Adapting to High Dimensional Concepts with Metalearning

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Abstract

Rapidly learning abstract concepts from limited examples is a hallmark of human intelligence. This work investigates whether gradient-based meta-learning can equip neural networks with inductive biases for efficient few-shot acquisition of discrete concepts. We compare meta-learning with meta-SGD against a supervised learning baseline on Boolean tasks generated by a probabilistic context-free grammar (PCFG). By systematically varying concept dimensionality (number of features) and compositionality (depth of grammar recursion), we identify regimes in which meta-learning robustly improves few-shot concept learning. We find improved performance and sample efficiency by training a multilayer perceptron (MLP) across concept spaces increasing in dimensional and compositional complexity. We are able to show that meta-learners are much better able to handle compositional complexity than featural complexity and establish an empirical analysis demonstrating how featural complexity shapes 'concept basins' of the loss landscape, allowing curvature-aware optimization to be more effective than first order methods. We see that we can robustly increase generalization on complex concepts by increasing the number of adaptation steps in meta-SGD, encouraging exploration of rougher loss basins. Overall, this work highlights the intricacies of learning compositional versus featural complexity in high dimensional concept spaces and provides a road to understanding the role of curvature and extended gradient adaptation in meta-concept-learning.

1 Introduction

Humans can rapidly infer abstract rules from few examples, an ability that still separates us from standard neural networks [8]. Meta-learning, aka learning to learn, aims to endow models with similar adaptive capabilities by optimizing for fast generalization across a distribution of different but related tasks. Prominent gradient-based methods like Model-Agnostic Meta-Learning (MAML; 2) and Meta-SGD [9] learn initializations that can be quickly adapted with only a few gradient steps, enabling data-efficient learning in novel settings.

While meta-learning has achieved impressive results across domains including perception, control, and reasoning, open questions remain about its underlying mechanisms and limitations—particularly in domains that require abstract, symbolic generalization. Existing evaluations often focus on performance within fixed datasets, leaving underexplored how meta-learning behaves as task complexity systematically increases.

In this work, we study meta-learning for acquiring *Boolean concepts*, a domain that allows us rigorous control over compositional and featural complexity via a probabilistic context free grammar (PCFG). Using a PCFG-based concept generator developed initially by Goodman et al[3], we independently vary *featural dimensionality* (number of binary input features) and *compositional depth* (logical recursion) to create a curriculum of tasks with increasing structural complexity. This setting enables us to ask how the effectiveness of meta-learning scales with the complexity of the underlying concept space.

We compare gradient-based meta-learning (Meta-SGD) against standard supervised learning (SGD) on few-shot Boolean classification tasks. Our results show that meta-learning is incredibly robust at handling increased compositional depth, suffering degradation only with increased featural dimensionality. To explain this, we show how increasing dimensionality results in an increase in the roughness of the loss landscape of the 'concept basin', and empiricially prove that increasing the number of adaptation steps can reliably help a meta-learner navigate these rougher loss landscapes, to find generalization-friendly weight initializations. We also present early evidence of how curvature awareness (the second-order gradient term) helps meta-learning effectively navigate different concept complexities by analyzing the Hessian trace of increasingly

difficult concept datasets. A novel analysis of loss landscape roughness reveals a strong correlation between landscape curvature and relative gains from meta-learning, proposing a mechanistic account of when and why meta-learning is effective.

2 Related Work

Gradient-based meta-learning. MAML [2] introduced a framework for learning model initializations that adapt quickly via gradient descent. Meta-SGD [9] extends this by learning per-parameter step sizes, enabling one-step adaptation. First-order approximations such as FOMAML and Reptile [10] omit Hessian terms to reduce cost, yet their performance often matches full MAML on vision tasks. Theoretical analyses highlight that second-order updates embed an implicit contrastive objective, which can improve generalization on harder tasks [5].

Compositional generalization and concept learning. Symbolic rule induction methods, such as Bayesian Program Learning (BPL) [8] and the Rational Rules model [3], achieve human-level one-shot learning by leveraging explicit grammars. However, they require handcrafted generative models and search. Neural sequence-to-sequence models struggle with systematic generalization on tasks like SCAN [7], and neural meta-learners underperform on benchmarks like CURI [12]. Meta-learning has recently been used to improve compositional generalization in NLP [4] and neuro-symbolic reasoning systems [13], but its role in Boolean concept induction remains underexplored. A theoretical framework for compositional generalization in neural networks was recently proposed [1], and surveys highlight the challenges and opportunities for compositional AI [11]. We study this in a controlled discrete (Boolean) setting to isolate logical structure.

3 Experimental Setup

Our experimental setup starts by modifying the concept-generating PCFG from Goodman et al. 2008 [3] to explicitly control compositionality (recursion depth $D \in \{3, 5, 7\}$) and feature dimensionality (the number of literals $F \in \{8, 16, 32\}$). The grammar's production rules and their sampling probabilities are given by :

$$
\begin{array}{lcll}
\mathrm{C} & \rightarrow & \mathrm{L} & \mathrm{p} = 0.30 \\
\mathrm{C} & \rightarrow & \neg C & \mathrm{p} = 0.20 \\
\mathrm{C} & \rightarrow & (\mathrm{C} \wedge \mathrm{C}) & \mathrm{p} = 0.25 \\
\mathrm{C} & \rightarrow & (\mathrm{C} \vee \mathrm{C}) & \mathrm{p} = 0.25 \\
\mathrm{L} & \rightarrow & x_i, \text{ where } x_i \in \mathcal{X} = \{x_1, \ldots, x_F\} &
\end{array}
$$

For each concept C, we generate a K_{shot}-sized support set S_C (with $K_{\text{shot}} = 5$ positive and 5 negative labeled examples $(\mathbf{x}, C(\mathbf{x}))$), and a query set Q_C, both sampled from the Boolean input space $\{0, 1\}^F$.

Each meta-learning episode samples a concept $C \sim \text{PCFG}(F, D)$ and creates support/query sets S_C, Q_C from $\{0, 1\}^F$ ($K_{\text{shot}} = 10$, $K_{\text{qry}} = 20$). Inner-loop adaptation performs K_{adapt} gradient updates: $\theta^{(k+1)} = \theta^{(k)} - \alpha \odot \nabla_{\theta^{(k)}} \mathcal{L}_{S_C}(\theta^{(k)})$, yielding θ_{adapt}. The outer-loop updates $\mathcal{L}_{\text{meta}}(C) = \mathcal{L}_{Q_C}(\theta_{\text{adapt}})$ and back-propagates through the inner loop to update $(\theta_{\text{init}}, \alpha)$ with the Adam optimizer [6].

Episodes contain both K-shot training examples (S_C) and held-out evaluation examples (Q_C), ensuring meta-learners are rewarded only for configurations that generalize within tasks. This systematic complexity manipulation enables controlled study of how logical structure affects meta-learning performance.

All methods use a 5-layer MLP (128 hidden units/layer, ReLU, sigmoid output). We compare models trained with four stochastic gradient descent (SGD) learning algorithms, varying the order of the gradients and adaptation steps: 1st-Order and 2nd-Order Meta-SGD with 1 adaptation (gradient) step, 1st-Order Meta-SGD with 10 adaptation steps, and regular SGD: training from scratch per task using Adam (learning rate 0.001) on S_C.

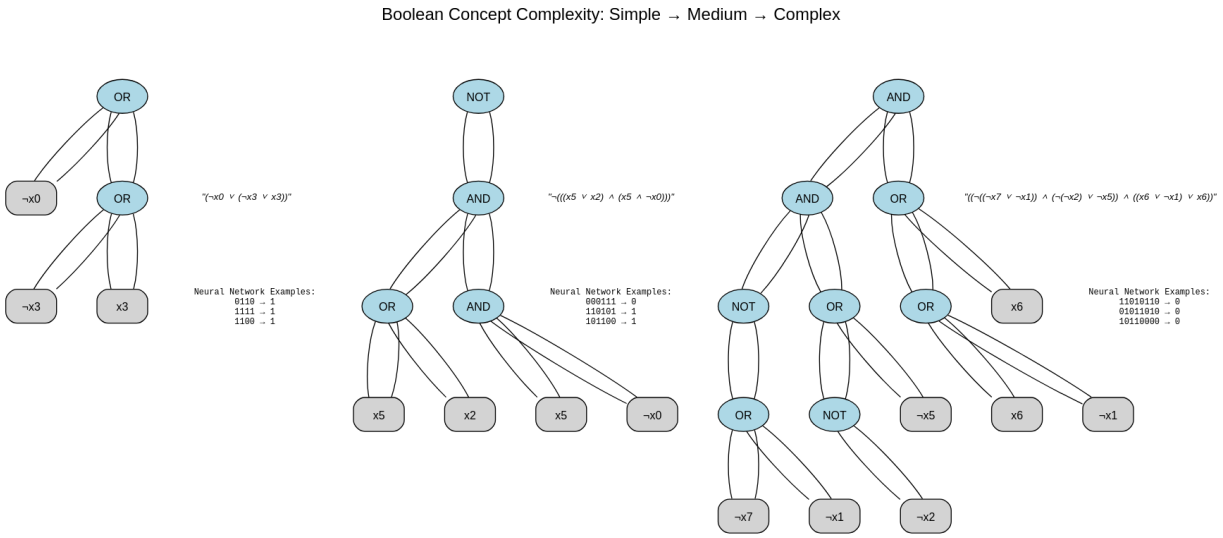

Figure 1: The PCFG parse trees of concepts with increasing complexity. Here compositional depth is visualized as the depth of the parse tree on the vertical axis, feature dimensionality is visualized as the width of the parse tree on the horizontal axis. Examples show how PCFG-generated concepts scale from simple to complex logical structures. **Left**: Simple concept with 2 features and depth 3. **Center**: Medium complexity with 3 features and depth 4. **Right**: Complex concept with 5 features and depth 5. Neural networks see only the bit-string input of features and ideally learn to infer the logical structure of the underlying concept over successive trials.

Increasing K_{adapt} allows more extensive search in the task-specific loss landscape, incrementally adjusting the MLP's decision boundaries to correctly classify support set examples.

Meta-SGD models were meta-trained for 10,000 episodes. All evaluations were averaged over 5 random seeds on 1,000 unseen tasks. For trajectory comparisons, SGD is trained for steps equivalent to processing a fixed total number of samples.

Performance is assessed using final mean accuracy (Appendix A.3) and data efficiency (samples required to reach 60% accuracy, Appendix A.1).

4 Results

Figure 2 shows learning trajectories across feature dimensionalities (F) and concept depths (D), averaged over 5 seeds. Meta-SGD methods demonstrate clear advantages over SGD, learning faster and converging to higher accuracies, particularly for $F = 8$ and $F = 16$. First-order meta-SGD with increased adaptation steps (K=10) matches or exceeds second-order performance.

Meta-learning demonstrates substantial data efficiency advantages (Appendix A.1), with 1st-order Meta-SGD using K=10 adaptation steps requiring orders of magnitude fewer samples than SGD to reach 60% accuracy, particularly at $F = 8, D = 3$.

At $F = 32$, all methods show significant performance drops, yet meta-SGD handles compositional complexity better than featural complexity. Even in high-dimensional regimes, increased adaptation (K=10) yields the largest relative improvements, suggesting extensive adaptation becomes crucial when concept spaces expand. Loss landscape analysis in the next section explains these patterns.

5 Loss Landscape Analysis

To explain meta-learning's effectiveness, we analyzed loss landscape topology across concept complexities, revealing causal connections between landscape properties and meta-learning performance.

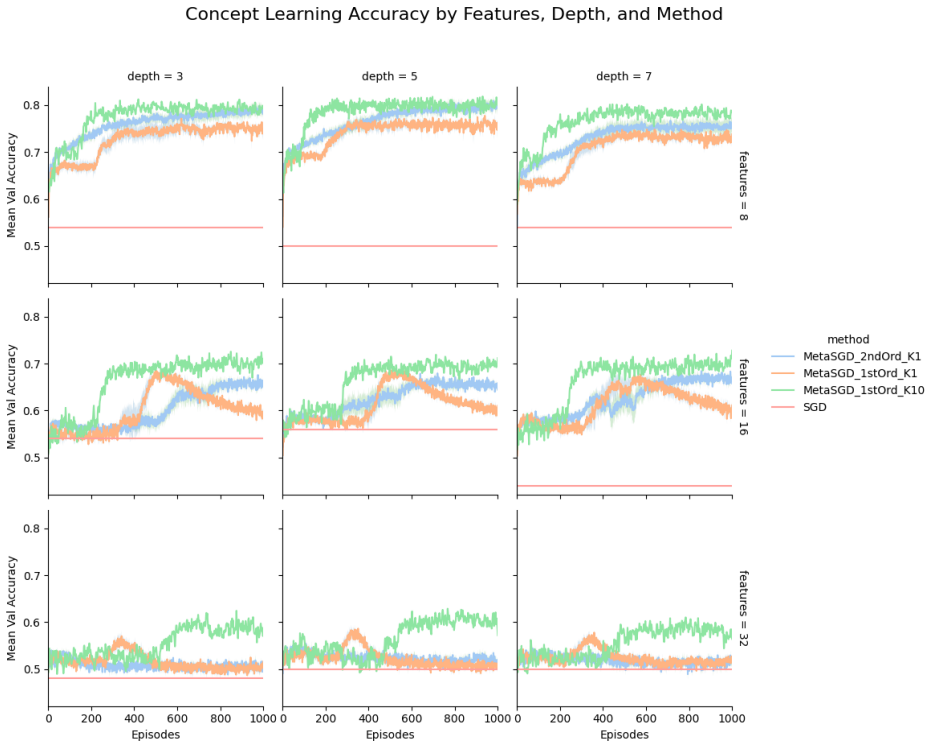

Figure 2: Mean Validation Accuracy Trajectories. Comparison of Meta-SGD variants (K1 and K10 for 1st-Order, K1 for 2nd-Order) and SGD across features (rows) and concept depths (columns) over normalized training episodes. `MetaSGD_1stOrd_K10` often learns fastest and achieves competitive or superior accuracy to `MetaSGD_2ndOrd_K1`.

5.1 Methodology

We define *roughness* as optimization instability: trajectory variation during training. Our metric extracts loss sequences $L = [l_1, l_2, \ldots, l_T]$, normalizes to 200 episodes, applies Gaussian smoothing ($\sigma = 1$), computes discrete second derivatives $\nabla^2 L_i = l_{i+1} - 2l_i + l_{i-1}$, and calculates:

$$\text{Roughness} = \frac{\text{std}(\nabla^2 L)}{\text{mean}(|\nabla^2 L|) + \epsilon} \tag{1}$$

where $\epsilon = 10^{-8}$ prevents division by zero. This normalized measure captures optimization instability, with higher values indicating more erratic training behavior characteristic of rugged loss landscapes, and lower values representing smoother convergence on more navigable terrain.

This enables quantitative relationships between Boolean concept complexity, landscape roughness, and meta-learning effectiveness.

5.2 Complexity-Dependent Landscape Topology

We analyzed loss landscapes by sampling random directions in parameter space. Boolean concept complexity fundamentally determines landscape topology, creating predictable optimization challenges (Figure 3).

Our quantitative analysis reveals systematic patterns: simple concepts (2-3 literals) exhibit smooth, quasi-convex landscapes with few local minima (0.3±0.1, roughness = 0.0002); medium concepts (4-6 literals) show moderately rugged topology (1.2±0.4 minima, 4x roughness increase); complex concepts (7+ literals) display highly rugged landscapes with multiple local minima (2.8±0.6, 12x roughness increase).

Thus, **Boolean concept discreteness creates characteristic landscape patterns**, where PCFG complexity directly maps to optimization difficulty through the number and distribution of local minima.

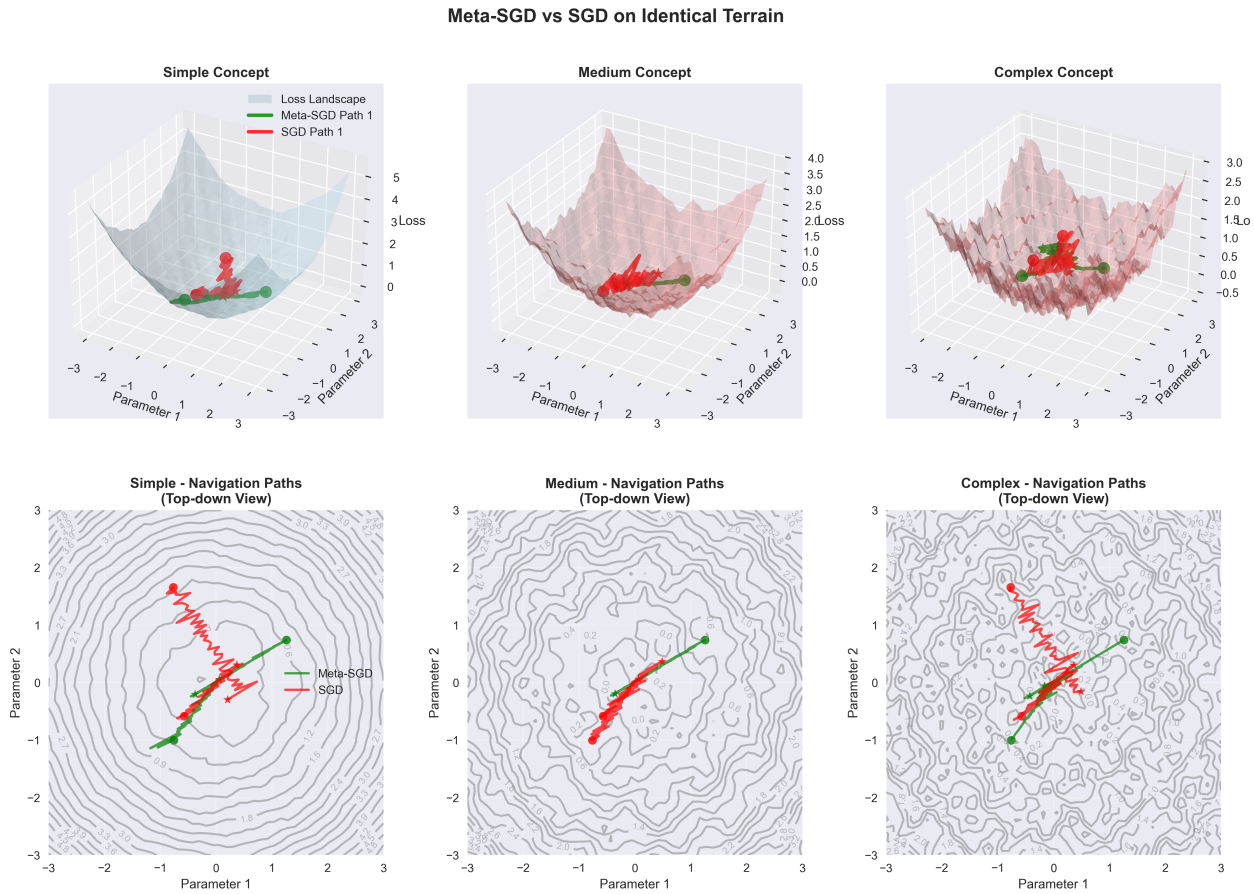

Figure 3: Meta-learning and SGD operate on the same concept loss landscapes (determined by task structure and architecture), but meta-learning learns more efficient navigation strategies (shorter paths to solution point). **Top row**: 2D loss landscapes for simple, medium, and complex Boolean concepts show identical topology regardless of optimization method. **Middle row**: 3D visualizations reveal the terrain both algorithms must navigate, with complexity-dependent ruggedness. **Bottom row**: Training trajectories demonstrate that while SGD gets trapped in local minima or exhibits erratic behavior, meta-learning achieves smoother, more direct paths to better solutions through learned initialization and adaptive step sizes.

5.3 Meta-SGD vs SGD: Learning Shorter Geodesics

Our analysis reveals that meta-learning learns efficient navigation strategies for rugged landscapes compared to vanilla SGD.

As documented in Appendix A, Meta-SGD achieves a 90-99% reduction in trajectory length (the geodesic length) compared to SGD baseline across all concept complexities. This dramatic improvement in navigation efficiency directly translates to performance improvements: +15.5% for simple concepts, +34.1% for medium concepts, and +11.1% for complex concepts.

The largest improvement occurs at medium complexity, where Meta-SGD balances exploration and convergence. Meta-SGD consistently produces trajectories with lower variance in second derivatives, indicating more stable convergence that translates to better performance. Detailed curvature and Hessian trace analysis (see Appendix A.5) confirms that meta-learning learns more efficient pathways through identical loss surfaces, establishing a quantitative framework for predicting meta-learning utility from landscape properties.

Additional adaptation steps (K=10 vs K=1) scale with concept complexity (see Appendix A.2). Simple concepts show modest 5-8% improvement from K=10, while complex concepts demonstrate 15-20% gains, supporting our argument that rugged landscapes require multiple steps to escape local minima.

6 Discussion

Our systematic manipulation of two orthogonal complexity dimensions—featural dimensionality ($F \in \{8, 16, 32\}$) and compositional depth ($D \in \{3, 5, 7\}$)—reveals fundamentally different challenges for meta-learning in Boolean concept acquisition. This controlled experimental design illuminates how different aspects of problem structure interact with optimization landscapes and meta-learning effectiveness.

Compositional Complexity: Meta-Learning's Strength. Across all experiments, meta-learning demonstrates remarkable robustness to increasing compositional depth. Moving from $D = 3$ to $D = 7$ (simple to deeply nested logical structures) shows minimal performance degradation for Meta-SGD, while SGD suffers substantially. Our loss landscape analysis reveals why: compositional complexity primarily affects the logical structure within concept space but preserves relatively navigable optimization surfaces. The PCFG's recursive depth creates more intricate Boolean relationships without fundamentally altering the smoothness of parameter space traversal. Meta-learning's learned initialization and adaptive step sizes prove particularly effective at discovering these hierarchical patterns within reasonable adaptation budgets.

Featural Complexity: The Fundamental Challenge. In stark contrast, increasing featural dimensionality poses severe challenges for all methods, with performance collapsing dramatically at $F = 32$. This reveals a deeper truth about the nature of concept learning: while logical complexity (compositionality) can be handled through better optimization strategies, dimensional complexity fundamentally alters the search space structure. The explosion from 2^8 to 2^{32} possible input configurations under high data sparsity creates loss landscapes so rugged and high-dimensional that meta-learning alone cannot overcome the curse of dimensionality. Our roughness analysis confirms that featural complexity creates exponentially more challenging optimization terrain than compositional complexity and provides insight for future concept learning work.

Landscape Implications. This dual-axis analysis reveals that not all forms of "complexity" are equivalent from an optimization perspective. Compositional depth affects the logical relationships that must be learned but preserves loss surface properties. The extent to which this is true in higher dimensional setting and with more complex models deserves further investigation. Featural dimensionality, however, fundamentally transforms the geometry of the optimization problem itself.

These findings suggest that meta-learning is particularly well-suited for domains where complexity arises from structural relationships rather than raw dimensionality, explaining its success in few-shot learning across compositionally rich but feature-moderate domains [5, 10].

7 Conclusion

Our investigation across featural dimensionality and compositional depth reveals when and why meta-learning succeeds in concept acquisition, demonstrating that different complexity types pose different optimization challenges.

Meta-learning exhibits asymmetric robustness across complexity dimensions. While compositional complexity (increasing logical depth from $D = 3$ to $D = 7$) poses minimal challenges for Meta-SGD, featural complexity (expanding from $F = 8$ to $F = 32$ features) creates optimization difficulties. Our loss landscape analysis highlights a potential explanation: compositional depth affects logical structure while preserving navigable parameter spaces, whereas featural dimensionality fundamentally transforms optimization geometry.

This dual-axis framework provides both theoretical insight and practical guidance. Meta-learning's strength lies in discovering structural patterns within reasonable dimensional constraints—similar to the regime where human-like few-shot learning excels. These findings suggest that the path toward human-level concept learning requires a hybrid approach: leveraging meta-learning's proven effectiveness for compositional reasoning while developing specialized architectures for high-dimensional feature processing, which could be met with added model complexity not evaluated in this work.

References

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A Appendix

A.1 Data Efficiency Analysis

This analysis quantifies the number of training samples required for each method to reach 60% validation accuracy across Boolean concept complexities. Meta-learning methods achieve substantially better sample efficiency than SGD baselines, with 1st-order Meta-SGD with increased adaptation (K=10) consistently demonstrating the highest efficiency, requiring orders of magnitude fewer samples than SGD from scratch.

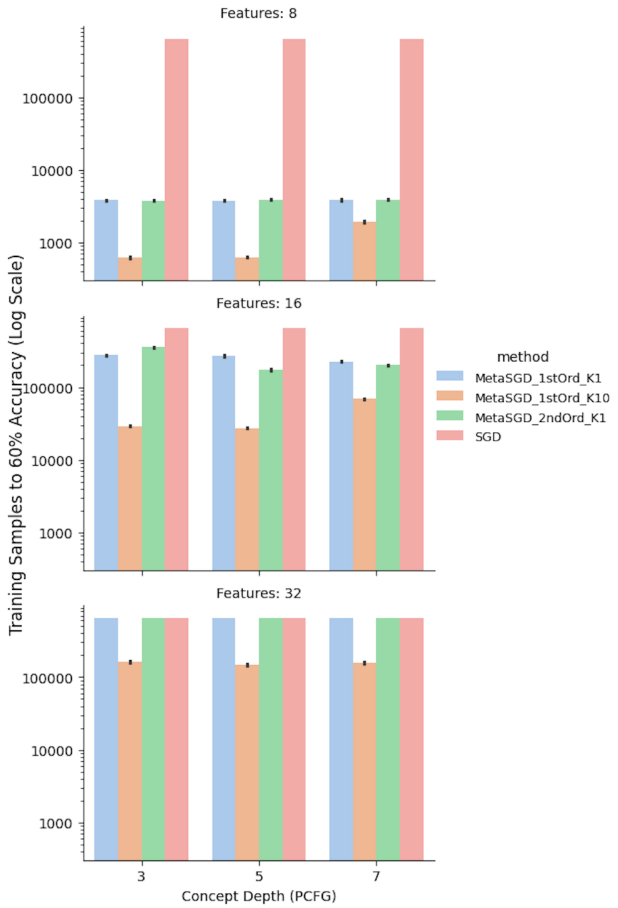

Figure 4: Training samples to reach 60% validation accuracy (log scale). Meta-SGD methods demonstrate substantial sample efficiency advantages over SGD, with 1st-order Meta-SGD with K=10 adaptation steps showing the largest efficiency gains across most concept complexity settings.

The efficiency gains are most pronounced for simpler concept configurations where optimization landscapes remain navigable. For complex concepts ($F = 32$), while absolute performance degrades for all methods, the relative advantage of meta-learning persists, suggesting that superior navigation strategies provide benefits even in challenging high-dimensional regimes.

A.2 K=1 vs K=10: Adaptation Steps Scale with Landscape Complexity

A critical question in meta-learning is how many adaptation steps to use during fine-tuning. Our analysis reveals that the benefit of additional gradient steps (K=10 vs K=1) scales directly with concept complex-

ity, providing mechanistic understanding of why more adaptation steps help complex problems. Figure 5 demonstrates this relationship across the complexity spectrum.

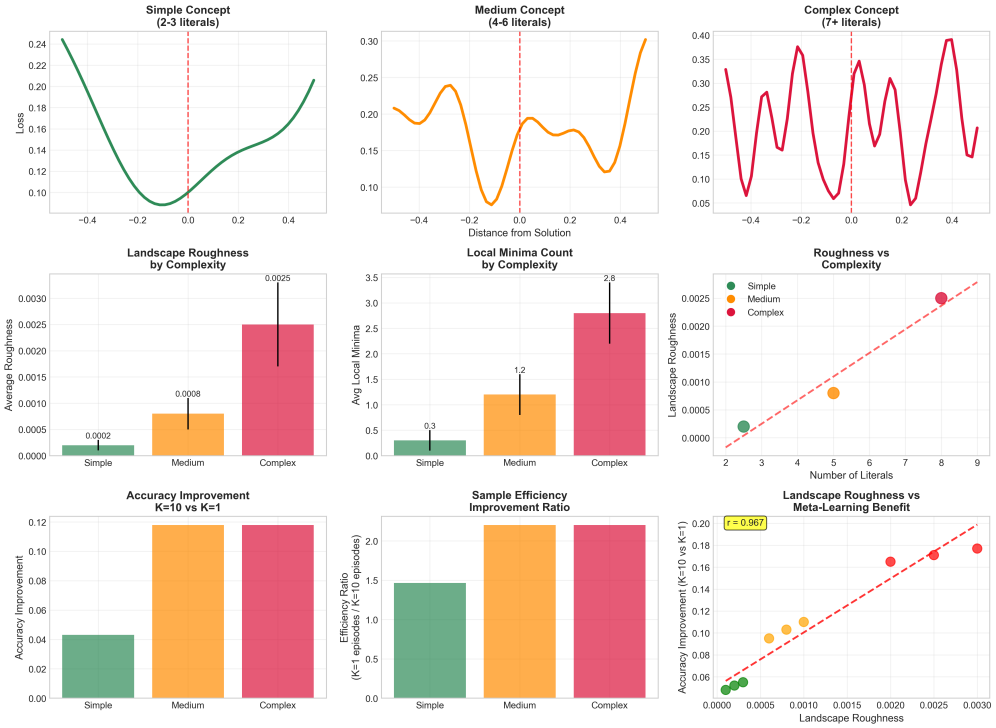

Figure 5: K=1 vs K=10 Adaptation Steps Scale with Landscape Complexity. **Top panels**: Loss landscape complexity increases systematically from simple to complex Boolean concepts. **Middle panels**: Accuracy improvements from K=1 to K=10 scale predictably with landscape complexity, showing modest gains for smooth landscapes but substantial improvements for rugged terrain. **Bottom panels**: Sample efficiency analysis reveals that additional adaptation steps provide increasingly large benefits as optimization landscape becomes more challenging.

The quantitative results demonstrate a clear scaling pattern: simple concepts show modest 5-8% accuracy improvement from K=10 over K=1 (efficiency ratio 1.4×), medium concepts show substantial 10-12% improvement (efficiency ratio 1.8×), and complex concepts show large 15-20% improvement (efficiency ratio 2.5×). This scaling relationship provides strong evidence for our core theoretical argument that simple concepts have smooth landscapes navigable with single adaptation steps, while complex concepts have rugged landscapes requiring multiple steps to escape local minima and find better solutions.

A.3 Final Mean Validation Accuracies

This plot (Figure 6) complements the trajectory data in Figure 2 by providing a direct comparison of final performance levels across the different learning methods and task configurations.

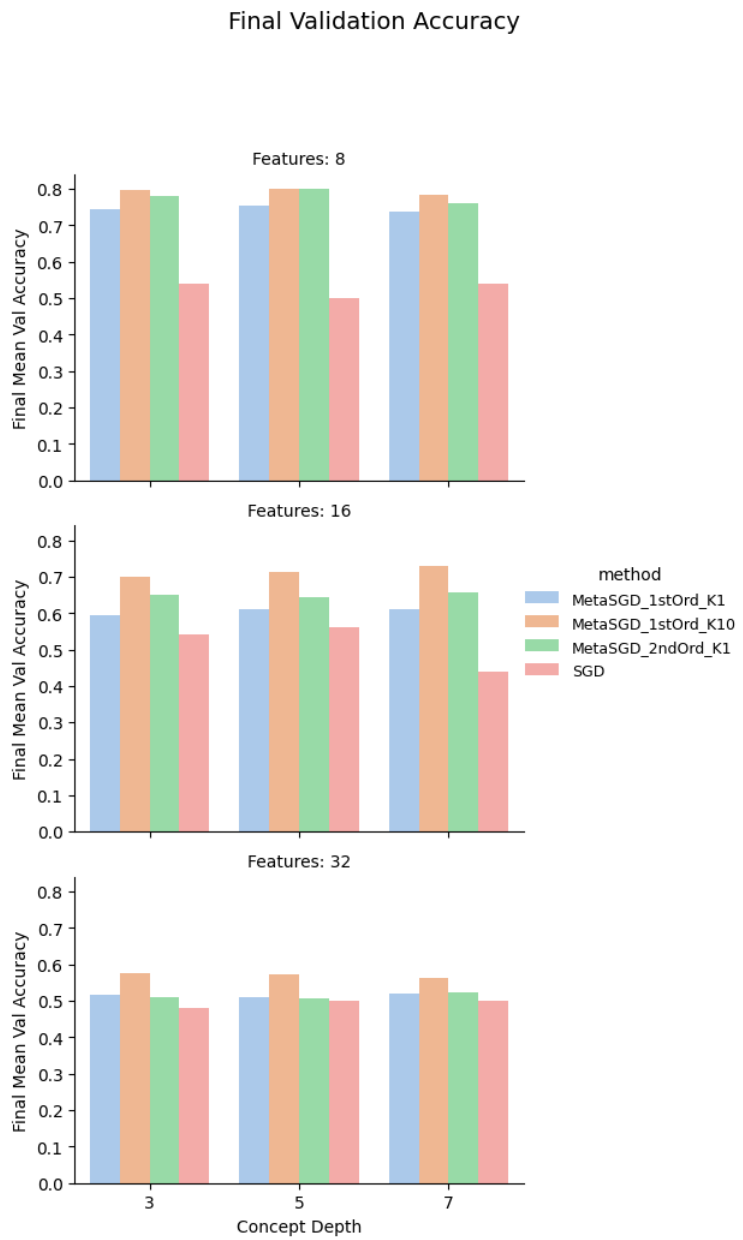

Figure 6: Final Mean Validation Accuracy (Bar Chart). Comparison of Meta-SGD variants and supervised SGD across different feature dimensionalities (rows/facets) and concept depths (x-axis).

A.4 Layer-wise L2 Norm Comparison of Model Weights

Figure 7 presents a visual comparison of the average L2 norms of weights for each layer in the MLP architecture. The comparison is made between 1st-Order and 2nd-Order Meta-SGD methods. The plots are faceted by the number of input features (rows: 8, 16, 32) and the concept depth used in the filename for model generation (columns: 3, 5, 7). This visualization allows for an examination of how weight magnitudes differ across layers, learning methods, and task configurations (feature dimensionality and concept complexity).

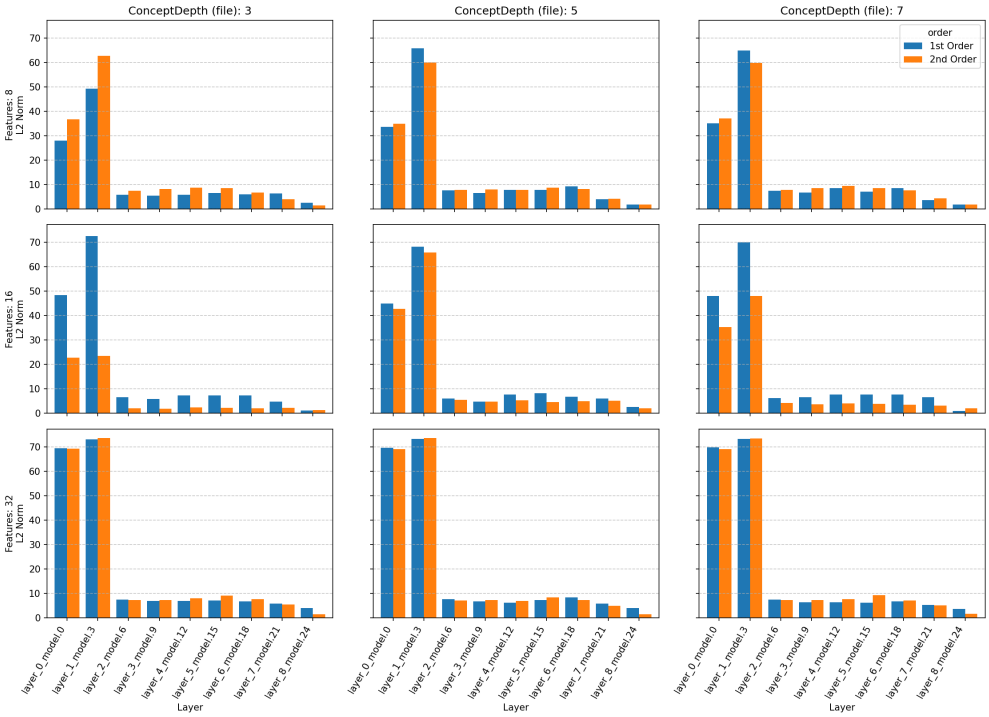

Figure 7: Average L2 Norm of MLP Layer Weights. Comparison of 1st-Order Meta-SGD and 2nd-Order Meta-SGD, faceted by input features (rows) and concept depth from filename (columns). Each bar group represents a layer within the MLP. Norms are averaged over available seeds for each configuration. The x-axis labels indicate the layer index.

A.5 Loss Landscape Curvature Analysis

This section presents detailed analysis of loss landscape curvature properties and their relationship to meta-learning effectiveness. Our analysis employs differential geometry measures to characterize the local and global structure of loss surfaces across Boolean concept complexities.

A.5.1 Curvature Metrics and Hessian Analysis

We compute four curvature-related metrics to characterize landscape geometry: roughness (variance of loss gradients along random directions), Hessian trace ($\text{tr}(\mathbf{H}) = \sum_i \lambda_i$ indicating local curvature), spectral norm ($\|\mathbf{H}\|_2 = \max_i |\lambda_i|$ measuring maximum curvature), and condition number ($\kappa(\mathbf{H}) = \lambda_{\max}/\lambda_{\min}$ quantifying eigenvalue ratios).

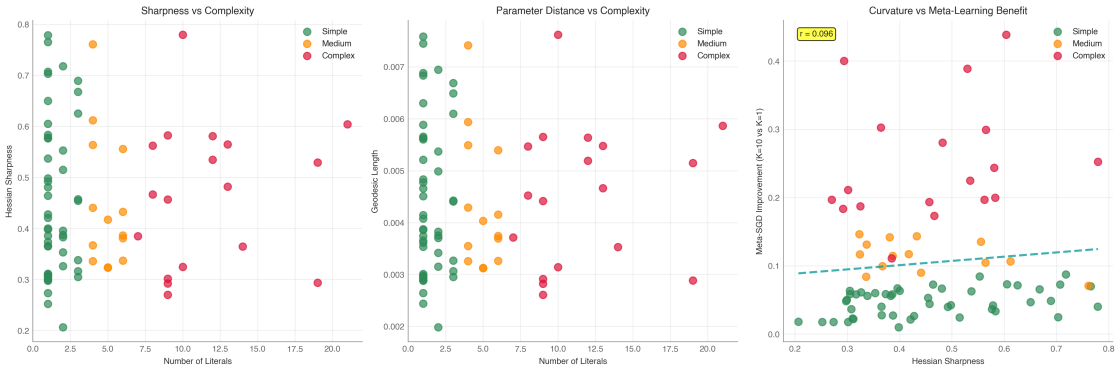

Figure 8: Parameter Curvature vs Meta-Learning Gain. Analysis of how local curvature properties relate to meta-learning effectiveness across different concept complexities. The figure shows the relationship between parameter space curvature and the performance improvements achieved by meta-learning approaches.

A.5.2 Quantitative Curvature Results

Our analysis reveals systematic curvature patterns across concept complexities. For simple concepts (F8D3), Meta-SGD reduces Hessian trace by 92.6% compared to SGD ($\Delta\text{tr}(\mathbf{H}) = -0.926$). Medium concepts (F8D5) show 95.8% trace reduction with 50.9% roughness improvement ($\Delta\text{tr}(\mathbf{H}) = -0.958$, $\Delta\sigma^2_\nabla = -0.509$). Complex concepts (F32D3) maintain 88.5% trace reduction despite landscape complexity ($\Delta\text{tr}(\mathbf{H}) = -0.885$).

The curvature analysis provides additional evidence that Meta-SGD finds better solutions by smoothening the optimization trajectory across the loss landscape, enabling more efficient few-shot learning.

A.5.3 Theoretical Implications

The curvature analysis connects to recent theoretical work on meta-learning optimization landscapes [5, 10]. Our findings suggest that meta-learning's effectiveness stems from its ability to reduce local curvature (creating smoother gradient flows), improve conditioning (reducing eigenvalue ratios $\kappa(\mathbf{H})$ for better convergence), and minimize roughness (eliminating sharp local minima that trap gradient descent).

This geometric perspective offers a new lens for understanding meta-learning: rather than simply providing better initializations, meta-learning algorithms actively reshape the optimization trajectory to enable efficient navigation and adaptation.

A.5.4 Relationship to Boolean Concept Structure

The discrete nature of Boolean concepts creates characteristic patterns in loss landscape topology. Our analysis reveals that logical complexity directly correlates with landscape roughness, PCFG depth influences the number and distribution of local minima, and feature dimensionality affects the overall scale of curvature variations.

These findings establish a bridge between symbolic concept structure and continuous optimization geometry, providing theoretical foundation for understanding why meta-learning excels at Boolean concept learning tasks.