dimension is not a new idea, and it was shown to be more efficient than the global eigenvalue approach (Pettis et al., 1979b). Ansuini et al. (2019) apply the TwoNN estimator (Facco et al., 2017) to the non-linear representation manifold of modern deep neural nets. They find that the intrinsic dimension is inversely correlated to the classification accuracy Their work is further extended to confirm that natural images lies in a low-dimension manifold (Pope et al., 2021), and that lower ID datasets leads to better generalization. Recanatesi et al. (2019) further highlights the connections between intrinsic dimension and the generalization properties, by comparing the models before and after supervised training. Intrinsic dimension can also be estimated locally with a Maximum Likelihood Estimator (Levina & Bickel, 2004), which Ma et al. (2018) propose to use as a regularizor against overfitting noisy labels. While the above work mainly focus on ID with supervised learning models, our work on SSL models presents a more nuanced view about ID. It is indeed the case that lower intrinsic dimension can lead to better accuracy, especially among supervised checkpoints (see "sup_RN18", "sup_RN34", "sup_RN50"in Figure 1). However, we hypothesize that when learning representation from scratch without labels, e.g., self-supervised learning, the representation manifold still need a certain amount of complexity in order to include enough information from the dataset.

5 Limitation and Potential Negative Societal Impacts

Limitations While our proposed evaluation scheme is designed to be general, a potential limitation is that it might be only suitable to the classification tasks, since we put emphasis on the emergence of learnable cluster structure. As a result, the proposed predictor might not be as useful for other downstream tasks like object detection or segmentation. While our results is true for a population of models, there are also some outliers. E.g., in Figure 1 "deepclusterv1" seems to have higher CL and ID than "simclrv2", but its accuracy is lower.

Societal impact This paper follows a line of work aiming at a better understanding of deep learning algorithms. Even though it does not directly contribute to any specific application, it promotes the development and dissemination of the deep learning technology, which, as any technology, can be used for harmful purposes. Moreover, we acknowledge that deep learning has been proved in the past to potentially reduce or amplify bias.

6 Conclusion & Discussion

We propose a unifying view to evaluate self-supervised representation learning through expressiveness and learnability. We propose to estimate expressiveness with intrinsic dimension (ID), and learnability with the acquisition speed of a KNN learner on the K-means clustering of the representation. We show that the proposed CLID evaluation scheme better predicts the ImageNet accuracy than other evaluation schemes. We further demonstrate that our CLID can also predict the transfer task performance.

Our work is a solid step towards understanding the current SSL algorithms and opens up interesting research directions. Future works could further explore the expressiveness-learnability in a more theoretical context, extend this framework to other field like pretrained language models, as well as devise new SSL algorithms that directly maximize intrinsic dimension or cluster learnability.

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