LOTTERY AWARE SPARSITY HUNTING: ENABLING FEDERATED LEARNING ON RESOURCE-LIMITED EDGE

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ABSTRACT

Limited computation and communication capabilities of clients pose significant challenges in federated learning (FL) over resource-limited edge nodes. A potential solution to this problem is to deploy off-the-shelf sparse learning algorithms that train a binary sparse mask on each client with the expectation of training a consistent sparse server mask. However, as we investigate in this paper, such naive deployments result in a significant accuracy drop compared to FL with dense models, especially under low client's resource budget. In particular, our investigations reveal a serious lack of consensus among the trained masks on clients, which prevents convergence on the server mask and potentially leads to a substantial drop in model performance. Based on such key observations, we propose *federated lottery aware sparsity hunting* (FLASH), a unified sparse learning framework to make the server win a lottery in terms of a sparse sub-model, which can greatly improve performance under highly resource-limited client settings. Moreover, to address the issue of device heterogeneity, we leverage our findings to propose *hetero-FLASH*, where clients can have different target sparsity budgets based on their device resource limits. Extensive experimental evaluations with multiple models on various datasets (both IID and non-IID) show superiority of our models in yielding up to $\sim 10.1\%$ improved accuracy with $\sim 10.26\times$ fewer communication costs, compared to existing alternatives, at similar hyperparameter settings.

1 INTRODUCTION

Federated learning (FL) McMahan et al. (2017) is a popular form of distributed training, which has gained significant traction due to its ability to allow multiple clients to learn a shared global model without the requirement to transfer their private data. However, clients' heterogeneity and resource limitations pose significant challenges for FL deployment over edge nodes, including mobile phones and IoT devices. To resolve these issues, various methods have been proposed over the past few years including efficient learning for heterogeneous collaborative training Lin et al. (2020); Zhu et al. (2021), distillation He et al. (2020), federated dropout techniques Horvath et al. (2021); Caldas et al. (2018b), efficient aggregation for faster convergence and reduced communication Reddi et al. (2020); Li et al. (2020b). However, these methods do not necessarily address the growing concerns of highly computation and communication limited edge.

Meanwhile, reducing the memory, compute, and latency costs for deep neural networks (DNNs) in centralized training for their efficient edge deployment has also become an active area of research. In particular, recently proposed *sparse learning* (SL) strategies Evci et al. (2020); Kundu et al. (2021); Mocanu et al. (2018); Dettmers & Zettlemoyer (2019); Raihan & Aamodt (2020) effectively train weights and associated binary *sparse masks* to allow only a fraction of model parameters to be updated during training, potentially enabling the lucrative reduction in both the training time and FLOPs Qiu et al. (2021); Raihan & Aamodt (2020), while creating a *model to meet a target parameter density denoted as d, and is able to yield accuracy close to that of the unpruned baseline*.

However, the challenges and opportunities of sparse learning in FL is yet to be fully unveiled. Only very recently, few works Bibikar et al. (2021); Huang et al. (2022) have tried to leverage sparse learning in FL primarily to show their efficacy in non-IID settings. Nevertheless, these works primarily used sparsity for non-aggressive model compression, limiting the actual benefits of sparse learning, and assumed multiple local epochs, that may further increase the training time for strugglers making the overall FL process inefficient Zhang et al. (2021).

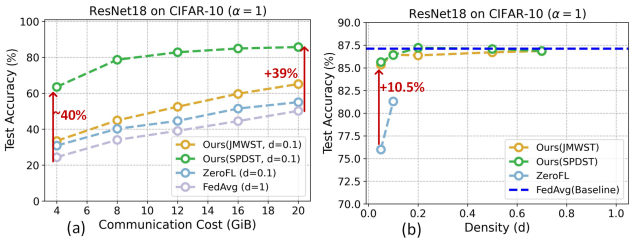

Figure 1: Comparison of (a) accuracy at different communication budget with, ZeroFL Qiu et al. (2021) and FedAvg. (w/ $d = 1.0$) (b) Accuracy vs. parameter density of each client. Proposed approaches can significantly outperform the existing alternative Qiu et al. (2021) at ultra-low target parameter density (d).

Moreover, the server-side pruning used in these methods may not necessarily adhere to the layers' pruning sensitivity[1] Ding et al. (2019) that often plays a crucial role in sparse model performance Kundu et al. (2021); Zhang et al. (2018); Ren et al. (2019). Another recent work, ZeroFL Qiu et al. (2021), has explored deploying sparse learning in FL settings with limited client epochs. However, Qiu et al. (2021) could not leverage any advantage of model sparsity in the clients' down-link communication cost and had to keep significantly more parameters active compared to a target d to yield good accuracy. Moreover, as shown in Fig. 1(b), for $d = 0.05$, ZeroFL still suffers from substantial accuracy drop of $\sim 14\%$ compared to the baseline.

Our Contributions. Our contribution is fourfold. In view of the above limitations, we first identify crucial differences between a centralized and the corresponding FL model, in learning the sparse masks for each layer. In particular, we observe that in FL, the server model fails to yield convergent sparse masks, primarily due to the lack of consensus among clients' later layers' masks. In contrast, the centralized model show significantly higher convergence trend in learning sparse masks for all layers. We then experimentally demonstrate the utility of pruning sensitivity and mask convergence in yielding good accuracy setting the platform to close the performance gap in sparse FL.

We then leverage our findings and present *federated lottery aware sparsity hunting* (FLASH), a sparse FL methodology addressing the aforementioned limitations in a unified manner. At the core, FLASH leverages a two-stage FL, a robust and low-cost layer sensitivity evaluation stage and a training stage that essentially supports the choice of *mask learnability* (m_{freez}). In particular, the first stage identifies a good initial sparse mask for the clients. We claim this to play a key role in communication-efficient learning of a model at the second stage with the sparse masks frozen to their initial values ($m_{freez} = 1$), yet yielding SOTA accuracy in sparse FL.

To deal with resource heterogeneity, we further extend our methodologies to *hetero*-FLASH, where we assume a critical scenario of individual clients having different d. Here, to deal with the unique problem of the server selecting different sparse models for clients, we present server-side gradual mask sub-sampling, that identifies sparse masks via a form of layer sensitivity re-calibration, starting for models with highest to that with lowest density support.

We conduct extensive experiments on MNIST, FEMNIST, and CIFAR-10 with different models for both IID and non-IID client data partitioning. Experimental results show that, compared to the existing alternative Qiu et al. (2021), at iso-hyperparameter settings, FLASH can yield up to $\sim 8.9\%$ and $\sim 10.1\%$, on IID and non-IID data settings, respectively, with reduced communication of up to $\sim 10.2\times$.

2 Related Works

Model Pruning. Over the past few years, a plethora of research has been done to perform efficient model compression via pruning, particularly in centralized training Ma et al. (2021); Frankle & Carbin (2018); Liu et al. (2021); You et al. (2019); He et al. (2018). Pruning essentially identifies and removes the unimportant parameters to yield compute-efficient inference models. More recently, sparse learning Evci et al. (2020); Kundu et al. (2021); Dettmers & Zettlemoyer (2019); Raihan & Aamodt (2020), a popular form of model pruning, has gained significant traction due to its popularity in yielding FLOPs advantage and potential speed-up even during training. In particular, it ensures only $d\%$ of the model parameters remain non-zero during the training for a target parameter density d, potentially enabling training complexity reduction.

[1] A layer with higher sensitivity demands higher % of non-zero weights compared to a less sensitive layer.

Dynamic network rewiring (DNR). We leverage DNR Kundu et al. (2021), to learn the sparsity mask of each client. In DNR, a model starts with randomly initiated mask following the target parameter density d. After an epoch, the client evenly prunes the lowest $p_r\%$ weights from each layer based on absolute magnitude, where p_r is prune rate. Note, this $p_r\%$ pruning happens on top of the sparse model with density d, allowing $p_r\%$ weights to be regrown. DNR then ranks each layer based on the normalized contribution of the summed non-zero weight magnitudes. Finally, the client regrows total $p_r\%$ weights in a non-uniform way, allowing more regrowth to the layers having higher rank. This process iteratively repeats over epochs to finally learn the mask.

Federated learning for resource and communication limited edge. To address device heterogeneity, existing works have explored the idea of heterogeneous training Horvath et al. (2021); Diao et al. (2020); Yao et al. (2021) allowing different clients to train on different fractions of full-model based on their compute-budget. On a parallel track, various optimizations are proposed in FL training framework to yield faster convergence, thus requiring fewer communication rounds Han et al. (2020); Gorbunov et al. (2021); Zhang et al. (2013); Li et al. (2019); Reddi et al. (2020); Islamov et al. (2021); Albasyoni et al. (2020).

To address the issue of client resource limitations, a few research have leveraged pruning in FL Li et al. (2020a); Jiang et al. (2022); Li et al. (2021). In particular, LotteryFL Li et al. (2020a) trained each client to have their personalized mask with which they are able to perform well only on their own data. Moreover, the clients often need to send full model costing bandwidth. PruneFL Jiang et al. (2022) also asks for significant communication costs as it demands participating clients to send all the gradient values to the server in the mask update rounds.

Only a few contemporary works Huang et al. (2022); Bibikar et al. (2021); Qiu et al. (2021) tried to leverage the benefits of sparse learning in federated settings. In particular, Huang et al. (2022) relied on a randomly initialized sparse mask, and recommended keeping it frozen Kundu et al. (2020) throughout the training, yet failed to provide any supporting intuition. FedDST Bibikar et al. (2021), on the other hand, leveraged the idea of RigL Evci et al. (2020) to perform sparse learning of the clients and relied on magnitude pruning at the server-side that does not necessarily adhere to the layer sensitivity towards a target density. Moreover, both the approaches assumed all clients can support a fixed d, a large number of local epochs, and focused primarily on only highly non-IID data without targeting ultra-low density d. More importantly, neither of these works investigated the key differences between centralized and FL sparse learning. With similar philosophy as ours, ZeroFL Qiu et al. (2021) first identified a key aspect of sparse learning in FL in terms of all clients' masks to be within 30% of the total model weights to yield good accuracy at high compression. However, ZeroFL suffered significantly in failing to exploit a proportional advantage in communication saving as even for low parameter density d, all clients had to download the dense model and send back at least a model with $d = 0.3$. Furthermore, these algorithms sacrifice significant accuracy at ultra-low d.

3 Revisiting Sparse Learning: Why Does it Miss the Mark in FL?

Table 1: FL training settings considered in this work.

Dataset	Model	#Params.	Data-partioning	Rounds (T)	Clients (C_N)	Clients/Round (c_r, c_d)	Optimizer	Aggregation type	Local epoch (E)	Batch size
MNIST	MNISTNet	262K	LDA	400	100	10, 10	SGD	FedAvg	1	32
CIFAR-10	ResNet18	11.2M		600						32
FEMNIST	Same as Caldas et al. (2018a)	6.6M	Reddi et al. (2020)	1000	3400	34, 34		McMahan et al. (2017)		16

Sparse learning uses proxies, including normalized momentum and normalized values of the non-zero weights Kundu et al. (2021); Dettmers & Zettlemoyer (2019), to decide the layers and weights that are more sensitive towards pruning and update the binary sparse mask accordingly. Note, centralized training has shown significant benefits with sparse learning with FLOPs reduction during forward operations Evci et al. (2020), and potential training speed-up of up to $3.3\times$ Qiu et al. (2021) while maintaining close to the baseline accuracy, even at $d \leq 0.1$. We now use a sparse learning, namely Kundu et al. (2021), in FL settings (refer to Table 1 for details) on CIFAR-10, where each client separately performs Kundu et al. (2021) to train a sparse ResNet18 and meet a fixed parameter density d, starting from a random sparse mask. After sending the updates to server, it aggregates them using FedAvg. We term this as *naive sparse training* (NST).

Observation 1. *At high compression $d \leq 0.1$, the collaboratively learned FL model significantly sacrifices performance, while the centralized sparse learning yields close to baseline performance.*

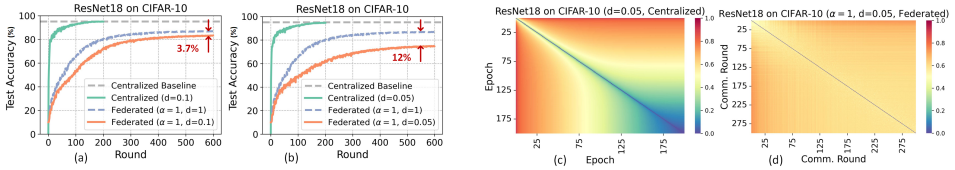

Figure 2: (a-b) Accuracy vs. round plot on deployment of off-the-shelf Kundu et al. (2021) sparse learning in FL for different d, (c-d) visualization of the Model's SM in terms of Jaccard distance while training with sparse learning for (c) centralized and (d) FL, respectively.

As shown in Fig.2(a), naive deployment of sparse learning significantly sacrifices accuracy in FL. In particular, for $d = 0.1$, the trained server-side model suffers an accuracy drop of 3.67%. At even lower $d = 0.05$, this drop significantly increases to 12.03%, hinting at serious limitations of sparse learning in FL. However, in centralized sparse learning, the model yields close to the baseline accuracy, even at $d = 0.05$.

Observation 2. *As the training progresses, the sparse masks in centralized training tend to agree across epochs, showing convergence, while server mask in FL does lack agreement across rounds.*

Definition 1. Sparse mask mismatch. For a model at round t, we define the *sparse mask mismatch* (SM) $\texttt{sm}^t$ as the Jaccard distance that is measured as follows.

$$\texttt{sm}^t = 1 - \frac{(\sum_{l=1}^{L} \mathcal{M}_l^t \cap \mathcal{M}_l^{t-1})}{(\sum_{l=1}^{L} \mathcal{M}_l^t \cup \mathcal{M}_l^{t-1})} \tag{1}$$

where $\mathcal{M}_l^t$ represents the sparse mask tensor for layer l at the end of round t. Interestingly, as depicted in Fig. 2(a), the SM for centralized learning (CL) tends to zero as the training progresses. In contrast, with the same model, dataset and d values, in FL, the SM remains > 0.4 indicating a substantial distinction in the sparse mask learning between CL and FL.

Observation 3. *At low target density, in federated sparse learning, throughout the training rounds, the disagreement on the later layer's masks remains more severe than the earlier ones.*

As the training progresses, in CL, mask for each layer shows significant convergence trend as measured by SM for the layer (Fig. 3(a)). However, Fig. 3(b) shows in FL, the later layers' masks differ significantly and continue to disagree over rounds with SM value as high as $\sim$0.8. This may be attributed to many possible mask choices due to the later layers' significantly fewer non-zero parameter allocation, compared to the initial layers, driven by their respective pruning sensitivities. For example, layer 1 requires 90% parameters to be present, compared to only 5% for layer 14, with the later costing an SM of $\sim$0.73.

Table 2: Performance based on the different levels of mask disagreement in CL.

Training type	Use sensitivity	Masks change at	Layer SM	Test acc%
Pre-defined w/ mask frozen	N	–	–	89.72
Pre-defined w/ mask frozen	Y	–	–	**91.66**
w/o mask frozen	Y	layer 9-16	0.8	88.88
	Y	layer 1-16	0.5	84.62
	Y	layer 1-16	0.8	82.32

To further investigate the impact of higher SM and layer sensitivity on a model's accuracy, we performed five different training in CL as described in Table 2. In particular, for the training in row1 we randomly generate sparse masks with uniform density for all the layers. For all other training, we first randomly create each layer's mask by following its pruning sensitivity[2] and then decide to keep the layer mask frozen for some or all the layers. For training described in rows 3-5, we allow a fraction of the mentioned layers' masks to differ between consecutive epochs such that they meet the target SM value, creating the situation of non-convergent masks. As Table 2 clearly shows that large SM for the layers can degrade the accuracy by up to 9.34%, we can safely conclude that *disagreement of masks across epochs can significantly affect the model's final performance*. Moreover, the model trained via sparse learning with sensitivity-driven pre-defined masks yields better performance than the one trained with uniform density sparse mask.

[2]For a sparse model it is evaluated as the ratio $\frac{\texttt{\# of non-zero layer parameters}}{\texttt{\# layer parameters}}$ Ding et al. (2019). We use another pre-trained model of the same architecture and target d for this evaluation.

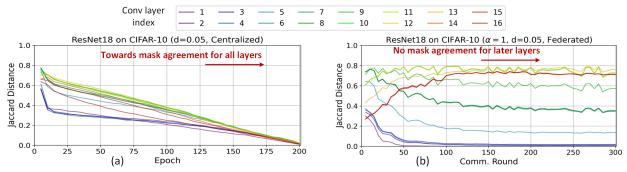

Figure 3: Layer-wise sparse mask mismatch (SM) vs. training epochs (rounds) plot for (a) centralized and (b) FL, respectively. In FL, the layer layers continue to have higher SM contrary to centralized, where every layer tend to reduce the SM as the training matures.

4 FLASH: METHODOLOGY

To win a lottery of having a sparse network yielding high accuracy at reduced parameters, we identify two key characteristics of sparse learning, namely, the pruning sensitivity and mask learnability. To explicitly adhere to these two important aspects, in FLASH, we present a two-stage sparse FL method, **stage1:** targeting sensitivity analysis to identify good initial sparse mask for each layer, **stage2:** targeting training to learn masks and weights. In particular, to evaluate layer sensitivity in stage1, the server randomly selects a small fraction of clients ($[\mathcal{C}_d]$), each locally sparse learning Kundu et al. (2021) for few warm-up epochs (E_d) (L4-8 in Algo. 1). Upon collection of layer-wise sensitivity from the clients, for each layer l, the server estimates average density[3] $\hat{d}^l$ as $\frac{\sum_{i=1}^{c_d} d_i^l}{c_d}$, where d_i^l is the density at layer l in i^{th} client. As these averaged layer-wise density values may not necessarily yield to the target density d, for a model with K parameters we follow the following *density re-calibration*

$$d_c^l = \hat{d}^l.r_f, \text{ where } r_f = \frac{d \times K}{\sum_{l=1}^{L} \hat{d}^l.k^l} \tag{2}$$

k^l is dense model's parameter size for layer l. For each layer l of the model, the server then creates a binary sparse mask tensor that is randomly initialized, with a fraction of 1s $\propto d_c^l$ (L10). In stage2, the server begins the training rounds starting with a sparse model initialization following the sparse mask computed in stage1 (L11). In particular, at each round, the clients perform sparse learning for E epochs (L21-26) with the provision to either train the mask or keep training on the weights with the mask frozen (L25). The later allows the clients to intermittently share masks saving up-link cost.

However, in FL settings, the masks often show poor convergence (section 3, Obs. 2). To address this, in stage 2, we present the idea of sparse FL with mask learnability either disabled or enabled (L9). The disabled scenario ($m_{freez} = 1$) essentially translates to sparse learning with pre-defined layer masks at initialization, allowing all the clients to have a forcefully convergent mask ($\texttt{sm}^t$ = 0 for all t). Moreover, as FLASH disentangles the sensitivity evaluation stage from the training, the pre-defined mask in this scenario benefits from the notion of layer sensitivity. We thus aptly name this scenario *sensitivity-driven pre-defined sparse training* (SPDST). Interestingly, earlier research Bibikar et al. (2021) hinted at poor model performance with pre-defined masks, contrasting ours where we see significantly improved model performance, implying the importance of stage1 (as will be elaborated in section 5).

In the enabled mask learning scenario ($m_{freez} = 0$), model masks and weights are jointly trained during clients' local learning, thus termed as *joint mask weight sparse training* (JMWST). However, as highlighted earlier, clients' naive sparse mask selection at the beginning of each round costs a considerable accuracy drop (section 3 Obs. 1). JMWST allows the server to select a sparse model for the clients at round t. For clients' target density d, the aggregated server model (L17) at the end of round $t-1$, generally has density $d_S > d$. To enable efficient sampling of sparse model, we leverage the density re-calibration strategy (Eq. 2) by taking the $(t-1)^{th}$ round's clients' sensitivity in to consideration. We then perform magnitude pruning to retain the top-d_c^l fraction of parameters for l^{th} layer at the server and send the pruned model to clients at round t (L18). The clients then perform local sparse learning, yielding another set of sparse models and so on. Note, the aggregation and sampling is simpler in SPDST, as the server model always remains at density d. In terms of yielding convergent masks, we indeed observed a lower SM for JMWST by $\sim 85\%$ compared to that in NST, evaluated after 300 rounds on CIFAR-10. Algorithm 1 details the FLASH training methods.

[3]which is same as sensitivity for a layer.

Algorithm 1: FLASH Training.

Data: Training rounds T, local epochs E, client set $[\mathcal{C}_N]$, clients per rounds c_r, target density d , sensitivity warm-up epochs E_d, density warm up client count c_d, initial value of freeze masks $m_{freez} = 0$, training algorithm t_{alg} and Aggregation type t_{aggr}.

$\mathcal{M}^{init} \leftarrow$ `createRandomMask`(d)
$\Theta^{init} \leftarrow$ `initMaskedWeight`$(\mathcal{M}^{init})$
`serverExecute`:
Calculate the layer-wise sensitivity in stage 1
Randomly sample c_d clients $[\mathcal{C}_d] \subset [\mathcal{C}_N]$
for *each client* $c \in [\mathcal{C}_d]$ ***in parallel*** **do**
  $\Theta_c \leftarrow$ `clientExecute`$(\Theta^{init}, E_d, 0)$ # m_freeze = 0
  $\mathcal{S}_c \leftarrow$ `computeSensitivity`(Θ_c)
end
Initialize a sensitivity-driven mask
$\mathcal{M}^0 \leftarrow$ `initMask`$([\mathcal{S}_c], d)$
$\Theta^0 \leftarrow$ `initMaskedWeight`$(\mathcal{M}^0)$
$m_{freez} \leftarrow$ `freezeMask`(t_{alg})
# Start Stage 2
for *each round* $t \leftarrow 1$ **to** T **do**
  Randomly sample c_r clients $[\mathcal{C}_r] \subset [\mathcal{C}_N]$
  for *each client* $c \in [\mathcal{C}_r]$ ***in parallel*** **do**
    $\Theta_c^t \leftarrow$ `clientExecute`$(\Theta^{t-1}, E, m_{freez})$
  end
  $\Theta_S^t \leftarrow$ `aggrParamUpdateMask`$([\Theta_c^t], t_{aggr})$
  $\Theta^t \leftarrow$ `subsampleServerModel`$(\Theta_S^t, [\Theta_c^t], d, m_{freez})$
end
`clientExecute`(Θ_c, E, m_{freez}) :
$\Theta_{c^0} \leftarrow \Theta_c$
for *local epoch* $i \leftarrow 1$ **to** E **do**
  $\Theta_{c^i} \leftarrow$ `doSparseLearning`$(\Theta_{c^{i-1}}, m_{freez})$
  $m_{freez} \leftarrow$ `checkUpdateMask`$()$
end
return Θ_{c^E}

Extension to support heterogeneous parameter density. To support different density budgets for different clients, we now present hetero-FLASH. Let us assume a total of N support densities $d_{set} = [d_1, .., d_M]$, where $d_i < d_{i+1}$. Now, for hetero-SPDST, we perform a sensitivity warm-up, to create the masks for the clients' with the highest density d_N. For any other density d_i, we sample a sparse mask from that with density d_{i+1}. Note, while creating the mask from d_{i+1} to d_i, we follow the layer-wise density re-calibration approach as mentioned earlier. For hetero-JMWST, at the beginning of each round, the server performs magnitude pruning to yield N sub models meeting N different density levels, contrasting to the creation of one model in JMWST. Participating clients of different densities use the corresponding sub models to start their local sparse training. In hetero-FLASH, server performs aggregation by following a form of *weighted fed averaging* (WFA). In particular, with similar inspiration as Diao et al. (2020), to give equal importance to each parameter update in such heterogeneous settings, WFA averages the values by their number of non-zero occurrences among the participating clients. For brevity, we have provided the algorithm for hetero-FLASH in the supplementary.

5 EXPERIMENTS

Datasets and Models. We evaluated the performance of FLASH on MNISTLeCun & Cortes (2010), Federated EMNIST (FEMNIST) Caldas et al. (2018a), and CIFAR-10 Krizhevsky et al. (2009) datasets with the CNN models described in McMahan et al. (2017), Caldas et al. (2018a), and ResNet18, respectively. Further model details are provided in the Supplementary. For data partitioning of MNIST and CIFAR-10, we use Latent Dirichlet Allocation (LDA)Reddi et al. (2020)

Table 3: Results with FLASH (SPDST, and JMWST) and its comparison with NST and PDST.

Dataset	Data Distribution	Density (d)	Baseline Acc %	NST Acc %	PDST Acc %	SPDST Acc %	JMWST Acc %
MNIST	IID ($\alpha = 1000$)	1.0	98.79 ± 0.06	–	–	–	–
		0.1	–	97.57 ± 0.11	97.09 ± 0.18	$\mathbf{98.21 \pm 0.06}$	97.95 ± 0.16
		0.05	–	95.19 ± 0.56	94.8 ± 1.04	$\mathbf{97.46 \pm 0.14}$	97.24 ± 0.21
	non-IID ($\alpha = 1.0$)	1.0	98.76 ± 0.06	–	–	–	–
		0.1	–	97.36 ± 0.19	96.82 ± 0.25	$\mathbf{97.96 \pm 0.13}$	97.72 ± 0.12
		0.05	–	95.75 ± 0.31	95.34 ± 0.77	$\mathbf{97.3 \pm 0.26}$	$\mathbf{97.38 \pm 0.11}$
	non-IID ($\alpha = 0.1$)	1.0	98.45 ± 0.17	–	–	–	–
		0.1	–	96.19 ± 0.22	94.41 ± 1.23	$\mathbf{97.22 \pm 0.43}$	96.53 ± 0.19
		0.05	–	91.66 ± 1.74	91.06 ± 1.1	95.7 ± 0.37	$\mathbf{95.91 \pm 0.84}$
CIFAR-10	IID ($\alpha = 1000$)	1.0	88.56 ± 0.06	–	–	–	–
		0.1	–	84.89 ± 0.26	86.72 ± 0.09	$\mathbf{88 \pm 0.28}$	87.62 ± 0.35
		0.05	–	77.48 ± 0.54	84.38 ± 0.12	$\mathbf{86.99 \pm 0.14}$	86.87 ± 0.08
	non-IID ($\alpha = 1.0$)	1.0	87.13 ± 0.18	–	–	–	–
		0.1	–	83.46 ± 0.19	85.07 ± 0.24	$\mathbf{86.42 \pm 0.49}$	$\mathbf{86.45 \pm 0.31}$
		0.05	–	75.1 ± 0.76	83.33 ± 0.14	$\mathbf{85.64 \pm 0.58}$	85.34 ± 0.27
	non-IID ($\alpha = 0.1$)	1.0	77.64 ± 0.49	–	–	–	–
		0.1	–	71.18 ± 1.23	74.82 ± 0.72	$\mathbf{76.74 \pm 1.46}$	74.74 ± 1.07
		0.05	–	61.29 ± 2.76	72.32 ± 1.05	$\mathbf{75.47 \pm 2.31}$	73.9 ± 1.45
FEMNIST	non-IID	1.0	84.68 ± 0.20	–	–	–	–
		0.1	–	76.92 ± 0.42	76.01 ± 1.26	82.70 ± 0.26	$\mathbf{83.02 \pm 0.21}$
		0.05	–	61.9 ± 2.6	63.65 ± 0.86	81.18 ± 0.36	$\mathbf{82.01 \pm 0.53}$

with three different α ($\alpha = 1000$ for IID and $\alpha = 1$ and 0.1 for non-IID). For FEMNIST, we employ the same setting as in Han et al. (2020), which partitions the data based on the writer into 3400 clients, making it inherently non-IID.

Training Hyperparameters. We use Clients' starting learning rate (η_{init}) as 0.1 that is exponentially decayed to 0.001 (η_{end}) at the end of training. Specifically, learning rate for participants at round t is $\eta_t = \eta_{init}(\texttt{exp}(\frac{t}{T}\texttt{log}(\frac{\eta_{init}}{\eta_{end}})))$. In all the sparse learning experiments, prune rate is set to 0.25[4]. Summary of the rest of the training hyperparameters can be found in 1. Furthermore, all the experiments were performed with three different seeds. We report the final results as the averaged accuracy with corresponding std deviation in the tables.

5.1 Experimental Results with FLASH

To understand the importance of stage1 in FLASH methodology, we identify a baseline training with uniform layer sensitivity driven *pre-defined sparse training* (PDST) in FL. Table 3 details the performance of FLASH at different levels of d, for various choices of sparse learning methods. In particular, as we can see in Table 3 column 5 and 6, the performance of both NST and PDST produced models cost heavy accuracy drop at ultra low parameter density $d = 0.05$. For example, on CIFAR-10 ($\alpha = 0.1$), models from NST and PDST sacrifice an accuracy of 16.35% and 5.32%, respectively. However, at comparatively higher density ($d = 0.1$), both can yield models with a lower accuracy difference from the baseline by around 6.46% and 2.82%. SPDST, on the other hand, can maintain **close to the baseline accuracy** at even ultra-low density for all data partitions. Interestingly, for majority of the cases, it even outperforms JMWST yielded models. *These results clearly highlight the efficacy of both sensitivity driven spare learning (as SPDST > PDST) and early mask convergence (as SPDST ≈ or > JMWST) in FL settings*. Fig. 4 shows the acc. vs. round comparison among the two proposed methods on different data distributions. Moreover, as in SPDST, the clients' do not need to send the mask at all, *allowing us to yield proportional communication advantage as the model sparsity*.

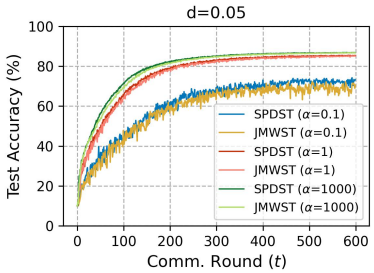

Figure 4: Test accuracy vs. round for different approaches on CIFAR-10.

Comparison with ZeroFL. Despite leveraging a form of sparse learning Raihan & Aamodt (2020), ZeroFL required significantly higher up-link/down-link communication cost compared to the target density d. This enables FLASH to gain a significant advantage in communication saving over ZeroFL, particularly for SPDST, as it only asks for the reduced size parameters to be communicated between the server and clients. In particular, we evaluate the communication saving as the ratio of the dense model size and corresponding sparse model size with the tensors represented in compressed sparse

[4]Prune rate controls the fraction of non-zero weights participating in the redistribution during sparse learning.

Table 4: Comparison of ZeroFL on various performance metrics with existing alternative sparse federated learning schemes.

Dataset	Data Distribution	Method	Density	Acc%	Down-link Savings	Up-link Savings
CIFAR-10	IID	ZeroFL Qiu et al. (2021)	0.1	82.71 ± 0.37	$1\times$	$1.6\times$
		FLASH (ours)	0.1	$\mathbf{88 \pm 0.28}$	$\mathbf{9.8}\times$	$\mathbf{9.8}\times$
		ZeroFL Qiu et al. (2021)	0.05	78.22 ± 0.35	$1\times$	$1.9\times$
		FLASH (ours)	0.05	$\mathbf{86.99 \pm 0.14}$	$\mathbf{19.5}\times$	$\mathbf{19.5}\times$
	non-IID ($\alpha = 1.0$)	ZeroFL Qiu et al. (2021)	0.1	81.04 ± 0.28	$1\times$	$1.6\times$
		FLASH (ours)	0.1	$\mathbf{86.42 \pm 0.49}$	$\mathbf{9.8}\times$	$\mathbf{9.8}\times$
		ZeroFL Qiu et al. (2021)	0.05	75.54 ± 1.15	$1\times$	$1.9\times$
		FLASH (ours)	0.05	$\mathbf{85.64 \pm 0.58}$	$\mathbf{19.5}\times$	$\mathbf{19.5}\times$
FEMNIST	non-IID	ZeroFL Qiu et al. (2021)	0.05	77.16 ± 2.07	$1\times$	$\mathbf{17.7}\times$
		FLASH (ours)	0.05	81.18 ± 0.36	$\mathbf{14.6}\times$	$14.6\times$
		FLASH (ours)	0.05	$\mathbf{82.01 \pm 0.53}$	$3.8\times$	$3.8\times$

Table 5: Performance of hetero-FLASH on various datasets where each client can have a density from the set $d_{set} \in [0.1, 0.15, 0.2]$.

Dataset	Data Distribution	Max Client Density (d_{set})	Hetero-SPDST Acc %	Hetero-JMWST Acc %
MNIST	IID ($\alpha = 1000$)	0.2	$\mathbf{98.29 \pm 0.05}$	97.44 ± 0.23
	non-IID ($\alpha = 1.0$)		$\mathbf{98.29 \pm 0.09}$	97.47 ± 0.22
	non-IID ($\alpha = 0.1$)		$\mathbf{97.63 \pm 0.22}$	96.11 ± 0.75
CIFAR-10	IID ($\alpha = 1000$)	0.2	$\mathbf{87.19 \pm 0.26}$	86.37 ± 0.2
	non-IID ($\alpha = 1.0$)		$\mathbf{86.16 \pm 0.04}$	84.67 ± 0.06
	non-IID ($\alpha = 0.1$)		$\mathbf{75.23 \pm 1.26}$	71.3 ± 2.75
FEMNIST	non-IID	0.2	$\mathbf{82.58 \pm 0.24}$	82.2 ± 0.42

row (CSR) format Tinney & Walker (1967). As depicted in Table 4[5], FLASH can yield an accuracy improvement of up to 10.1% at a reduced communication cost of up to $10.26\times$ (computed at up-link when both send sparse models).

5.2 Experimental Results with Hetero-FLASH

Table 5 shows the performance of hetero-FLASH where the clients can have three possible density budgets as defined by the d_{set}. We assume the maximum capacity clients' density budget of 0.2. To train on all the density values, we first create three sets, each having 40%, 30%, and 30% of total clients, and corresponds to density 0.2, 0.15, and 0.1, respectively. Now, during every round, we sample 10% from each set with corresponding target density. Similar to the trend in FLASH, hetero-SPDST outperforms the JMWST counter-parts by up to 5.93% evaluated on the three datasets.

5.3 Quantitative Analysis

Dependence of initial sensitivity warm-up on participating clients. To understand the importance of the clients' participation in the warm-up, we experimented with six different scenarios. In particular, we used two different values of participating clients ([10, 20]) each corresponding to three different local epoch choices ([10, 20, 40]). As shown in Fig. 5(a), the yielded pruning sensitivity follows a similar trend. Moreover, an SPDST training with mask chosen from any of these sensitivity lists finally yield FL models with similar performances (Fig. 5(b)), clearly demonstrating the robustness of our warm-up based sensitivity evaluation stage.

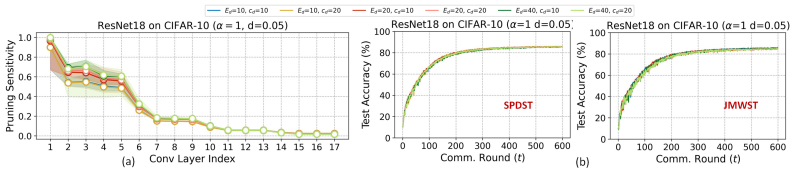

Figure 5: (a) Layer sensitivity evaluated at the end of sensitivity warm-up stage for different client participation size and their local epochs, (b) Comparison of server side model performance with the initialized sparse mask based on different sensitivity evaluated from (a).

[5]We understand for FEMNIST, ZeroFL reported significantly higher up-link saving, however, to the best of our understanding it should be similar to their report on other datasets, i.e. $\sim 1.9\times$.

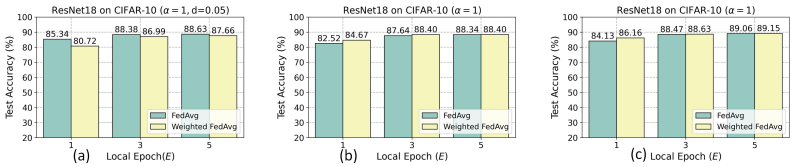

Figure 6: Performance comparison of fedavg with weighted fedavg for different local epoch choices, in (a) FLASH (JMWST), and hetero-FLASH for (b) JMWST and (c) SPDST.

Table 6: ResNet18($d = 0.05$) accuracy (%) on CIFAR-10($\alpha = 1.0$) at different up-link limits.

Method	4GB	8GB	12GB	16GB
NST	38.97	46.96	52.02	57.81
PDST	76.14	82.24	83.17	83.33
SPDST	**78.60**	**84.54**	**85.6**	**85.64**
JMWST	43.36	57.56	67.86	74.27
FedAvg (dense)	25.7	34.28	40.61	44.15

Table 7: ResNet18($d = 0.05$) accuracy (%) on CIFAR-10($\alpha = 0.1$) at different up-link limits.

Method	4GB	8GB	12GB	16GB
NST	26.31	35.89	40.28	43.94
PDST	62.32	70.57	72.29	72.32
SPDST	**63.62**	**73.13**	**75.47**	**75.47**
JMWST	32.4	43.21	51.24	55.59
FedAvg (dense)	16.97	25.91	29.89	34.74

Importance of parameters' weighted fed averaging at the server. Earlier literature Bibikar et al. (2021) suggested the use of weighted averaging, for sparse learning on pathologically non-IID data. Inspired by that, we now investigate the necessity of WFA in FLASH. In particular, we performed experiments on CIFAR-10 ($\alpha = 1.0$), both with and without WFA, during server aggregation. As shown in Fig. 6(a), WFA models performs consistently inferior to the fed averaged models. On the contrary, hetero-FLASH, enjoys consistent superior performance with WFA (Fig. 6(b-c)). Thus, further investigations on the utility and use case of such weighted averaging is an interesting future research direction.

Impact of local epoch count per round. Fig. 6 shows the model performances with three different local epoch scenarios ([1, 3, 5]). Specifically, for both FLASH and hetero-FLASH, increased local epochs consistently yield better accuracy, hinting at the better learning of the sparse model.

Performance at limited communication budget. Table 6 and 7 show the performance of FL models when the clients are communication limited. In particular, we see both PDST and SPDST can significantly outperform other approaches in yielding a significantly well-trained model at low comm. budget. This can be attributed to their significantly smaller model sizes, helping them to run for higher number of rounds than others, on a limited bandwidth scenario.

6 Conclusions

This paper presented federated lottery aware sparsity hunting methodologies to yield sparse server models with low parameter density while costing insignificant accuracy drop compared to the unpruned counterparts. In particular, we demonstrated two efficient sparse learning solutions specifically tailored for FL, enabling better computation and communication benefits over existing sparse learning alternatives. We experimentally demonstrated the superiority of our models in yielding up to $\sim 10.1\%$ improved accuracy with $\sim 10.26\times$ fewer communication costs, compared to the existing alternatives Qiu et al. (2021), at similar hyperparameter settings.

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A Appendix

A.1 Model Architectures

Table 8 shows the model architectures used for MNIST and FEMNIST datasets. For CIFAR-10 we used ResNet18 He et al. (2016) with the first CONV layer kernel size as 3×3 instead of original 7×7.

Table 8: Architecture used for MNIST and FEMNIST datasets

MNIST	**FEMNIST**
CONV$5 \times 5(C_o = 10)$	CONV$5 \times 5(C_o = 32)$
max_pool	max_pool
CONV$5 \times 5(C_o = 20)$	CONV$5 \times 5(C_o = 64)$
max_pool	max_pool
FC$(5120, 50)$	FC$(3136, 2048)$
FC$(50, 10)$	FC$(2028, 62)$

A.2 Hetero-FLASH Algorithm

Algorithm 2 details the training algorithm in hetero-FLASH. Note that and `aggrParamUpdateMask` and `subSampleServerModel` are the two functions that play key role in supoorting heterogeniety in sparsity ratios for different clients. The details of these two functions are elaborated in Algortihm 3 and 4, respectively. We plan to open-source our code upon acceptance of the paper.

A.3 More Quantitative Analysis

1. Ablation with the mask update interval rounds (r_{int}). As mentioned in the original manuscript, in the case of JMWST, to save communication energy we often can choose not to update the mask every round. We thus performed ablation with an increased frequency of mask update interval round r_{int} from the default value of 1 (similar to Qiu et al. (2021)). Table 9 shows the results with different r_{int}. In particular, as we can see in the table, less frequent update intervals does not degrade the final model performance. Moreover, a less frequent update can provide additional savings in terms of up-link cost for the models as the masks do not change every round.

Algorithm 2: Hetero-FLASH Training.

Data: Training rounds T, local epochs E, client set $[[\mathcal{C}_{N_1}], ..., [\mathcal{C}_{N_M}]]$, clients per rounds c_r, target density set d_{set} = $[d_1, ..., d_M]$, sensitivity warm-up epochs E_d, density warm up client count c_d, initial value of freeze masks $m_{freez} = 0$, training algorithm t_{alg} and aggregation type t_{aggr}.

$\mathcal{M}^{init} \leftarrow$ `createRandomMask()`
$\Theta^{init} \leftarrow$ `initMaskedWeight`$(\mathcal{M}^{init})$
`serverExecute`:
Randomly sample c_d clients $[\mathcal{C}_d] \subset [\mathcal{C}_{N_M}]$
for *each client* $c \in [\mathcal{C}_d]$ ***in parallel*** **do**
  $\Theta_c \leftarrow$ `clientExecute`$(\Theta^{init}, E_d, 0)$
  $\mathcal{S}_c \leftarrow$ `computeSensitivity`(Θ_c)
end
$\mathcal{M}^0 \leftarrow$ `initMask`$([\mathcal{S}_c], d_{set})$
$\Theta^0 \leftarrow$ `initMaskedWeight`$(\mathcal{M}^0)$
$m_{freez} \leftarrow$ `freezeMask`(t_{alg})
for *each round* $t \leftarrow 1$ **to** T **do**
  Randomly sample c_r clients $[\mathcal{C}_r] \subset [\mathcal{C}_N]$
  for *each client* $c \in [\mathcal{C}_r]$ ***in parallel*** **do**
    $\Theta_c^t \leftarrow$ `clientExecute`$(\Theta^{t-1}, E, m_{freez})$
  end
  $\Theta_S^t \leftarrow$ `aggrParamUpdateMask`$([\Theta_c^t], t_{aggr})$
  $\Theta^t \leftarrow$ `subSampleServerModel`$(\Theta_S^t, [\Theta_c^t], d_{set}, m_{freez})$
end
`clientExecute`(Θ_c, E, m_{freez}) :
$\Theta_{c^0} \leftarrow \Theta_c$
for *local epoch* $i \leftarrow 1$ **to** E **do**
  $\Theta_{c^i} \leftarrow$ `doSparseLearning`$(\Theta_{c^{i-1}}, m_{c^{freez}})$
  $m_{freez} \leftarrow$ `checkUpdateMask()`
end
return Θ_{c^E}

Algorithm 3: `aggrParamUpdateMask`

Data: Round t, aggregation type $t_{aggr} \in$ [`fedAvg`, `weightedFedAvg`], clients updates $[\Theta^t]$ = $[\Theta_{c_1}, ..., \Theta_{c_r}]$, client data size $[ds_{c_1}, ..., ds_{c_r}]$

if t_{aggr} *is* `fedAvg` **then**
  $\Theta_S^t \leftarrow \frac{1}{\Sigma_{c_i=1}^{c_r} ds_{c_i}} \Sigma_{c_i=1}^{c_r} ds_{c_i} \cdot \Theta_{c_i}^t$
else
  //For hetero-FLASH
  $\mathcal{W}^t \leftarrow$ `initWeightFactor()`
  for *each update* $\Theta_{c_i} \in [\Theta^t]$ **do**
    $\mathcal{W}_{c_i}^t \leftarrow ds_{c_i} \times$ `retrieveMask`(Θ_{c_i})
    $\mathcal{W}^t \leftarrow \mathcal{W}^t + \mathcal{W}_{c_i}^t$
  end
  //`safeDivide`(a,b): gives zero anywhere the b is queal to zero
  $\Theta_S^t \leftarrow \Sigma_{c_i=1}^{c_r}[$`safeDivide`$(\mathcal{W}_{c_i}^t, \mathcal{W}^t) \cdot \Theta_{c_i}^t]$
end

2. Impact of Number of participating clients per round. Fig. 7 (a), shows that JMWST and SPDST follow the same pattern at the baseline model ($d = 1.0$) with FedAvg. In other words, similar to FedAvg, as the c_r increases, the performance enhances. Also, for a specific c_r, JMWST and SPDST perform better than PDST and NST.

3. Impact of Batch-Normalization layer statistics. Fig 7 (b), shows the performance comparison between batch bormalization (BN) and static batch normalization (static BN, as suggested in Diao et al. (2020)). In particular in our setting, using BN layer statistics always outperform the static BN.

Algorithm 4: `subsampleServerModel`

Data: Current round id t, client set $[\mathcal{C}_r]$, aggregated Weight Θ_S^t of model with L layers, support density set $d_{set} = [d_1, ..., d_M]$ where $d_i < d_{i+1}$, model layer-wise parameter count $[k] = [k^1, ..., k^L]$.

if $size(d_{set})$ *is* 1 **then**
  //JMWST subsampling in FLASH
  $\mathcal{M} \leftarrow$ `initMaskWithZeros()`
  $[\hat{d}^1, ..., \hat{d}^L] \leftarrow$ `avgLayerWiseDensity`$([\mathcal{C}_r])$
  $r_f \leftarrow \frac{d_1 \times K}{\sum_{l=1}^{L} \hat{d}^l . k^l}$
  for *layer* $l \leftarrow 1$ **to** L **do**
    `idx` $\leftarrow$ `getSortedWeightIndeices`(Θ_S^t, l)
    $n_z \leftarrow$ `int`$(r_f \times \hat{d}^l \times k^l)$ //number of non-zeros
    $\mathcal{M}^l[\text{idx}[: n_z]] \leftarrow 1$
  end
else
  //For hetero-FLASH
  for $d_i \in d_{set}$ **do**
    $\mathcal{M}_i \leftarrow$ `initMaskWithZeros()`
  end
  $\mathcal{D}_s^t \leftarrow$ `getCurrentDensity`(Θ_S^t)
  $[\hat{d}^1, ..., \hat{d}^L] \leftarrow$ `getLayerWiseDensity`(Θ_S^t)
  for *layer* $l \leftarrow 1$ **to** L **do**
    `idx` $\leftarrow$ `getSortedWeightIndeices`(Θ_S^t, l)
    for $d_i \in d_{set}$ **do**
      $r_{f_i} \leftarrow \frac{d_i}{\mathcal{D}_s^t}$
      $n_z \leftarrow$ `int`$(r_{f_i} \times \hat{d}^l \times k^l)$
      ${\mathcal{M}_i}^l[\text{idx}[: n_z]] \leftarrow 1$
    end
  end
end

Table 9: Ablation with different mask update intervals for JMWST for a target density $d = 0.1$ on CIFAR-10.

Model	Data distribution	Mask update interval rounds (r_{int})		
		$r_{int} = 1$	$r_{int} = 2$	$r_{int} = 5$
ResNet18	IID ($\alpha = 1000$)	87.62 ± 0.35	87.76 ± 0.07	87.86 ± 0.12
	non-IID ($\alpha = 1$)	86.45 ± 0.31	86.26 ± 0.07	86.36 ± 0.13
	non-IID ($\alpha = 0.1$)	74.74 ± 1.07	73.73 ± 1.18	75.47 ± 1.18

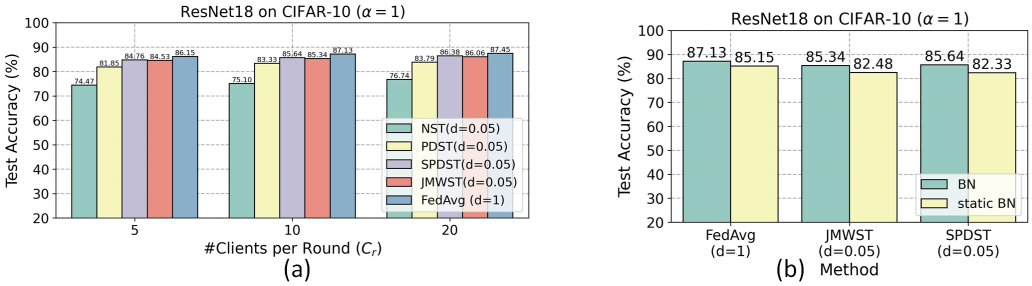

Figure 7: (a) Performance of the final trained model for different participating clients per round, (b) Significance of BN and Static BN in final model performance.

4. Revisiting sparse mask mismatch for FLASH. As demonstrated in Figs 8, the sparse mask mismatch in the case of JMWST significantly reduces, helping the mask train in a convergent way, significantly faster than that in NST.

Fig. 9 shows the layer-wise SM, for centralized trained model (Fig. 9a) and FL trained model with sparsity (Fig. 9b-c). In particular, the SM at later layer can significantly reduce in the case of JMWST as compared to NST, further demonstrating the convergence ability even at the later layers.

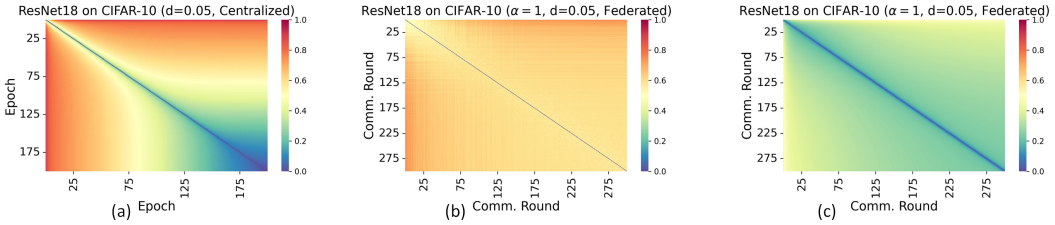

Figure 8: Sparse mask mismatch (SM) for (a) centralized sparse learning, (b) NST, and (c) JMWST in federated settings.

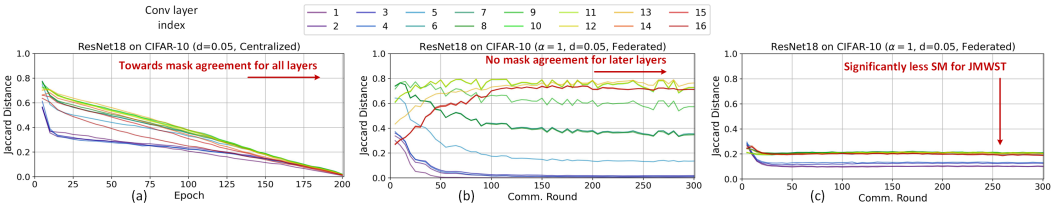

Figure 9: Layer-wise sparse mask mismatch (SM) vs. training epochs (rounds) plot for (a) centralized and (b) FL with NST, and (c) FL with JMWST.

5. Convergence trend of proposed algorithms. Fig. 10 shows the test accuracy vs FL rounds for NST, PDST, SPDST, and JMWST algorithms on CIFAR-10 dataset with non-IID data distribution ($\alpha = 1$). As shown in the plots, for $d = 0.05$ and $d = 0.1$, NST has slower convergence with lower final accuracy. Introducing consensus among the clients for the sparse mask accelerates the convergence and enhances the final performance.

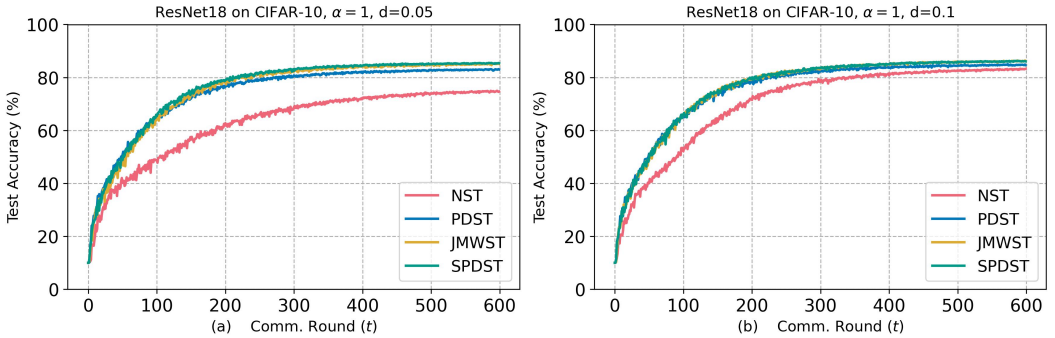

Figure 10: Performance of proposed algorithms vs. comm. rounds on CIFAR-10 dataset for (a) $d = 0.05$ (b) $d = 0.1$.

6. Communication saving and FLOPs evaluations. Employing sparse learning in FL helps participating clients reduce both the communication and compute costs (FLOPs) for training. Without the loss of generality, we now evaluate the convolutional layer training FLOPs for FLASH, and demonstrate the relation of parameter density d with FLOPs and communication saving. Let us assume a layer l has a weight tensor $\boldsymbol{\theta}^l \in \mathbb{R}^{C_o \times C_i \times h \times w}$, where h and w are the height and width of the convolutional kernel, and C_o and C_i represent the number of filters and channels per filter, respectively. It takes an input tensor $\boldsymbol{I} \in \mathbb{R}^{C_i \times H \times W}$ to produce an output tensor $\boldsymbol{O} \in \mathbb{R}^{C_o \times R \times S}$. Let us also assume d to be the density of non-zero in the weight tensor for all the layers. During training, FLOPs associated to each weight tensor update can be partitioned in to three component, namely, forward pass FLOPs ($FLOP_{fwd}$), backward input grad FLOPs ($FLOP_{back_in}$), and backward weight grad FLOPs ($FLOP_{back_wt}$). The weight sparsity helps both $FLOP_{fwd}$ and $FLOP_{back_in}$

to reduce proportionally as given below.

$$FLOP_{fwd} = d \times C_i \times (h \times w) \times (R \times S) \times C_o \tag{3}$$

$$FLOP_{back_in} = d \times C_i \times (h \times w) \times (H \times W) \times C_o \tag{4}$$

Finally, if the zero weights' gradients flow is computed for the purpose of mask learning, then $FLOP_{back_wt}$ can't leverage the advantage of low parameter density. Thus during mask training of JMWST, as the gradient needs to be dense, $FLOP_{back_wt}$ is same as that in dense computation. However, for SPDST, zero weights remain as zero, allowing us to safely skip the associated gradient computation. This essentially helps SPDST to extract benefits of sparsity during all the three stages of FLOPs computation. Following Eq. shows the $FLOP_{back_wt}$ in FLASH.

$$FLOP_{back_wt} = s_1 \times C_i \times (h \times w) \times (H \times W) \times C_o \tag{5}$$

where,

$$s1 = \begin{cases} 1, & \text{for JMWST} \\ d, & \text{for SPDST} \end{cases} \tag{6}$$

For similar density, clients in ZeroFL also enjoys similar benefits in $FLOP_{fwd}$ and $FLOP_{back_in}$. However, $FLOP_{back_wt}$ can be reduced only via introduction of activation sparsity, a. It is well surveyed in literature that having sparse activation density as low as parameter density may significantly impact model performance. Thus generally, $a > d$ that allows SPDST to enjoy a FLOPs benefit of $\frac{a}{d}$ for $FLOP_{back_wt}$.

These computations can be easily extended to linear layers. Also, we can safely ignore the FLOPs associated to the BN layers due to their negligible contribution to the total FLOPs.

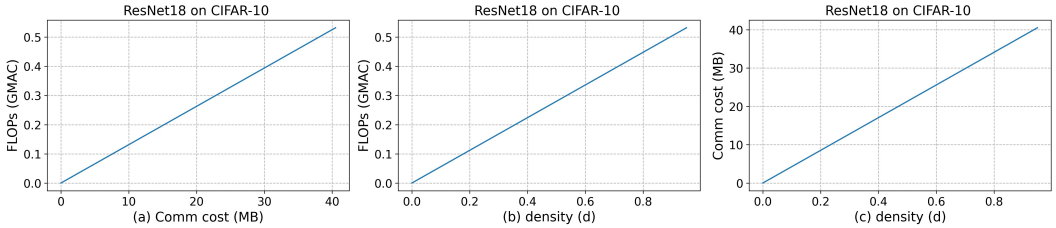

Figure 11: Computation and Communication relation with (a) each other (b, c) with different density levels for SPDST algorithm.

7. Discussion on compute benefits at the edge. To extract FLOPs benefits for irregular pruning in FLASH, we assume that the compute energy for the sparse network can be avoided via the means of clock-gating Yang & Kim (2018) of the zero-valued weights. Moreover, there has been recent development of sparsity-friendly DNN accelerators Qin et al. (2020) that can efficiently reduce the compute cost by a significant margin. Such accelerators can leverage the yielded sparse FL models to deploy at compute constrained edges.

8. FLOPs vs. communication cost for different density budget. To reach a target accuracy value, we now plot the FLOPs to uplink communication cost for different density budget.