Recursive Binary Neural Network Learning Model with 2-bit/weight Storage Requirement

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Abstract

This paper presents a storage-efficient learning model titled Recursive Binary Neural Networks for embedded and mobile devices having a limited amount of on-chip data storage such as hundreds of kilo-Bytes. The main idea of the proposed model is to recursively recycle data storage of synaptic weights (parameters) during training. This enables a device with a given storage constraint to train and instantiate a neural network classifier with a larger number of weights on a chip, achieving better classification accuracy. Such efficient use of on-chip storage reduces off-chip storage accesses, improving energy-efficiency and speed of training. We verified the proposed training model with deep neural network classifiers and the permutation-invariant MNIST benchmark. Our model achieves data storage requirement of as low as 2 bits/weight while the conventional binary neural network learning models require data storage of 8 to 16 bits/weight. With same amount of data storage, our model can train a bigger network having more weights, achieving ~1% better classification accuracy than the conventional binary neural network learning model. To achieve the similar classification error, the conventional binary neural network model requires 3-4× more data storage for weights than our proposed model.

1. Introduction

Deep Neural Networks (DNN) have demonstrated the state-of-the-art results in a wide range of cognitive workloads such as computer version [1] and speech recognition [2], achieving better-than-human performance for the tasks often considered too complex for machines. The success of DNN has indeed motivated scientists and engineers to implement a DNN in mobile and embedded devices, dubbed as Internet of *Smart* Things [3-4]. The recent works in this area however, mostly implement inference function of DNN, without function of training DNN, while training DNN is done in cloud computers and post-training weights are downloaded to mobile and embedded devices [5].

On-device learning, however, becomes increasingly important for the mobile and embedded devices for the following three reasons. First, intelligent device benefits to have the model that is custom-built for the device and its user and environment. This is because the model tends to be more accurate and effective if constructed with the consideration of those factors. Second, the training data from mobile and embedded sensing devices can contain security-sensitive information, e.g., personal health data from wearable medical devices. At the risk of being leaked, users typically do not want to upload such data onto cloud computers. Finally, in the era of Internet of Things (IoT), we anticipate a drastic increase in the number of deployed devices, which can proportionally increase the number of learning tasks to be done in the cloud. Coupled with the complexity of learning, even for powerful cloud computers, this can be a computationally challenging task.

On-device learning, however entails various challenges, in algorithms, data, and systems [6-7]. The most eminent challenge regarding computing systems is high energy consumption

caused by dense computation and data access of DNN system which is prohibited to limited resources of embedded devices. The high overhead of data access is caused by fetching DNN weights from DRAM (or FLASH) external to a computing chip on an embedded device. Since the data storage size is limited for such computing chip, the parameters of a DNN have to be stored in external DRAM and FLASH during training. For example, ARM Cortex M3 processor, a processor widely used in commercial wearable devices such as FitBit, has only 64 kilo-Byte (kB) on-chip data storage. This can only fit in very small size of DNN especially if each weight is 32-bit float point number. Compared to accessing on-chip SRAM, accessing off-chip DRAM incurs 3 to 4 orders of magnitudes more energy and delay overhead. Therefore, fetching weights every time for each data makes training prohibitive to be implemented on a mobile and embedded device [8].

Recently several techniques on pruning, distilling, and binarizing weights have been proposed to compress the parameters of a DNN. This makes it more feasible to fit weights in on-chip SRAM [8-12]. These techniques can also reduce computation overhead. However, these works focused on weight size compression *after training is finished*. The data storage requirement during training remains the same.

Similarly, several learning models, which belong to so-called binary neural networks, have been proposed [9-11]. These model uses sign bits (or binary information) of weights in several parts of the learning model notably the part of multiplying and accumulating weights with inputs/activations. Although this greatly reduces computational complexity, each weight still needs to be represented in high precision number with multiple bits during the end-to-end training process since it has to be iteratively fine-tuned in the weight update part. Therefore, this so-called binary neural network models have not demonstrated to scale storage requirement below 32 bits/weight.

Our goal is, therefore, to efficiently use the limited amount of on-chip data storage *during training*. We also aim to scale computational complexity. Toward this goal, we propose a new learning model, *Recursive Binary Neural Network (RBNN)*. This model is based on the process of training of a neural network, weight binarization, recycling storage of non-sign-bit portion of weights to add more weights to enlarge the neural network for performance improvement. We recursively perform this process until either accuracy stop improving or we use up all the storage on a chip.

We verified the proposed RBNN model on a multi-layer perceptron (MLP)-like classifier and the MNIST benchmark. We considered typical storage constraints of embedded sensing devices in the order of hundreds of kB. The experiment confirms that the proposed model (i) demonstrates ~1% classification accuracy improvement over the conventional BNN learning model specifically following [9] for the same storage constraints or (ii) scale on-chip data storage requirement by 4X for the same classification test error (~2%), marking the storage requirement of 2 bits/weight. The conventional BNN model ([9] but also [10,11]) exhibits a significantly larger storage requirement of 8 to 32 bits/weight.

The remainder of the paper is as follow. In Sec. 2 we will introduce the works related to this paper, including comparison to existing works on DNN compression, BNN, and low-precision DNN. In Sec. 3 we will describe the intuitive and details of our proposed model. Sec. 4 will present the experimental results and comparisons to the conventional BNN model. Finally, in Sec. 5, we will conclude the paper.

2. Related Work

2.1. Distillation and Compression of DNN parameters

Knowledge distillation [12] is a technique to compress knowledge of an ensemble of DNNs into one small DNN while maintaining the accuracy. Although this technique can scale the number of weights for deployment systems post-training, it cannot scale data storage requirement for training. Specifically, during training, each of weights is represented in high-precision number, which needs to be stored in multi-bit data storage.

Another technique is to compress the data size of weights by exploiting redundancies in them. In Ref. [8], the authors combine four sub-techniques, namely weight pruning, quantization,

sharing, and compression coding to reduce the data size of weights. Similar to the knowledge distillation, this technique can be applied to the weights that are already trained, and cannot scale data storage requirement of weights during training.

2.2. Binary Neural Network (BNN)

Recent works proposed to use binary information of weights [9], activations [10,11], and even inputs [11] in some parts of learning and post-learning operations. The use of binary information of weights notably in multiply-and-accumulate (MAC) operation can drastically reduce computational complexity. However, those BNN techniques still cannot scale the storage requirement of weights during training. In these works, each weight is represented in 32 bits. This is because mainstream training models such as stochastic gradient decent requires fine-grained update of weights.

2.3. Low-precision fix-point weight representation

Several studies have demonstrated moderately lowering the precision of weights (i.e., quantization) has a tolerable impact on training and post-training operations of DNN [13,14]. In Ref. [13], the authors trained a DNN having 16-bit fixed-point weights with the proposed stochastic rounding technique, and demonstrated little to no degradation in classification accuracy. In Ref. [14], the authors proposed the dynamic fixed-point representation (i.e., dynamically changing the position of decimal point over computation sequences) to further reduce the precision requirement down to 10 bits per synapse. Using fix-point representation help to reduce storage requirement and fixed-point arithmetic is more hardware friendly [8].

3. Recursive Binary Neural Network (RBNN) Model

3.1. Key idea

Table I shows which information of weights are used in each step of training in both conventional BNN [9-11] and our proposed RBNN. The conventional BNN works [9-11] use sign bits of weights during multiply-and-accumulate (MAC) operation in feed-forward and back propagation. However, the weight update has to be done in high precision. This mandates to store multi-bit (32 bit if using float point number) weights in data storage during learning, resulting in no savings in weight storage requirement.

Table I Comparisons of weight information usage in BNNs and RBNN

Steps	BNN [16-18]	Proposed RBNN
MAC in forward prop.	Sign bits of weights	Sign bits of weights
MAC in back prop.	Sign bits of weights	Sign bits of weights
Weight update	All bits of weights	All bits of weights
Recursive recycling	N/A	Keep sign bits and recycle storages of the other bits for more plastic weights

However, it has been studied that in the trained neural networks we can use only the sign bits of weights to perform inference [9-11]. This vast different requirements of weight precision between learning and post-learning inspires us to create our RBNN model. As shown in the third column in Table I, we also use only the sign bits for MAC operations to reduce computational complexity. The main difference is that after training and binarization of weights (keeping only sign bit), we recycle the data storages that are used to store non-sign bits of weights to add more multi-bit train-able weights to the neural network. We then trained this new network having both the binarized non-trainable weights and the newly-added trainable weights. We perform this process recursively, which makes the neural networks larger and more accurate but using the same amount of data storage for weights.

Figure 1 depicts the process of our proposed RBNN learning model with an example of

multi-layer fully-connected neural network. In the beginning the neural network has one input, two set of two hidden, and one output neurons and eight weights each of which has *n* bits. We first train this $1\times2\times2\times1$ network using the conventional back-propagation training algorithm. After that, we discard all bits except the sign bit in each weight (binarization), resulting in a $1\times2\times2\times1$ trained network having binary weights (*trained_BNN*). Then we continue the second iteration of training (the second subfigure of Fig. 1). Specifically, we recycle the storage that is used to store the *n-1* non-sign bits of weights in the $1\times2\times2\times1$ network. Using this data storage, we add eight additional weights (W_{21} to W_{28}) to the *trained_BNN*, expanding the network to a $1\times4\times4\times1$. In this *enlarged_BNN,* each of the newly-added weights is *n-1* bits. In other words, the *enlarged_BNN* comprises of one *trained_BNN* that has eight weight (W^b_{11} to W^b_{18}) that are trained (binary, *non-plastic*, marked as solid lines in Fig. 1) and one *incremental_BNN* with eight weights (W_{21} to W_{28}) that are under training (n-1 bits, *plastic,* marked as dash lines in Fig. 1). The *incremental_BNN* is trained together with the *trained_BNN* but only the weights of *incremental_BNN* are updated.

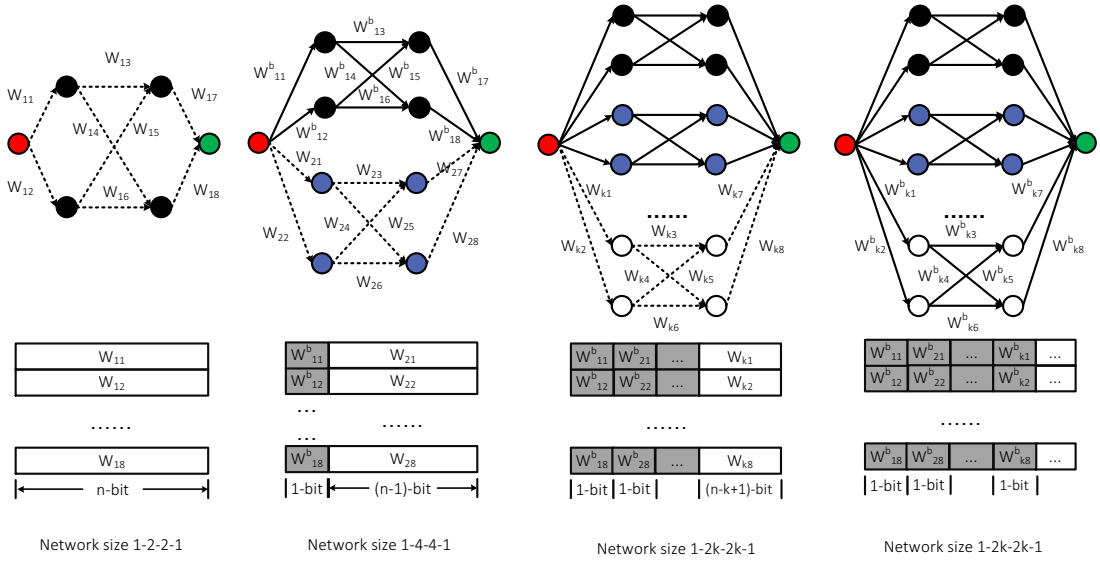


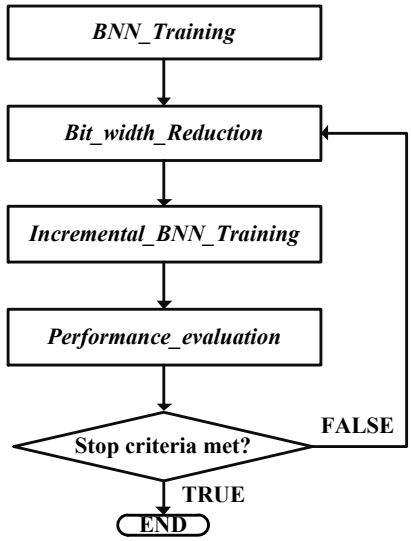
Figure 1: RBNN learning model with an example neural network. The recursive operation increases the number of weights in the neural network (top) while using the same amount of storage for weights (bottom)

We repeat the same process of binarization and recycling. In every iteration, the *enlarged_BNN* integrates 8 more weights, and the bit-width of newly-added plastic weights in the *incremental_BNN* is reduced by one. At the k iterations, the trained_BNN has 8(k-1) neurons and the plastic weights have (n-k+1) bit-width. After the k-th training is finished, as shown in the rightmost in Figure 1 the neural network becomes a $1\times2k\times2k\times1$ with $8\cdot k$ binary weights. This network has k times more weights than the first $1\times2\times2\times1$ network. However, the data storage of weights remains the same, scaling the storage requirement per weight to n/k (=4·n/4·k), which is k times smaller than that of the first network. Thus the proposed RBNN can either achieve better classification accuracy - enabled by the more number of weights – with the same amount of weight storage, or reduce weight storage requirement for the same classification accuracy level.

3.2. Model details

Figure 2 depicts the details of the proposed RBNN model. The left part of Figure 2 depicts the flow chart of RBNN model, while right part explains the function of each stage. In the beginning of the training procedure, conventional BNN training algorithm *BNN_Training* is used to train a BNN. After training, we got a *trained_BNN* with binary synaptic weights. The synaptic

bit-width is reduced by 1. And then we use the rest synaptic bits as weights of *incremental_BNN*. After training the *incremental_BNN* with algorithm *Incremental_BNN_Training*, the performance of the *enlarged_BNN* is tested. If the performance doesn't stop improving and there are still available synaptic bits after weight binarzation, the rest synaptic bits will be reused to further enlarge current trained BNN.



***BNN_Training*:** Training initial BNN using conventional BNN training method.

***Bit-width_Reduction*:** Reduce bit-width of all synaptic weights by 1

***Incremental_BNN_Training*:** Training *incremental_BNN* with previously *Trained_ BNN*

Performance_evaluation: Test the performance of trained enlarged BNN

Stop criteria: When the performance stops improving or improves too slowly with synaptic bits reuse, the process should stop.

Figure 2: Detail of Training Method for RBNN Model

The method *Incremental_BNN_Training* is designed to train the *incremental_BNN* to improve performance of *enlarged_BNN*. To meet this goal, the conventional BNN training method is adjusted as shown in Algorithm 1. The main idea of this training method is: both *trained_BNN* and *incremental_BNN* are used to calculate output of the *enlarged_network* in feedforward stage. While during back-propagation and parameter-update, only plastic weights in *incremental_BNN* are updated. Similar to the conventional BNN training algorithm [16], binary weights are used in both feed forward and back propagation in *Incremental_BNN_Training*, to reduce computational overhead. Since weights in *trained_BNN* are binary, the multiplication related to weights can all be simplified as shift.

Algorithm 1 *Incremental_BNN_Training. C* is the cost function for mini-batch, η the learning rate and L the number of layers. The function $Binarize()$ specifies how to binarize the weights. $Act_hid()$ and $Act_out()$ are activation function of hidden layers and output layer, respectively.

Require: a minibatch of inputs and targets (a_0 , a^*), previous weights of incremental BNN $W(I)$, weights of trained BNN $W(T)$,
Ensure: updated weights of incremental BNN $W(I)^{t+1}$
{**1. Forward Propagation**}
{1.1 Computing outputs of hidden layers in trained BNN and incremental BNN}
for k = 1 to L-1 **do**
  $a(\mathrm{T})_\mathrm{k} = Act_hid(W(T)_k \cdot a(T)_{k-1})$
  $W(I)_k^b \leftarrow Binarize(W(I)_k^b)$
  $a(\mathrm{I})_\mathrm{k} = Act_hid\left(W(I)_k^\mathrm{b} \cdot a(I)_{k-1}\right)$
end for
{1.2 Computing outputs of enlarged BNN}
$a_L = Act_out(W(T)_L \cdot a(T)_{L-1} + W(I)_L \cdot a(I)_{L-1})$
{**2. Back propagation**}
{Please note that only gradients of **incremental BNN** are computed.}
Compute $g_{a\mathrm{L}} = \frac{\partial C}{\partial a_L}$ knowing a_L and a^*
for k = L to 1 **do**
  $g_{W(I)_k^b} \leftarrow \left(g_{a(I)_k} \cdot W(I)_k^b\right) \circ a'(I)_{k-1}$

10: $g_{W(I)_k^b} \leftarrow g_{a(I)_k}^T \cdot a(I)_{k-1}$
11: **end for**
{**3 Parameter Update**}
{Please note that only weights of **incremental BNN** are updated.}
12: **for** $k = L$ to 1 **do**
13: $W(I)_k^{t+1} \leftarrow W(I)_k^t + \eta \cdot g_{W(I)_k^b}$
14: **end for**

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4. Experiment Setup

4.1. Permutation-invariant MNIST benchmark

We used the permutation-invariant MNIST to test the performance of the proposed RBNN model. We use the original training set of 60,000 28-by-28 pixel gray-scale images and the original test set of 10,000 images. The training and testing data x is normalized to be within the interval [-1, 1] and exhibits zero mean. Following the common practices, we use the last 10,000 images of the training set as a validation set for early stopping and model selection. As we use the permutation-invariant MINST, i.e., ignoring the 2-dimentonal image structure of the image, we did not consider convolutional computation. We also did not consider data augmentation, pre-processing, and unsupervised pre-training during our experiment.

4.2. Neural network configuration and data format

We considered the storage constraints of mainly hundreds of kB based on the typical embedded system designs [15]. We considered a feed-forward fully-connected neural network with one or two hidden layers. We considered several different numbers of neuron units in the hidden layer ranging from 200 to 800. The numbers of the input and output units are 784 and 10, respectively. We used the *tanh_opt()* for the activation function of the hidden layer and the *softmax()* or linear output for that of the output layer. We use the classical Stochastic Gradient Descent (SGD) algorithm for cross-entropy or hinge loss minimization without momentum. We use a small size of batch (1,000) and a single static learning rate which is optimized for each BNN. Any other advanced techniques such as dropout, Maxout, and ADAM are not used for both the proposed and the baseline models. We recorded the best training and test errors associated with the best validation error after up to 1,000 epochs. The results are averages of results from 20 independent experiments for each case.

We used the fixed-point arithmetic for all the computation and data load and access. The fix-point intermediate computations, such as gradient calculation have sufficient precision. The translation from wide fixed-point number to narrow fixed-point and binary number is performed with simple decimation without using advanced techniques such as stochastic rounding [14]. We saturated values in the event of overflow in weight update. The dynamic range of fix-point representation is optimized to achieve better accuracy performance.

5. Results and Discussion

5.1. Accuracy improvement

Figure 3 depicts the classification errors of the proposed learning model across three recursive iterations. The initial bit-width of weights is 8 bits. In each series of data points in Figure 3, the leftmost point is the initial neural network, i.e., with 2 layers of 200 hidden units and 198,800 synaptic weights (= 784·200+200·200+200·10). At this point, the *synaptic bit requirement*, defined as the ratio of total storage bits to the number of weights, is 8-bit/weight. At this point the network is equivalent to one trained with the conventional BNN model specifically following [9]. In the second leftmost data point in the series is the neural network after the first recursive iteration. The network size is enlarged by twice, resulting in the 784-400-400-10 network. This reduces synaptic bit requirement to 4 bits/weight. Compared to original BNN, the enlarged BNN respectively achieves ~0.7% and ~0.4% reduction in training and test error rate. Finally, after three recursive iterations, the neural network implements BNN with size 784-800-800-10 (555,800 synaptic weights). It

requires only 198.8 kB data storage for weights, marking the weight storage requirement of 2b/weight and the test error of ~2.17%. This accuracy is as good as network with 4× times of synaptic data storage in [9].

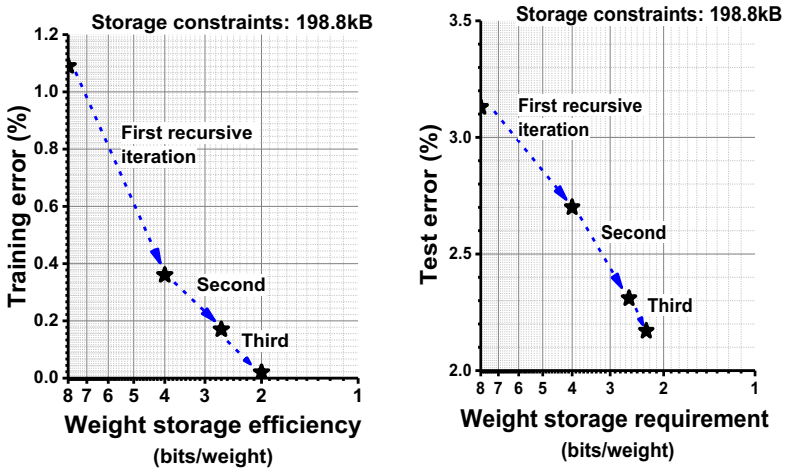

Figure 3: (left) Training error and (right) testing error across recursive iterations in the proposed RBNN model. The total weight storage used is 198.8 kB.

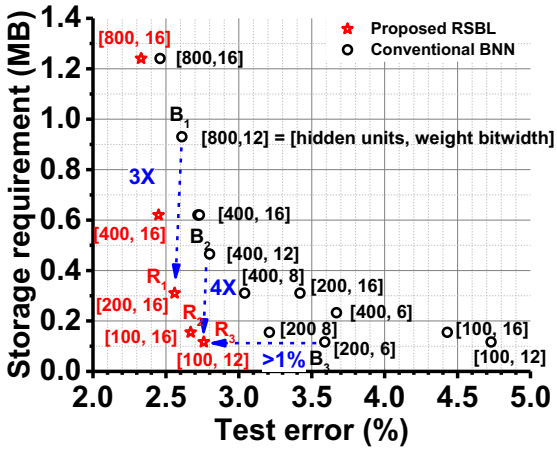

Figure 4: The storage requirement and test error trade-offs achieved by the proposed RBNN model and the conventional BNN model. The proposed model achieves >3× data storage savings for the same test error and >1% lower error for the same data storage.

5.2. Storage savings

We trained DNNs of a range of configuration using our RBNN and the conventional BNN learning model [9]. We used fix-point number for weights. For the conventional model, we considered BNN containing from 100 to 800 hidden neurons combined with 16-b to 6-b synaptic weights. For the proposed model, we considered from 100 to 800 *initial* hidden neurons and 12 to 16-bit *initial* weight precisions. Those DNNs require 116 kB to 1.2 MB data storage for weights. Figure 4 shows the results of this experiment: the proposed model can achieve ~1% lower test error than the conventional model using the similar amount of data storage. To achieve the similar test error, the proposed RBNN model requires 4× less data storage than the conventional BNN model. It is noticed that the accuracy improvement is not as significant when hidden neurons are as many as 800. We observe that at this point, the training data has been modeled well (0 training error and very small cost function value), so we think at this point, regularization technique is needed to further improve performance, which is beyond the scope of this paper.

Table II shows the detail comparisons of six neural networks, three from the proposed RBNN model (R_1, R_2, R_3) and three from the conventional BNN model (B_1, B_2, B_3) [9]. The complexity of computation for learning and inferring one data is also listed. R_1 and B_1, R_3 and B_2 achieve similar test error, but R_1 outperforms B_1 in the storage requirement by 3× and R_3 outperforms B_2 by 4×, respectively. The downside of R_1 and R_3 is the increase in computations during training: it requires more shift and add operations, but it still needs the same number of multiplications which is much more complex than add and shift, as compared to B_1 and B_2. On the other hand, the computation complexity for inference operation are the same.

Table II. Detail comparisons of RBNNs and BNNs

	R_1	R_2	R_3	B_1	B_2	B_3
Initial hidden neurons	200	100	100	800	400	200
Final hidden neurons	800	700	400	800	400	200
Final synapses	635,200	555,800	317,600	635,200	317,600	155,600
Initial weight bitwidth	16	16	12	12	12	6
Storage req (b/weight)	4	2.28	3	12	12	6
Test error (%)	2.56	2.65	2.76	2.61	2.80	3.60
Comp., learning ***Shift / Multiply / Add***	2,223,200 635,200 2,223,200	2,779,000 555,800 2,779,000	1,111,600 317,600 1,111,600	1,270,400 635,200 1,270,400	635,200 317,600 635,200	317,600 158,800 317,600
Comp., inference ***Shift, Add***	635,200 635,200	555,800 555,800	317,600 317,600	635,200 635,200	317,600 317,600	158,800 158,800
Storage for weights	310kB	155kB	116kB	930kB	465kB	114kB

5.3. Discussion on Convolutional Neural Networks

Although we mostly focus on a fully-connected DNN in this paper, we want to make a brief discussion on the application of our proposed RBNN model on the Convolutional Neural Networks (CNN). A CNN employs many convolution layers, each of which uses a smaller number of weights than a fully-connected one. But a convolution layer uses the same weights many times. This makes the operation of convolutional layers dominated by multiply and add operation rather than weight data storage operation. Our RBNN model can still mitigate the complexity of multiply and add operations since it uses only the sign bit of a weight. Moreover, as the computational complexity of convolution layers is scaled, the computation and data-storage overhead associated with the last fully-connected layers in a CNN becomes more pronounced. Our proposed model can reduce the overhead of weight data storage of them. We are currently exploring the application of our RBNN model on a CNN.

6. Conclusion and Future Work

This paper presents a learning model regarding local on-device learning with limited data on-chip storage. The proposed RBNN model efficiently uses limited on-chip data storage resources by recycling data storage that would have been wasted, to add and train more synaptic weights to a neural network classifier. We verified the proposed model with the neural network classifier and the permutation-invariant MNIST benchmark under the typical embedded device storage constraints. The results show that the proposed model achieves 2b/weight storage requirement while achieving ~1% better classification error as compared to the conventional binary-weight learning model for the same storage constraint. Our proposed model also achieves 3-4× less data storage than the conventional model for the same classification error. We expect the future work that extends the application of the learning model to other neural network topologies and datasets, such as BNN with binary activation function [10], and also more complicated model like CNN. We also expect to apply the RBNN model to the ensembles of neural networks [16], the mixture of experts [17], and the incremental learning [18].

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