GRADIENT INFORMATION MATTERS IN POLICY OPTIMIZATION BY BACK-PROPAGATING THROUGH MODEL

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Paper under double-blind review

ABSTRACT

Model-based reinforcement learning provides an efficient mechanism to find the optimal policy by interacting with the learned environment. In addition to treating the learned environment like a black-box simulator, a more effective way to use the model is to exploit its differentiability. Such methods require the gradient information of the learned environment model when calculating the policy gradient. However, since the error of gradient is not considered in the model learning phase, there is no guarantee for the model's accuracy. To address this problem, we first analyze the convergence rate for the policy optimization methods when the policy gradient is calculated using the learned environment model. The theoretical results show that the model gradient error matters in the policy optimization phrase. Then we propose a two-model-based learning method to control the prediction error and the gradient error. We separate the different roles of these two models at the model learning phase and coordinate them at the policy optimization phase. After proposed the method, we introduce the directional derivative projection policy optimization (DDPPO) algorithm as a piratical implementation to find the optimal policy. Finally, We empirically verify the effectiveness of the proposed algorithm and yield the state-of-the-art performance on sample efficiency through benchmark continuous control tasks.

1 INTRODUCTION

Reinforcement learning (RL) is a powerful technique for solving the sequential decision making problems (Li, 2018; Sutton & Barto, 1998). The sequential decision-making problem is always formulated as the Markov decision process (MDP) framework. The agent uses policy to determines its action according to the current state of the environment, interacts with the environment, and gets the immediate reward from the environment as feedback. Then, the environment changes to the next state. The goal for reinforcement learning is to find the optimal policy that can achieve the highest expectation of the summation of the future reward.

Recent work on model-based RL (Nagabandi et al., 2018; Luo et al., 2018; Kurutach et al., 2018; Wang et al., 2019; Janner et al., 2019; Pan et al., 2020), has shown the power of first learning the environment model and then use it to do the policy optimization. These methods all achieve similar performance by using fewer data compare to their model-free counterparts. Several methods are proposed to achieve the goal, such as ensembles (Kurutach et al., 2018), probabilistic models (Chua et al., 2018), and meta-learning (Clavera et al., 2018).

In addition to treating the learned environment like a black-box simulator, a more effective way of using the model is to exploit its differentiability (Heess et al., 2015; Clavera et al., 2019; D' Oro & Jaśkowski, 2020; Amos et al., 2021), which is mainly focused in this paper. These methods compute the gradient of the objective with respect to the policy directly by back-propagating through the model. Note that there is a big difference between these methods and the rollout-based model-based methods that the model gradient is crucial when calculating the policy gradient in these methods. However, since the traditional model learning only aims to get the accurate prediction for the next state and the reward given current state and action by using the function approximation techniques such as neural networks, there is no accuracy guarantee for the model gradient.

In this paper, to address these problems, we first theoretically analyze the problem and then propose our solution based on the theoretical results. First of all, we present the convergence rate analysis

for the policy optimization algorithms in what the policy gradient is calculated using the learned environment model. By taking the model gradient error into account, we can see that the transition and reward model gradient matter in the policy optimization. Specifically, the bias of the estimated policy gradient used to update the policy is not only introduced by the prediction error of the learned model but also introduced by the gradient error of the learned model. Furthermore, the policy gradient bias due to the different types of model error will finally influence the convergence rate of the policy optimization process.

Then, inspired by the theoretical results, we propose the two-model-based learning method. According to the policy gradient bias and convergence rate analysis, in order to optimize the policy efficiently and accurately, we need the learned environment model both with small prediction error and small gradient error. Therefore, we propose to separate different requirements for the model into different models and use the prediction model together with the gradient model as our learned model. In the model learning phase, the prediction model aims to reduce the prediction error, and the gradient model focuses on minimizing gradient error. In the policy optimization phase, we will use the prediction model to rollout the data and use the gradient model to calculate the policy gradient.

To make the proposed method applicable, we introduce the directional derivative projection policy optimization (DDPPO) algorithm. Our first goal is to use data to estimate the gradient or Jacobian matrix for the environment model and use the estimator to learn the model's gradient explicitly. The challenge is that the state and action are usually with high dimensions and directly estimating the gradient or Jacobian matrix using data is intractable. Thus, we first estimate the directional derivative which is a scalar by using data and then project the model's gradient or Jacobian matrix into these directions. Using the error between the estimated directional derivative and the projection value, we can learn the model's gradient or Jacobian matrix. Secondly, after we can learn the environment model with a more accurate gradient, we can leverage two models to do the policy optimization. We adopt the prediction model to rollout data and use the generated data together with the real data to calculate the policy gradient according to the learned gradient model.

Finally, we conduct experiments on the simple environments and the benchmark MuJoCo continuous control environments. The experimental results verify our theoretical findings and demonstrate the effectiveness of the proposed method.

Our main contribution can be summarized as follows:

1. We theoretically depict how the different model errors influence the convergence rate of the model-based policy optimization algorithm. The result shows that the gradient error of the model indeed matters in the convergence of the policy optimization.

2. We propose the two-model-based learning method and the practical DDPPO algorithm. Motivated by the theoretical results, we propose to learn and to use two environment models for the model-based policy optimization. One is called the prediction model which aims to rollout accurate trajectories. Another one is called the gradient model which aims to provide the gradient information in the policy optimization phase.

3. Empirically, we can achieve state-of-the-art sample efficiency in the MuJoCo continuous control tasks.

2 PRELIMINARIES

Reinforcement Learning: We consider a discrete-time Markov decision process (MDP) $\mathcal{M}$, defined by tuple $(\mathcal{S}, \mathcal{A}, f, r, \gamma, p_0)$, where $\mathcal{S}$ and $\mathcal{A}$ are the state and action spaces, respectively. Here, $f : s_{t+1} = f(s_t, a_t, \epsilon_t)$ is the transition distribution, $r : s_t = r(s_t, a_t)$ is a reward function, p_0 is the initial state distribution and γ represents the discount factor. We define the return (J) as the expected sum of discounted rewards along a trajectory ($R = \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t)$). The goal of reinforcement learning is to find a policy π_ϕ that maximizes the expected sum of discounted rewards, i.e.,

$$\max_{\phi} J(\phi) = \max_{\phi} \mathbb{E}_{\pi_\phi} R = \max_{\phi} \mathbb{E}_{\pi_\phi} \left[\sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \right]. \tag{1}$$

Model-based RL is characterized by learning the transition model using data collected from interaction with the environment. Typically, we use a parametric function $\tilde{f}_\theta$ to denote the transition

model trained by a neural network. The state predicted by the learned model is defined as $\tilde{s}_{t+1}$ and $\tilde{s}_{t+1} \sim \tilde{f}_\theta$. Similarly, we use a parametric function $\tilde{r}_\theta$ to denote the reward model. Generally we use M to denote the true model and $\tilde{M}$ the learned model. Then $\tilde{s}_{t+1}, \tilde{r}_t \sim \tilde{M}_\theta(s_t, a_t)$.

The model is trained by supervised learning, typically, via maximum likelihood:

$$\max_\theta J_{\tilde{M}}(\theta) = \max_\theta \mathbb{E}\left[\log \tilde{M}(s_{t+1}, r_t | s_t, a_t)\right], \tag{2}$$

where (s_t, a_t, r_t, s_{t+1}) are data collected from interaction with the environment.

Policy Optimization According to the Policy Gradient Calculated Using Model: After we learned the model, we can use the model to rollout trajectories and use the data to calculate the policy gradient for the purpose of policy optimization. Specifically, denote the policy paramters as ϕ, we can calculate the return J using generated data as $J_\pi(\phi) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t \tilde{r}_t\right]$, where $\tilde{s}_{t+1}, \tilde{r}_t \sim \tilde{M}(\tilde{s}_t, \tilde{a}_t)$ and $\tilde{a}_t \sim \pi_\phi(\tilde{s}_t)$.

To optimize the objective, One can use the traditional policy optimization methods such as A3C (Mnih et al., 2016), PPO (Schulman et al., 2017), SAC (Haarnoja et al., 2018). Beyond that, many previous works argue that we can use the model to calculate the policy gradient by back-propagating through the model (Heess et al., 2015; Clavera et al., 2019; Amos et al., 2021) leveraging the reparameterization trick (Kingma & Welling, 2014).

Specifically, since we have learned the environment model, we can calculate the gradient of the objective w.r.t ϕ by chain rule as shown in Equation 3 in which the gradients $\frac{dr_t}{da_t}, \frac{dr_t}{ds_t}, \frac{ds_k}{ds_{t+1}}, \frac{ds_{t+1}}{da_t}$ need to be calculated using the learned model.

$$\frac{dJ}{d\phi} = \mathbb{E}\sum_{t=0}^{\infty} \frac{dJ}{da_t}\frac{da_t}{d\phi} = \mathbb{E}\sum_{t=0}^{\infty}\left(\gamma^t \frac{dr_t}{da_t} + \sum_{k=t+1}^{\infty} \gamma^k \frac{dr_k}{ds_k}\frac{ds_k}{ds_{t+1}}\frac{ds_{t+1}}{da_t}\right)\frac{da_t}{d\phi}. \tag{3}$$

The expectation is taking over randomness from random policy and transition. Usually, we can only estimate the policy gradient using stochastic samples and even using biased model and sample denoted as $\frac{dR^{\hat{M}}}{d\phi}$, we update the policy parameters towards the gradient directions using learning rate α_t,

$$\phi_{t+1} = \phi_t + \alpha_t \frac{dR^{\hat{M}}}{d\phi}. \tag{4}$$

3 Related Work

Due to the space limitation, we clarify several highly related works in this section and put the detailed related work into the Appendix A. Overall speaking, most of the model-based policy optimization algorithms aim to calculate the policy gradient accurately and efficiently. These works can be divided into two categories, i.e., how to learn the model and how to use the model.

Learn the better model: Chua et al. (2018) design an ensemble-based neural network to learn the true model. Zhang et al. (2021a) design the dropout mechanisms to improve the robustness of the learned model. Zhang et al. (2020b) design the auto-regressive network structure to improve the accuracy of the model prediction. **Consider the decision-making phase when learning the model:** These works are always called Decision-aware model learning (DAML). D'Oro et al. (2020) use policy gradient to re-weighted the model prediction error for each sample. Farahmand et al. (2017) use the value function to re-weighted the model prediction error for each sample **Use the learned model to argument data:** Janner et al. (2019) use the learned model to rollout the augmented trajectories and use both the generated trajectories and the true trajectories to train the SAC algorithm. Yu et al. (2020); Lee et al. (2020) extend the similar idea to the offline model-based RL setting. **To calculate the policy gradient by back-propagating through the model:** SVG (Heess et al., 2015) presents a framework for learning continuous control policies using back-propagating through the model. More recent works (Clavera et al., 2019; Amos et al., 2021) extend the idea by combining the learned Q-function and the SOTA policy optimization algorithms and thus achieve better performance. MAGE (D' Oro & Jaśkowski, 2020) computes the policy gradient using value gradient and proposes to reduce the value gradient through TD learning.

All the above methods do not consider the influence of the model gradient error with respect to the convergence of the policy optimization. And obviously, they do not use multiple function approximators to separate different requirements for the model at the learning and using phase. Therefore, these works are different from our proposed methods. Moreover, we can leverage the insight inside of these related works to better design our model-based algorithms.

4 MODEL-BASED POLICY OPTIMIZATION BY CONSIDERING THE MODEL GRADIENT ERROR

In this section, we will analyze the influence of the model error in the policy optimization process. First of all, for the policy optimization algorithms that using policy gradient calculated according to the learned model, we theoretically show that both the prediction error and the gradient error of the learned model matter for the policy gradient bias and the convergence rate. Secondly, based on the theoretical findings, we propose our method to learn the accurate model for policy optimization. Thirdly, we show the implementation details of our method and propose the DDPPO algorithm.

4.1 CONVERGENCE RATE FOR MODEL-BASED POLICY OPTIMIZATION

In this section, we will prove the finite sample bound for the model-based policy optimization algorithms. Due to the space limitation, we put the full proof into the Appendix C.

First of all, recall the policy gradient formulation (Equation 3) in the Preliminary section. In the model-based setting, we need to use the learned model to calculate the gradient value ($\frac{dr_t}{da_t}, \frac{dr_t}{ds_t}, \frac{ds_k}{ds_{t+1}}, \frac{ds_{t+1}}{da_t}$) in Equation 3. Since the model error is unavoidable, it is necessary for us to analyze the bias of the policy gradient introduced by the model error.

For simplicity and clarity, we will use the following notations. We use fraction notation $\frac{d\cdot}{d\cdot}$ to represent the gradient calculated by taking gradient on the true model and use the "hat" version $\widehat{\frac{d\cdot}{d\cdot}}$ to represent the gradient calculated on the learned model. We use the "tilde" notation $\tilde{\cdot}$ to represent the state, action or reward generate using the learned model and the without-tilde notation to represent those variables generate using the true model. For example the notation $\frac{ds_{t+1}}{ds_t}$ to denote the gradient function value calculated by taking gradient for the output of the transition model s' ($s' = f(s, a, \epsilon)$) with respect to its input s given input value equals $s = s_t$. We use the notation ϵ_f to represent the transition model prediction error which means, for $\forall\ s, a$, we have $\left\|\tilde{f}(s,a) - f(s,a)\right\| \leq \epsilon_f$. We use the notation ϵ_f^g to represent the gradient error of the transition model. For $\forall\ s, a$, we have

$$\left\|\widehat{\frac{ds'}{ds}} - \frac{ds'}{ds}\right\| + \left\|\widehat{\frac{ds'}{da}} - \frac{ds'}{da}\right\| = \left\|\frac{d\widehat{f}(s,a)}{ds} - \frac{df(s,a)}{ds}\right\| + \left\|\frac{d\widehat{f}(s,a)}{da} - \frac{df(s,a)}{da}\right\| \leq \epsilon_f^g. \tag{5}$$

Similarly, we use the notation ϵ_r^g to represent the gradient error of the reward model.

Theorem 1. *Suppose assumptions hold. The policy gradient bias ϵ_M can be bounded as*

$$\epsilon_M = \left\|\mathbb{E}\left(\frac{dR^{\widehat{M}}}{d\phi} - \frac{dR^M}{d\phi}\right)\right\| \leq C_6\epsilon_r^g + C_7\epsilon_f^g + C_8\epsilon_f, \tag{6}$$

where C_6, C_7, C_8 are three constants related to the Lipschitz constants of the transition model and the reward model and γ.

Remark: The above theorem shows that the difference between the policy gradient calculated using two different models can be upper bounded by the difference of the gradient of the reward model, the gradient of the transition model, and the prediction of the transition model. The result suggests that gradient error matters in estimating the policy gradient. Now, one natural conjecture is that the gradient error will influence the convergence of the policy optimization process. The next theorem depicts the influence rigorously.

Recall the update rule (Equation 4) in Preliminary section. We update the policy parameters by following the estimated policy gradient direction. Intuitively, the model error will influence the convergence.

Theorem 2. *Suppose assumptions 1, 2, 3, 4,5, and 6 holds. L_J is the Lipschitz constant of the objective function for the parameter ϕ and L_s^g is the smoothness constant of the objective function for the parameter ϕ. Superpose the policy parameters are updated as shown in Equation 4 and the learning rate is set as α_t. We have*

$$\min_{0<t<T} E\left\|\frac{dJ}{d\phi_t}\right\|^2 \leq \frac{2}{\sum_{t=1}^T \alpha_t}\left(2U_J + 2\tau\alpha_0 L_J^2 + \sum_{t=1}^T \alpha_t^2\left(L_J^2 {L_J^g}^2 \tau + L_J^g \alpha_t^2 L_R^2\right)\right) + 2L_J m\rho^\tau + \epsilon_M^2. \tag{7}$$

Remark: The above theorem shows the rate of the model-based policy gradient method converge to its stationary point. This setting is commonly used in the previous papers that analyze the convergence rate for the actor critic method with function approximation and non-convex optimization (Wu et al., 2020; Kumar et al., 2019; Qiu et al., 2021; Ghadimi & Lan, 2013). The results show that if we set the learning rate α_t that satisfies the condition $\lim_{T\to\infty}\frac{\sum_{t=1}^T \alpha_t^2}{\sum_{t=1}^T \alpha_t} = 0$, the policy optimization process will converge to near its stationary point with rate $\mathcal{O}\left(\frac{\sum_{t=1}^T \alpha_t^2}{\sum_{t=1}^T \alpha_t}\right)$. As an example, if we set the learning rate as $\alpha_t = \frac{1}{\sqrt{t}}$, we can get the convergence with rate $\mathcal{O}(\frac{1}{\sqrt{T}})$.

Remark: The bias is introduced by the policy gradient bias and the Markov noise. The bias introduced by the Markov noise can be arbitrarily small if we set τ sufficiently large which doesn't change the main order of the convergence rate. The bias introduced by the policy gradient bias is analyzed in Theorem 1. According to the theoretical results, to reduce policy gradient bias, we need to control the model prediction error and the model gradient error in the model learning phase.

We summarize the necessary assumptions here. These assumptions are commonly used for analyzing reinforcement learning algorithms.

Assumption 1 (Bounded region)**.** *There exist a finite $D < \infty$ such that: (Wu et al., 2020; Wang et al., 2017)*

$$\|s_1 - s_2\| \leq D \text{ for } \forall s_1, s_2 \in \mathcal{S}. \tag{8}$$

Assumption 2 (Lipschitz)**.** *The transition model and the reward model are both Lipschitz continuous with Lipschitz constant L_f and L_r respectively (Clavera et al., 2019).*

Assumption 3 (Uniform periodicity)**.** *For a fixed policy $\pi_\theta(\cdot|s_0)$, denote μ_θ as the stationary distribution induced by the policy and the transition probability measure $P(\cdot|s,a)$. Then there exists $m < \infty$, $C_0 < \infty$, and $\rho \in (0,1)$ such that:(Wu et al., 2020; Zhang et al., 2020a; Qiu et al., 2021)*

$$d_{TV}\left(P(s_{t+\tau} \in \cdot|s_0 = s), \mu_\theta(\cdot)\right) \leq m\rho^\tau \ \forall\tau \geq 0, \ \forall s \in \mathcal{S}. \tag{9}$$

Assumption 4 (Lipschitz policy)**.** *Let $\pi_\phi(a|s)$ be a policy parameterized by θ. there exist constants such that for all given state s and action a we have:(Wu et al., 2020; Clavera et al., 2019; D' Oro & Jaśkowski, 2020)*

$$\left\|\frac{d\pi_\phi(a|s)}{d\phi}\right\| \leq L_\pi. \tag{10}$$

Assumption 5 (Step-size)**.** *The step-size α_t is non-increasing and non-negative (Wang et al., 2017; Wu et al., 2020).*

Assumption 6 (Bounded reward)**.** *The reward is bounded in the region $[0, r_{max}]$ (Clavera et al., 2019; Wu et al., 2020).*

4.2 Two-Model-Based Learning Policy Optimization

The theoretical result in the previous subsection indicates that both the model prediction error and the model gradient error are crucial for policy optimization. Therefore, to do efficient policy optimization, we need to consider both errors in the model learning phase. In this subsection, we will discuss how to learn and how to use the model taking both errors into account. Different from the classical model-based methods, we design a two-model-based learning method to achieve the goal. We called these two models prediction models and the gradient models respectively. Our goal is to use the prediction models to predict the next state and the reward and to use the gradient models to

calculate the policy gradient. In the following part, we will show how to learn and how to use these two models. Specifically, our approach repeatedly alternates between collecting samples from the environment, training the model, and optimize the policy.

Model Learning: Training of the prediction model is commonly through supervised learning, e.g., maximum likelihood with early stopping on a validation set (Janner et al., 2019; Clavera et al., 2019). We use the data collected from the environment to construct the mean square prediction error and use the error as learning signals for the parameter optimization. Training of the gradient model is more complex. For the purpose of reducing the gradient error, we need to use the data collected from the environment to estimate the gradient of the environment model and construct the mean square gradient error for the parameter optimization.

Policy Optimization: To update the policy, firstly, we generate trajectories by unrolling the latest policy on the prediction model. Then we calculate the gradient of the objective by back-propagating through the gradient model. Finally, we update the parameters in the policy by following the policy gradient direction.

4.3 Directional Derivative Projection Policy Optimization: A Practical Implementation

In this subsection, we propose Directional Derivative Projection Policy Optimization (DDPPO) as an implementation of our proposed two-model-based learning method. The key question is how to use the data collected from the environment to estimate the gradient or Jacobian of the environment model. Note that the dimension of the Jacobian of the learned environment model is the product of the state's dimension and the action's dimension, which is usually large. To directly estimate the gradient or Jacobian matrix by perturbation of the elements in the state or action is intractable and not efficient. In particular, for model-based RL, we cannot estimate gradients by perturbing one dimension of a state or action and calculating the difference in output, because we can only sample data from the environment, and this is contrary to the purpose of efficiency. Although the model's gradient or Jacobian matrix is difficult to estimate directly on account of the high dimensions, the directional derivative which is a scalar can be estimated by finite difference using data sampled. Meanwhile, the directional derivative of the learned model can be calculated by projecting the model's gradient or Jacobian matrix into a certain direction.

Hence in this paper, we use the error between the estimated directional derivative and the projection value to constraint the learned model's gradient. We will also discuss how to design two models to rollout and calculate gradients respectively when optimizing the policy. At last the overall algorithm pseudo-code is demonstrated in Algorithm 1 in Appendix B.1.

Model Learning: Prediction loss and the gradient loss. In our approach, there are two models to be trained, i.e., prediction model $\tilde{M}^p$ and gradient model $\tilde{M}^g$. The prediction model is trained following Janner et al. (2019) via maximum likelihood:

$$J_{\tilde{M}^p}(\theta_p) = \mathbb{E}\left[\log \tilde{M}^p(s_{t+1}, r_t | s_t, a_t)\right]. \tag{11}$$

To improve the ability of models to portray complex environment, we use a bootstrap ensemble of models $\{\tilde{M}_{\theta_1}, \ldots, \tilde{M}_{\theta_B}\}$ which is consistent with Janner et al. (2019); Clavera et al. (2019). Each member of the ensemble is learned by Equation 2. To generate a prediction from the model ensemble, we select a model uniformly at random and use the selected model to perform a model rollout at each time step.

Different from the prediction model, the gradient model aims to better characterize the gradient of the environment instead of an accurate prediction. Hence, we hope to trade-off the original maximum likelihood objective by a gradient loss term that constraints the Jacobian of the learned model. To construct the gradient loss term, we utilize the finite-difference. Note that for an input $\boldsymbol{x} = (s, a)$, and the corresponding output $M(\boldsymbol{x})$ with dimension d, the projection of the Jacobian along a vector v, i.e., $\nabla_{\boldsymbol{x}} M(x) \cdot \frac{\boldsymbol{v}}{||\boldsymbol{v}||}$, can be estimated by finite difference, $\frac{M(\boldsymbol{x}+\boldsymbol{v}) - M(\boldsymbol{x})}{||\boldsymbol{v}||}$. Therefore we use the finite difference as a target to constraint the Jacobian of the gradient model. Since the finite difference converges to the directional derivative when the norm of $\boldsymbol{v}$ goes to zero, the data point $\boldsymbol{x} + \boldsymbol{v}$ should be close to $\boldsymbol{x}$. So in this paper, we sample the nearest n data points of x to calculate n directional derivatives in n different directions and use the finite difference to constraint

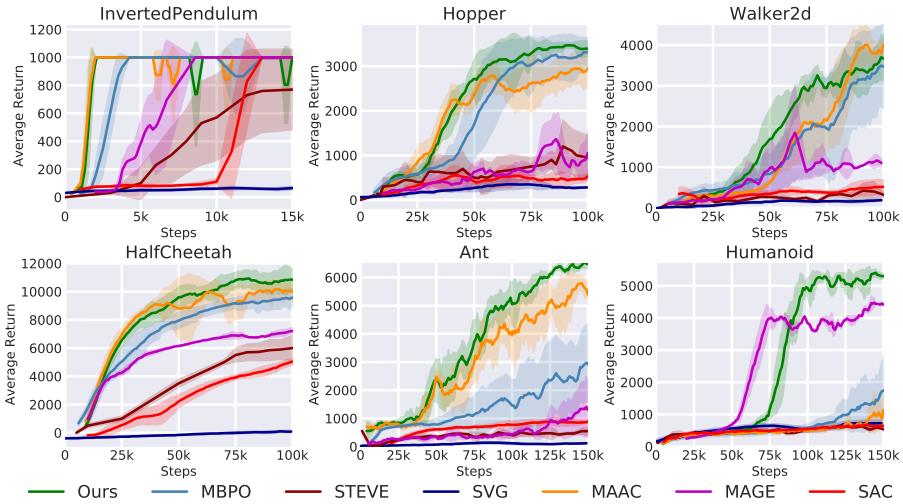

Figure 1: Learning curves of our algorithm and baselines on six different MuJoCo environments. Solid curves express the mean of five trials. Shaded regions correspond to standard deviation among trials. Our method has better sample efficiency than the model-based baselines. The improvement of performance over previous algorithms is large on complex environments, i.e., Ant and Humanoid.

the Jacobian of the learned model. The gradient loss item can be constructed as follows:

$$R = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{d}||\nabla_{\boldsymbol{x}}\tilde{M}(\boldsymbol{x})\cdot\boldsymbol{v}_i - (\tilde{M}(\boldsymbol{x}_i) - \tilde{M}(\boldsymbol{x}))||^2, \tag{12}$$

where $\boldsymbol{v}_i$ denotes directions $\boldsymbol{x}_i - \boldsymbol{x}$, and $\boldsymbol{x}_i$ are the nearest n points of $\boldsymbol{x}$ in the buffer. Intuitively, the gradient at $\boldsymbol{x}$ can be constrained if there are enough points close enough to $\boldsymbol{x}$. By leveraging the gradient loss term, the gradient mode $\hat{M}^g$, is trained via a multi-objective loss:

$$J_{\tilde{M}^g}(\theta_g) = \mathbb{E}\left[\log \tilde{M}^g(s_{t+1}, r_t|s_t, a_t)\right] + w\cdot R. \tag{13}$$

Finally, the trained gradient model and the prediction model will be used for policy optimization.

Model Usage: Policy optimization by back-propagating through model. In our DDPPO algorithm, to train the policy, we use the objective function that constructed by following SAC (Haarnoja et al., 2018), since SAC can achieve outstanding performance. The policy is learned by rolling out the prediction model for H steps and maximizing the following objective function:

$$J_\pi(\phi) = \mathbb{E}\left[\sum_{t=0}^{H-1}\gamma^t(\tilde{r}_t - \alpha\log\pi_\phi(a_t|\tilde{s}_t)) + \gamma^H(\tilde{Q}(\tilde{s}_H, a_H) - \alpha\log\pi_\phi(a_H|\tilde{s}_H))\right], \tag{14}$$

where $\tilde{s}_{t+1} \sim \tilde{M}^p_{\psi_p}$, $\tilde{s}_0$ random sampled from the environment buffer $\mathcal{D}_{env}$, $a \sim \pi_\phi$, α is the temperature hyperparameter proposed in SAC. Note that the policy can be learned by just computing the derivative of the objective since the learned prediction model $\tilde{M}^p$ is differentiable. However, in our approach, we replace the gradient $\nabla_{\tilde{s}_t,a_t}\tilde{M}^p(\tilde{s}_t, a_t)$ by $\nabla_{\tilde{s}_t,a_t}\tilde{M}^g(\tilde{s}_t, a_t)$ to calculate the full gradient of the objective , believing in that the gradient model can capture the true gradient better than the prediction model.

As for the Q-value function, it can be trained by minimizing the Bellman residual. In practice, we follow SAC (Haarnoja et al., 2018) that trains two Q-functions to mitigate positive bias which is known to degrade the performance of value-based methods (Hasselt, 2010; Fujimoto et al., 2018), and make use of target Q-function which has been shown to stabilize training (Mnih et al., 2015).

5 Results

The goal of our experimental evaluation is to answer the following questions: (1) How does our algorithm perform on benchmark reinforcement learning tasks, compared to state-of-the-art model-based and model-free algorithms? (2) Does it matter to explicitly learn the gradient of the model

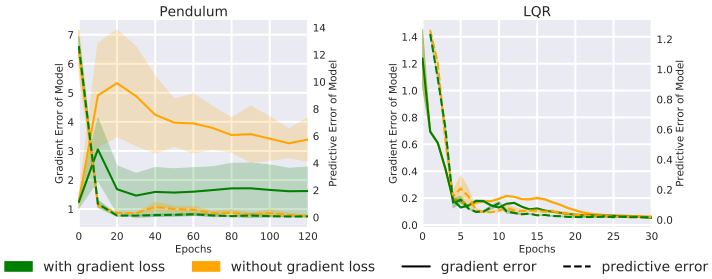

Figure 2: Learning curves of models with and without gradient loss on two simple environments. Data used to train each model is collected by a random policy. Solid curves express the mean of gradient error of five trials. Dotted lines express the mean of prediction error during the training of the model. Shaded regions correspond to standard deviation among trials. The model with gradient loss has a lower gradient error than the model without gradient loss. The improvement is more pronounced in nonlinear cases, i.e., Pendulum environment.

by using the gradient loss? (3) How does the sensitivity of our algorithm when changing the hyper-parameters n and w that are related to the gradient model learning? (4) Why build two models? To answer these questions, we evaluate our approach on model-based continuous control benchmarks in the MuJoCo simulator (Todorov et al., 2012). Experimental details can be found in Appendix B.2.

5.1 Comparison with State-of-the-Arts

To answer the first question, we compare our method against state-of-the-art model-based and model-free methods in terms of sample complexity and asymptotic performance. Model-free baseline includes SAC (Haarnoja et al., 2018), which is a widely accepted baseline. For model-based methods, we select MBPO (Janner et al., 2019) and STEVE (Buckman et al., 2019) as our baseline which both use short-horizon model-based rollouts. We also compare to model-based methods SVG (Heess et al., 2015), which firstly uses the derivative of models to learn the policy, and MAAC, (Clavera et al., 2019) similar to SVG, however entirely relies on the predictions of the model, removing the need for likelihood ratio terms. MAGE (D' Oro & Jaśkowski, 2020) is added to comparison since this method focuses on the gradient of the Q-function.

The learning curves for all methods, shown in Figure 1, highlight the strength of our method in terms of performance and sample complexity. In all the MuJoCo simulator environments, especially in two higher-dimensional tasks, Ant and Humanoid, our method learns faster and has better efficiency than previous model-based and model-free methods. Note that MAGE learns fastest on Humanoid, and the trick of this algorithm can also be applied to our method. However, even without the help of tricks in MAGE, we can still achieve better asymptotic properties. We get the near-optimal policies in the Humanoid environment using just over 100 thousand interactions with the environment, while around three times as many interactions are needed for MBPO to achieve the same performance.

5.2 Gradient Error Analysis

In this subsection, we answer the second question, i.e., Does it matter to explicitly learn the gradient of the model by using the gradient loss? Firstly, we aim to analyze the difference between the gradient of the true model and the learned model. We collect experience using a random policy and train the model with and without gradient loss respectively. Then we calculate the MSE error of the gradient of the model. Figure 2 shows how the gradient error and the prediction error changes during training of the model on two simple environments that the true gradient is analytical available. From the results in Figure 2 we can see that the prediction error for both models decreases to zero while the gradient error of the model without gradient loss keeps larger than that of the model with gradient loss. The learning curves highlight the strength of adding a gradient loss to the model in terms of high prediction accuracy and accurate gradient estimation under the simple case.

5.3 Ablation Study

To answer the third question, i.e., How does the sensitivity of our algorithm when changing the hyper-parameters, we conduct two sets of experiments in both Hopper and Ant environments. Figure

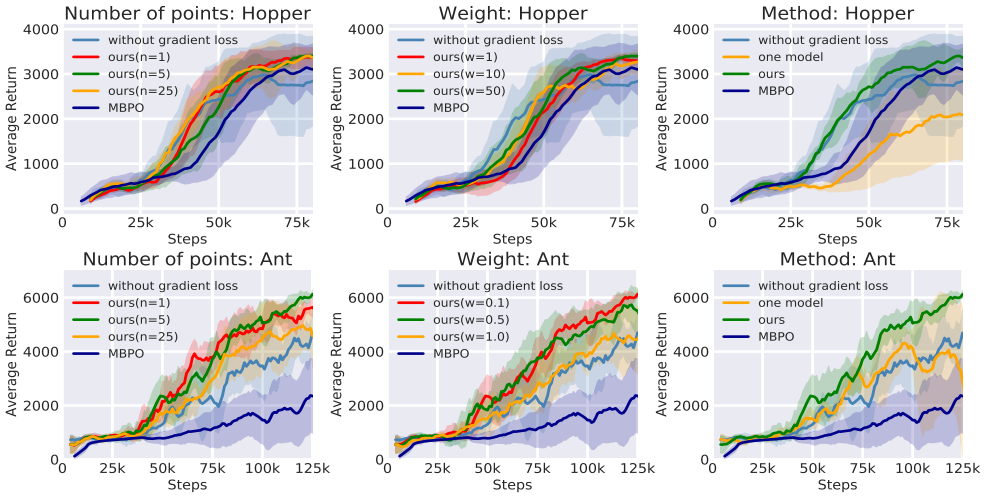

Figure 3: **Left:** The performances of our method with different n indicate our method is not sensitive to the number of directional derivatives used to construct the gradient loss. **Middle:** The performances of our method with different w, the coefficient in front of the gradient loss. The weight is recommended to be smaller for a higher-dimensional environment. **Right:** Our method outperforms the variant (without gradient loss) which underpins the importance of considering the gradient error when training the model. And the higher performances than the variant (One model) that only uses one model highlight the strength of the two-model-based structure.

3 shows the results. The left column shows how the efficiency of our algorithm changes by tuning n. The results demonstrate that, regardless of the value n, our method is better than our variant without gradient loss. The middle column shows the results for tuning w. The results indicate that the weight is recommended to be smaller for a higher-dimensional environment and the performance is not sensitive to the number of directional derivatives used to construct the gradient loss, even only one directional derivative used can also help to improve the performance.

Now we answer the last question, i.e., why build two models. To investigate the importance of considering the gradient loss and the important role of two-model-based, we carry out an ablation test on Hopper and Ant. Specifically, we remove the gradient loss in our method and retrain the policy (without gradient loss) to see the importance of considering the gradient loss. To see the impact of training two models for prediction and gradient calculation respectively, we design a variant of our method that trains only one model with gradient loss used for both purposes (one model). The experimental results are shown in the right column of Figure 3. The results underpin the importance of considering the gradient error when training the model if back-propagating through it. On the other hand, the results highlight the strength of the two-model-based structure. We also draw the prediction error of both models in our algorithm and the results are shown in Appendix B.3. The prediction error of the prediction model is lower than that of the gradient model, which further validates the idea of two-model-based learning.

6 Conclusions and Future Work

In this paper, we theoretically analyze how the model gradient error and the model prediction error influence the policy gradient bias and the convergence rate of model-based policy optimization. The theoretical results motivate us to design a two-model-based learning method for policy optimization. We design the learning and using methods for the two models respectively by taking the model gradient information into account and propose DDPPO algorithms as a practical implementation. Our algorithm can achieve superior sample efficiency than state-of-the-art model-based reinforcement learning algorithms on challenging high-dimensional continuous control tasks. For future work, it is enticing to investigate and design a better gradient estimator in order to optimize the gradient model more efficiently. The idea about learning the model according to its usage instead of only considering its prediction error is also general enough to be applied to other problems beyond reinforcement learning.

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A RELATED WORK

Model-free and Model-based Method: Model-free approaches, as a general-purpose tool for learning complex policies (Mnih et al., 2015; Lillicrap et al., 2016; Haarnoja et al., 2018), has the problem of low-efficiency (Janner et al., 2019), which limits the application in real-world physical systems where data collection can be an arduous process. Model-based Reinforcement Learning attempts to reduce sample complexity while maintaining the asymptotic performance. The learned model can be viewed as a black-box simulator and then used for training a model-free policy (Nagabandi et al., 2018; Luo et al., 2018; Kurutach et al., 2018; Wang et al., 2019; Janner et al., 2019; Pan et al., 2020). Tools like ensembles (Kurutach et al., 2018), probabilistic models (Chua et al., 2018), and meta-learning (Clavera et al., 2018) are widely used in model-based RL. A major problem of model-based RL is model-bias (Deisenroth & Rasmussen, 2011) caused by a compounding error effect of long-term predictions. To address this problem, a popular approach is to use interpolation between different horizon predictions (Buckman et al., 2019; Janner et al., 2019) and interpolating between model and real data (Kalweit & Boedecker, 2017). Different from the traditional model based method, we argue to learn the model beyond the accurate prediction and to use the model more efficiently by calculating the policy gradient analytical through the learned model.

Improved Model-based Methods: Several issues have been studied to achieve better performance in model-based RL. Malik et al. (2019) explores which uncertainties are needed for model-based RL. Further research implements a masking mechanism based on the model's uncertainty estimation to decide whether the model should be used or not (Pan et al., 2020). Dropout mechanisms are also applied to model-based RL algorithms to improve the robustness while maintaining high sampling efficiency (Zhang et al., 2021b). Efficient explorations is also considered in model-based RL (Curi et al., 2020; Song & Sun, 2021; Yao et al., 2021). For model learning, Ke et al. (2018) use latent variables to carry future information that improves long-term predictions, Tomar et al. (2021) use model-invariance state abstraction to improve generalization. Zhang et al. (2021a) shows that tuning of hyperparameters dynamically improves the performance compared to using static hyperparameters which are kept fix for the whole training. This operation can be seen as a simple treatment of model uncertainty. Considering that we also need the prediction model at the model learning and using phase, we can leverage the useful ideas inside of these methods to help us to build a robust prediction model and use it efficiently.

Decision-aware Model Learning: Decision-aware model learning (DAML) (Lovatto et al., 2020; Nair et al., 2020; Abachi et al., 2021; D'Oro et al., 2020; Farahmand et al., 2017; Modhe et al., 2021) considers the problem that how we can learn the model that can be better leveraged by the policy optimization or Q learning. D'Oro et al. (2020) using policy gradient to re-weighted the model prediction error for each sample. Farahmand et al. (2017) use the value function to re-weighted the model prediction error for each sample. Please note that, although the main goal about learning the model by considering the follow-up decision making problems seems very similar, our ideas and methods are totally different from DAML. DAML is still the traditional rollout-based model-based method with a re-weighting prediction loss.

Differentiable Model-based Method: A more effective way of using the model is to exploit its differentiability. SVG (Heess et al., 2015) presents a framework for learning continuous control policies using back-propagating through the model. However, SVG just uses real trajectories leveraging likelihood ratio term which in turn increases the variance of the gradient estimate. Amos et al. (2021) extend SVG further leveraging the soft update (Haarnoja et al., 2018) and model-based value expansion (Feinberg et al., 2018). Different from SVG, MAAC (Clavera et al., 2019) entirely relies on the predictions of the model, removing the need for likelihood ratio terms, and achieves better performance than MBPO leveraging model-based value expansion for updating policy and Q-function. The plan horizon in these methods allows to trade-off between the accuracy of the learned model and the accuracy of the learned Q-function. In addition to back-propagating through the path, MAGE (D' Oro & Jaśkowski, 2020) uses the differentiability of the learned model to compute gradient targets in temporal difference learning. Our method is different from these methods. Instead of Different from these methods, our main contribution is that we propose to use two models to learn the model prediction and the model gradient respectively. By explicitly construct the gradient estimator, our gradient model can produce a more accurate policy gradient and thus benefits for the policy optimization.

Outside of RL: Several algorithms outside of RL adopt gradient penalties into the loss function when training a neural network, known as double backpropagation (Drucker & Le Cun, 1992). Sobolev training (Czarnecki et al., 2017) is most related to this topic, which tries to learn both the value and gradient of a target function during supervised training. However, Sobolev training requires the ground-truth gradient available. In the model-based reinforcement learning setting, we do not know the ground-truth gradient.

B ALGORITHM DETAILS

B.1 IMPLEMENTATION OF DDPPO

Algorithm 1 Directional Derivative Projection Policy Optimization

Initialize the policy π_ϕ, predictive model $\tilde{M}^p_{\theta_p}$, gradient model $\tilde{M}^g_{\theta_g}$, value funcrion Q_ψ
Initialize environment replay buffer $\mathcal{D}_{env}$, model replay buffer $\mathcal{D}_{model}$
repeat
  Take an action in real environment with policy π_ϕ, and add transition (s, a, s') to $\mathcal{D}_{env}$
  Train predictive model $\tilde{M}^p_{\theta_p}$ and update θ_p by maximizing $J_{\tilde{M}^p}(\theta_p)$
  Train gradient model $\tilde{M}^g_{\theta_g}$ and update θ_g by maximizing $J_{\tilde{M}^g}(\theta_g)$
  for $N_{rollouts} steps$ **do**
    Sample s_t uniformly from $\mathcal{D}_{env}$
    Perform k-step model rollout on $\tilde{M}^p_{\theta_p}$ starting from s_t with policy π_ϕ
    Add samples to $\mathcal{D}_{model}$
  end for
  $\mathcal{D} \leftarrow \mathcal{D}_{env} \cup \mathcal{D}_{model}$
  for $N_{update\ Q}\ steps$ **do**
    Update ψ using data form $\mathcal{D}$: $\psi \leftarrow \psi - \lambda_Q \nabla_\psi J_Q(\psi)$
  end for
  for $N_{update\ Policy}\ steps$ **do**
    Sample trajectories of length H with policy π and predictive model $\tilde{M}^p_{\theta_p}$
    Update ϕ using sampled trajectories and gradient model $\tilde{M}^g_{\theta_g}$: $\phi \leftarrow \phi + \lambda_\pi \nabla_\pi J_\pi(\phi)$
  end for
until The policy performs well in real environment M
return Optimal policy π_{ϕ^*}

Our algorithm can be divided into three main parts: model learning, policy optimization, and Q-function learning. And our approach alternates between collecting samples from the environment, training the model, updating Q-function, and policy optimization.

Firstly, we collect data using the current policy and add it into the environment replay buffer $\mathcal{D}_{env}$. Then the prediction model is trained using data from $\mathcal{D}_{env}$. The gradient model is also trained with the multi-objective loss. The third step is to perform k step model rollout on the prediction model $\tilde{M}^p_{\theta_p}$ with the current policy, where k is increased over time which proposed by Janner et al. (2019) to achieve better performance. Those samples are appended to the replay buffer $\mathcal{D}_{model}$, together with samples from $\mathcal{D}_{env}$, used to update the Q-function. To update the policy, firstly, we obtain imaginary trajectories by unrolling the latest policy on the prediction model $\tilde{M}^p_{\theta_p}$ for H steps from a randomly sampled state in $\mathcal{D}_{env}$. Then we update policy by calculating the gradient of the objective in equation 14, while using the gradient model $\tilde{M}^g_{\theta_g}$ instead of $\tilde{M}^p_{\theta_P}$.

In our approach, we apply the soft Q-update (Haarnoja et al., 2018) which minimizes the following objective:

$$J_Q = \mathbb{E}\left[\left(\tilde{Q}(s_t, a_t) - \left(r(s_t, a_t) + \gamma \tilde{Q}(s_{t+1}, a_{t+1}) - \alpha \log(\pi_\phi(a_{t+1}|s_{t+1}))\right)\right)^2\right]. \tag{15}$$

Table 1: Hyperparameter settings for DDPPO results shown in Figure 1.

Environment Name	**InvertedPendulum**	**Hopper**	**Walker2D**	**HalfCheetah**	**Ant**	**Humanoid**
epochs	15	100	100	100	150	150
environment steps /epoch	1000					
ensemble size	7					
G_1 /environment step	10					
G_2 /environment step	10					
H	3	2	3			
n	10	25	25	5		
w	10	50	0.1	1.0	0.1	0.1

The temperature hyperparameter is automatically tuned by adjusting the expected entropy over the visited states to match a target value (Haarnoja et al., 2018) with the following objective:

$$J_\alpha = \mathbb{E}\left[-\alpha \log \pi_\phi(a_t, s_t) - \alpha\bar{\mathcal{H}}\right] \tag{16}$$

where $\bar{\mathcal{H}}$ is the target entropy.

B.2 Experiment Settings

B.2.1 Environment Settings

We evaluate our approach on six continuous control benchmark tasks in the MuJoCo (Todorov et al., 2012) simulator in our experiments: InvertedPendulum-v2, Hopper-v2, Walker2d-v2, HalfCheetah-v2, Ant-v2, Humanoid-v2. Standard full-length versions of these tasks are used. Note that the Ant and Humanoid environments are truncated observations which is consistent with MBPO (Janner et al., 2019). The code for building Ant and Humanoid environment is provided by Janner et al. (2019).[1]

B.2.2 Hyperparameter Settings

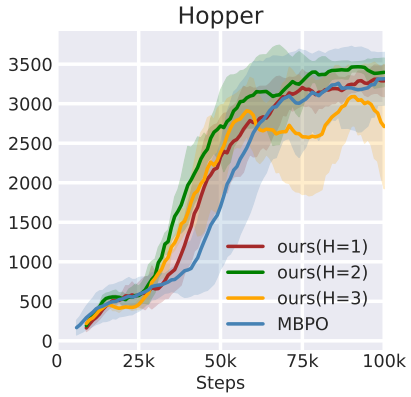

Table 1 shows the hyperparameters used for DDPPO to achieve results shown in Figure 1. G_1 is the number of updates for policy per environment step, G_2 is the number of updates for critic per environment step. H is the rollout length for updating policy. n is the number of directional derivatives used to construct the gradient loss. w is the coefficient in front of the gradient loss when updating the gradient model. Note that H is used to trade-off between the accuracy of the learned model and the accuracy of the learned Q-function. As shown in the right figure, the results by setting a small H is close to the results of MBPO, while a larger H may cause worse results. We try different steps and propose 2 for Hopper and 4 for other environments. We employ 5000 (200 for the InvertedPendulum environments) warmup steps of interaction with the environment before starting to update the actor and the critic. The network structure of both the prediction model and the gradient model is the same as MBPO. Both models are trained after every 250 steps of interaction with the environment. Every time after model training, we perform rollout for k steps and add the collected data to the buffer used for updating the critic. k is increased with the number of interactions. For InvertedPendulum, Ant, HalfCheetah, and Walker2D k keeps fixed. For Hopper k increases from 1 to 15 between epoch 25 to epoch 155. For Humanoid k increases from 1 to 25 between epoch 25 to epoch 155. The policy and Q-value function are updated 10 times per environment step which is less than the number of MBPO. For updating the policy we rollout on the prediction model for H steps with batch size 256.

[1]https://github.com/JannerM/mbpo/tree/master/mbpo/env

B.3 More results for ablations

The results in Figure 3 show the importance of considering the gradient error when training the model if back-propagating through it and highlight the designed two-model-based learning procedure. We extend the experiments in Section 5.3 to more environments. The results in Figure 4 show the performance of our method compared to the variants of our method. The results in Figure 5 show that the prediction models in our algorithm have lower prediction error than the gradient models, which confirm our conjecture that the prediction and gradient calculation should be done by the corresponding model.

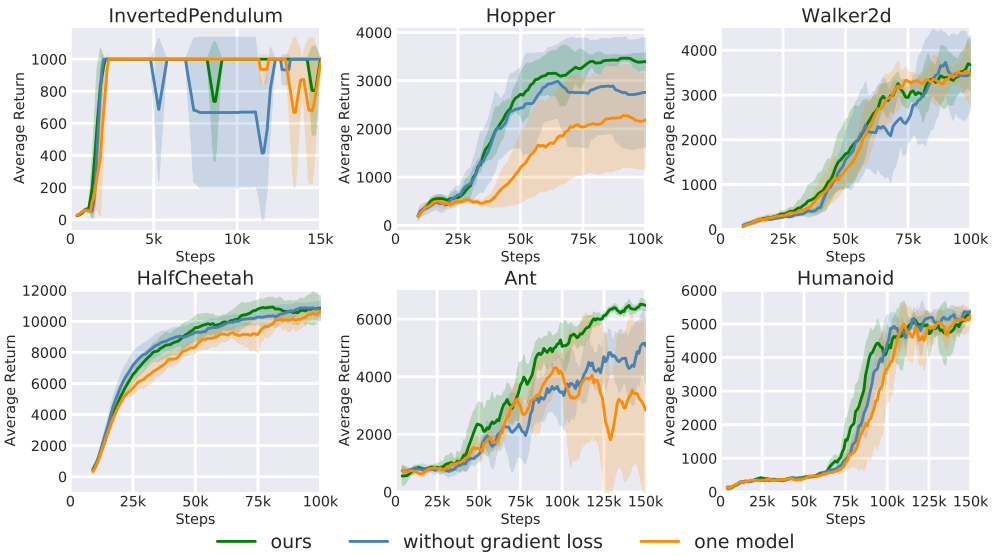

Figure 4: The impact of adding the gradient loss and the importance of two-model-based learning.

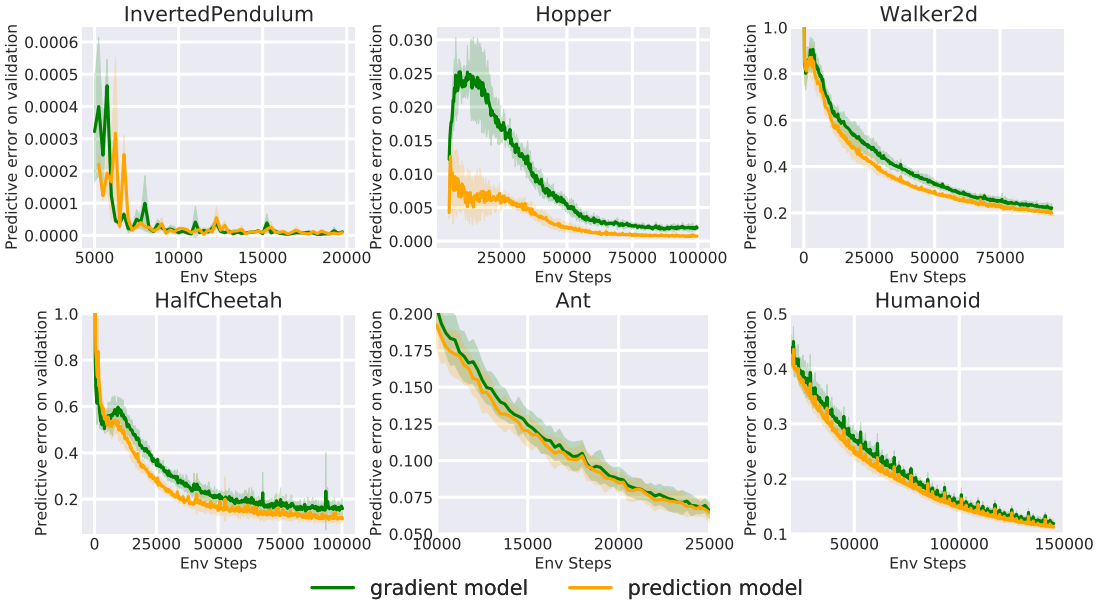

Figure 5: Prediction error of the prediction model and the gradient model on validation during the training of the agent.

C PROOFS OF THE THEOREM

In this section, we present the proof of the main theorems in section 3.

First of all, we show some assumptions and lemmas that is important for the proof.

The first assumption is about the stationary property of the trajectory. We assume that two trajectories started from different initial point will not be arbitrary far from each other even after sufficient longer time.

Assumption 7 (Stationary)**.** *For $\forall t_1 < t_2$ and $s^0_{t_1}, s^1_{t_1}$, denote $\tau = t_2 - t_1$ there exist $L_1(\tau) < \infty$, $\tau = t_2 - t_1$, such that $\left\| \mathbb{E}\left(\frac{s^1_{t_2} - s^0_{t_2}}{s^1_{t_1} - s^0_{t_1}}\right) \right\| \leq L_1(\tau)$. And $L_1(\tau)$ can be uniformly upper bounded by $L_1 < \infty$.*

Lemma 1. *Suppose assumptions hold, denote the $C_0 = \min(\frac{D}{\epsilon_f}, \frac{1-L_f^{t+1}}{1-L_f})$ for $\forall t$ we have*

$$\|\tilde{s}_t - s_t\| \leq C_0 \epsilon_f \tag{17}$$

Lemma 2. *Denote $C_1 = tL_1^2$, $C_2 = tC_0 L_f L_1^2$. We have*

$$\delta\left(\mathbb{E}\left(\frac{ds_t}{ds_0}\right)\right) = \left\| \mathbb{E}\left(\widehat{\frac{ds_t}{ds_0}} - \frac{ds_t}{ds_0}\right) \right\| \leq C_1 \epsilon_f^g + C_2 \epsilon_f \tag{18}$$

Lemma 3. *If constent $0 < \gamma < 1$, for arbitrary constant C, we have:*

$$\sum_{t=0}^{\infty} t\gamma^t C = \frac{\gamma}{(1-\gamma)^2} C \tag{19}$$

Lemma 4. *Denote constants $C_3 = \frac{L_1}{1-\gamma}$ and $C_4 = \frac{L_r L_1^2 \gamma}{(1-\gamma)^2}$,$C_5 = \frac{C_0 L_1 L_f}{(1-\gamma)} + \frac{C_0 L_f L_r \gamma}{(1-\gamma)^2}$, then we have:*

$$\delta\left(\frac{dR}{ds}\right) = \left\| \mathbb{E}\left(\frac{d\widehat{R^M}}{ds} - \frac{dR^M}{ds}\right) \right\| \leq C_3 \epsilon_r^g + C_4 \epsilon_f^g + C_5 \epsilon_f \tag{20}$$

Lemma 5. *Considering a Markov process M with stationary distribution μ. Data ξ_t are sampled from the M. There are two functions of parameter ϕ: $G(\phi_t, \xi_t)$ and $g(\phi_t)$ where*

$$g(\phi_t) = \mathbb{E}_{\xi \sim \mu} G(\phi_t, \xi). \tag{21}$$

Denote the upper bound for the function G as L_1. Denote the Lipschitz constant for the function G as L_2.

$$\phi_{t+1} = \phi_t - \alpha_t G(\phi_t, \xi_t) \tag{22}$$

For all $0 < T < \infty$, we can proof that:

$$\left\| \sum_{t=1}^{T} \alpha_t \left\langle g(\phi_t), G(\phi_t, \xi_t) - g(\phi_t) \right\rangle \right\| \leq 2\tau\alpha_0 L_1^2 + \sum_{t=\tau+1}^{T} \alpha_t^2 L_1^2 L_2 \tau + \sum_{t=1}^{T} \alpha_t L_1 m \rho^\tau \tag{23}$$

Lemma 6. *Denote U_J as the upper bound of the expected sum of discounted rewards J. Then $U_J \leq \frac{r_{max}}{1-\gamma}$*

Lemma 7. *There exist two constants L_J and L_J^g such that for all ϕ_1, ϕ_2, we have*

$$\|J(\phi_1) - J(\phi_2)\| \leq L_J \|\phi_1 - \phi_2\| \tag{24}$$

$$\left\| \frac{dJ}{d\phi_1} - \frac{dJ}{d\phi_2} \right\| \leq L_J^g \|\phi_1 - \phi_2\| \tag{25}$$

Lemma 8. *There exist constants L_R such that for all ϕ_1, ϕ_2, we have*

$$\mathbb{E}\|R(\phi_1) - R(\phi_2)\| \leq L_R \|\phi_1 - \phi_2\| \tag{26}$$

$$\tag{27}$$

Lemma 9. *Suppose the function J is smooth with smoothness constant L_J^g*

$$J(\phi_2) \geq J(\phi_1) + \langle \frac{dJ}{d\phi_1}, \phi_2 - \phi_1 \rangle - \frac{L_J^g}{2} \|\phi_2 - \phi_1\| \tag{28}$$

Lemma 10. *If $x, y, z > 0$ and $\sqrt{x} \leq \sqrt{y} + \sqrt{z}$, then we have*

$$x \leq 2y + 2z \tag{29}$$

C.1 PROOF OF THEOREM 1

Overall speaking, the main idea behind the proof of Theorem 1 can be summarized as follows.

Considering that to calculate a gradient value of a function output w.r.t its input, we need to specify the function and the input value. Therefore, as in the Equation 3, both the function($\tilde{f}$ and $\tilde{r}$) to be taking gradient and the input value $\tilde{s_t}$ are different from the ground-truth. So we need to first bound the error of the prediction state(Lemma 1) and then bound the error of the gradient of the learned model in true state (Lemma 4).

Denote $C_6 = \frac{L_f L\pi + L_f C_3 L\pi\gamma}{1-\gamma}$, $C_7 = \frac{\gamma L_f L_\pi C_4 + L_1 L_\pi}{1-\gamma}$, $C_8 = \frac{C_0 L_f + \gamma C_5 L_f L_\pi + L_1 L_\pi L_f C_0}{1-\gamma}$, we can prove the theorem by using previous lemmas:

$$\left\| \mathbb{E}\left(\frac{dR^{\widehat{M}}}{d\phi} - \frac{dR^M}{d\phi} \right) \right\| \tag{30}$$

$$= \left\| \mathbb{E}\left(\sum_{t=0}^{\infty} \gamma^t \left(\widehat{\frac{d\tilde{r}_t}{d\phi}} - \frac{dr_t}{d\phi} \right) + \gamma \sum_{t=0}^{\infty} \gamma^t \left(\widehat{\frac{dR}{d\tilde{s}_{t+1}}} \widehat{\frac{d\tilde{s}_{t+1}}{d\tilde{a}_t}} \frac{d\tilde{a}_t}{d\phi} - \frac{dR}{ds_{t+1}} \frac{ds_{t+1}}{da_t} \frac{dat}{d\phi} \right) \right) \right\| \tag{31}$$

$$\leq \frac{1}{1-\gamma}(L_f C_0 \epsilon_f + \epsilon_r^g) L_\pi + \left\| \mathbb{E}\left(\gamma \sum_{t=0}^{\infty} \gamma^t \left(\delta\left(\frac{dR}{ds}\right) \widehat{\frac{d\tilde{s}_{t+1}}{d\tilde{a}_t}} + \frac{dR}{ds_{t+1}} \delta\left(\frac{ds}{da}\right) \right) L_\pi \right) \right\| \tag{32}$$

$$\leq \frac{1}{1-\gamma}(L_f C_0 \epsilon_f + \epsilon_r^g) L_\pi + \frac{\gamma}{1-\gamma} L_f L_\pi (C_3 \epsilon_r^g + C_4 \epsilon_f^g + C_5 \epsilon_f) + \frac{1}{1-\gamma} L_1 L_\pi (L_f C_0 \epsilon_f + \epsilon_f^g) \tag{33}$$

$$= C_6 \epsilon_r^g + C_7 \epsilon_f^g + C_8 \epsilon_f. \tag{34}$$

The first equation is the definition. The second inequality follows the assumption 7, the right term . The third term follows the lemma 4

C.2 PROOF OF THE THEOREM 2

According to the lemma 8, we can have:

$$J(\phi_{t+1}) \geq J(\phi_t) + \alpha_t \left\langle \frac{dJ}{d\phi_t}, \frac{dR^{\hat{M}}}{d\phi} \right\rangle - \frac{L_J}{2} \|\phi_{t+1} - \phi_t\|^2, \tag{35}$$

where $\frac{dR^{\hat{M}}}{d\phi}$ is the estimated policy gradient calculated using learned model and used to update the policy parameter from ϕ_t to ϕ_{t+1}.

Denote $h_t^1 = \frac{dR^{\hat{M}}}{d\phi} - \frac{dR^M}{d\phi}$, $h_t^2 = \frac{dR^M}{d\phi}$.

We can further decompose the value improvement as

$$J(\phi_{t+1}) \geq J(\phi_t) + \alpha_t \left(\left\langle \frac{dJ}{d\phi_t}, h_t^1 \right\rangle + \left\langle \frac{dJ}{d\phi_t}, h_t^2 \right\rangle \right) - \frac{L_J^g}{2} \|\phi_{t+1} - \phi_t\|^2. \tag{36}$$

According to the Cauchy–Schwarz inequality we have

$$\mathbb{E}\left\langle \frac{dJ}{d\phi_t}, h_t^1 \right\rangle \geq -\sqrt{E\left\| \frac{dJ}{d\phi_t} \right\|^2 \mathbb{E}\left\| h_t^1 \right\|^2} \geq -\sqrt{E\left\| \frac{dJ}{d\phi_t} \right\|^2} (C_6 \epsilon_r^g + C_7 \epsilon_f^g + C_8 \epsilon_f). \tag{37}$$

Rearrange the term we have

$$\mathbb{E}\left\langle \frac{dJ}{d\phi_t}, h_t^2 \right\rangle = \mathbb{E}\left\langle \frac{dJ}{d\phi_t}, \frac{dR^M}{d\phi_t} - \frac{dJ}{d\phi_t} \right\rangle + \mathbb{E}\left\langle \frac{dJ}{d\phi_t}, \frac{dJ}{d\phi_t} \right\rangle. \tag{38}$$

Note that the first term in the right hand side is 0 if the data is I.I.D. sampled from the stationary distribution. If the data is sampled from the markov process, the data is non-i.i.d.. we can further bound the first term using the Lemma 5

$$\sum_{t=1}^{T}\alpha_t\mathbb{E}\left\langle\frac{dJ}{d\phi_t},\frac{dR^M}{d\phi_t}-\frac{dJ}{d\phi_t}\right\rangle\geq-\left(2\tau L_J\alpha_0+\sum_{t=1}^{T}\alpha_t^2L_J^2L_J^g\tau+\sum_{t=1}^{T}\alpha_tL_Jm\rho^\tau\right). \tag{39}$$

The second term is

$$\mathbb{E}\left\langle\frac{dJ}{d\phi_t},\frac{dJ}{d\phi_t}\right\rangle=\mathbb{E}\left\|\frac{dJ}{d\phi_t}\right\|^2. \tag{40}$$

Noticing that

$$\phi_{t+1}-\phi_t=\alpha_t\frac{dR^{\hat{M}}}{d\phi_t}. \tag{41}$$

Combine Equations (37), (39), (40), (41), we can get

$$\alpha_tB_1-\alpha_tB_3\sqrt{B_1}\leq J(\phi_{t+1})-J(\phi_t)+\alpha_tB_4+B_5, \tag{42}$$

where $B_1=\left\|\frac{dJ}{d\phi_t}\right\|^2$, $B_2=2\tau L_J\alpha_0+\sum_{t=1}^{T}\alpha_t^2\left(L_J^2L_J^g\tau+\frac{L_J^gL_R{}^2}{2}\right)+\sum_{t=1}^{T}\alpha_tL_Jm\rho^\tau$, $B_3=C_6\epsilon_r^g+C_7\epsilon_f^g+C_8\epsilon_f$, $B_4=\mathbb{E}\left\langle\frac{dJ}{d\phi_t},\frac{dR^M}{d\phi_t}-\frac{dJ}{d\phi_t}\right\rangle$ $B_5=\frac{L_J^g\alpha_t^2L_R{}^2}{2}$ Notice B_1,B_4,B_5 are functions of t.

Taking summarizing on both side, we have

$$\sum_{t=1}^{T}\alpha_tB_1-\sum_{t=1}^{T}\alpha_tB_3\sqrt{B_1}\leq J(\phi_{T+1})-J(\phi_0)+\sum_{t=1}^{T}\alpha_tB_4+\sum_{t=1}^{T}B_5, \tag{43}$$

$$\frac{\sum_{t=1}^{T}\alpha_tB_1}{\sum_{t=1}^{T}\alpha_t}-\frac{\sum_{t=1}^{T}\alpha_tB_3\sqrt{B_1}}{\sum_{t=1}^{T}\alpha_t}\leq\frac{2U_J+B_2}{\sum_{t=1}^{T}\alpha_t}. \tag{44}$$

We have

$$\frac{\sum_{t=1}^{T}\alpha_t(\sqrt{B_1}-\frac{B_3}{2})^2}{\sum_{t=1}^{T}\alpha_t}\leq\frac{2U_J+B_2}{\sum_{t=1}^{T}\alpha_t}+\frac{B_3^2}{4}. \tag{45}$$

Therefore,

$$\min_{1\leq k\leq T}(\sqrt{B_1}-\frac{B_3}{2})^2\leq\frac{2U_J+B_2}{\sum_{t=1}^{T}\alpha_t}+\frac{B_3^2}{4}. \tag{46}$$

According to the lemma 10 We can prove that

$$\min_{1\leq k\leq T}B_1\leq\frac{4U_J+2B_2}{\sum_{t=1}^{T}\alpha_t}+B_3^2. \tag{47}$$

C.3 Proof of the useful lemmas

Proof of Lemma 1. According to the transition model definition and its Lipschitz continuous assumption, On the one hand, since the state are bound in the range with diameter D, we can see that $\|\tilde{s_t}-s_t\|\leq D$ On the other hands,

$$\|\tilde{s_t}-s_t\|\leq\epsilon_f+L_f\|\tilde{s}_{t-1}-s_{t-1}\|\leq\left(\frac{1-L_f^{t+1}}{1-L_f}\right)\epsilon_f. \tag{48}$$

□

Proof of the Lemma2.

$$\left\|\mathbb{E}\left(\widehat{\frac{d\tilde{s}_t}{d\tilde{s}_{t-1}}}\cdot\cdots\cdot\widehat{\frac{d\tilde{s}_1}{d\tilde{s}_0}}\right)\mathbb{E}\left(\frac{ds_t}{ds_{t-1}}\cdot\cdots\cdot\frac{ds_1}{ds_0}\right)\right\| \tag{49}$$

$$=\left\|\mathbb{E}\sum_{i=1}^{t}\left(\Pi_{j=0}^{i-1}\frac{ds_j}{ds_{j-1}}\right)\left(\widehat{\frac{d\tilde{s}_i}{d\tilde{s}_{i-1}}}-\frac{ds_i}{ds_{i-1}}\right)\left(\Pi_{j=i+1}^{t}\widehat{\frac{d\tilde{s}_j}{d\tilde{s}_{j-1}}}\right)\right\| \tag{50}$$

$$\leq\mathbb{E}\sum_{i=1}^{t}\left(\left\|\Pi_{j=0}^{i-1}\frac{ds_j}{ds_{j-1}}\right\|\right)\left(\left\|\widehat{\frac{d\tilde{s}_i}{d\tilde{s}_{i-1}}}-\widehat{\frac{ds_i}{ds_{i-1}}}\right\|+\left\|\widehat{\frac{ds_i}{ds_{i-1}}}-\frac{ds_i}{ds_{i-1}}\right\|\right)\left(\left\|\Pi_{j=i+1}^{t}\widehat{\frac{ds_j}{ds_{j-1}}}\right\|\right) \tag{51}$$

$$\leq\sum_{i=1}^{t}L_1^2(\epsilon_f^g+C_0\epsilon_f L_f) \tag{52}$$

$$\leq tL_1^2\epsilon_f^g+tC_0L_fL_1^2\epsilon_f. \tag{53}$$

□

Proof of the Lemma 3. Let $s=\sum_{t=0}^{\infty}t\gamma^tC$, then we can get that $\gamma s=\sum_{t=0}^{\infty}t\gamma^{t+1}C=\sum_{t=1}^{\infty}(t-1)\gamma^tC$. So, $s-\gamma s=(1-\gamma)s=\sum_{t=1}^{\infty}\gamma^tC=\frac{\gamma}{1-\gamma}C$. At last we can get $s=\frac{\gamma}{(1-\gamma)^2}C$. □

Proof of the Lemma 4.

$$\left\|\mathbb{E}\left(\frac{d\widehat{R^M}}{ds}-\frac{dR^M}{ds}\right)\right\| \tag{54}$$

$$=\left\|\mathbb{E}\sum_{t=0}^{\infty}\gamma^t\widehat{\frac{d\tilde{r}_t}{d\tilde{s}_t}}\widehat{\frac{d\tilde{s}_t}{\tilde{s}_0}}-\mathbb{E}\sum_{t=0}^{\infty}\gamma^t\frac{dr_t}{ds_t}\frac{ds_t}{s_0}\right\| \tag{55}$$

$$\leq\mathbb{E}\sum_{t=0}^{\infty}\gamma^t\left(\left\|\widehat{\frac{d\tilde{s}_t}{d\tilde{s}_0}}\delta\left(\frac{dr_t}{ds_t}\right)\right\|+\left\|\frac{dr_t}{ds_t}\delta\left(\frac{ds_t}{ds_0}\right)\right\|\right) \tag{56}$$

$$\leq\sum_{t=0}^{\infty}\gamma^t\left(L_1(\epsilon_r^g+L_rC_0\epsilon_f)+L_r\left(C_1\epsilon_f^g+C_2\epsilon_f\right)\right) \tag{57}$$

$$=\frac{1}{1-\gamma}\left(L_1(\epsilon_r^g+L_rC_0\epsilon_f)\right)+L_r\sum_{t=0}^{\infty}\gamma^t\left(C_1\epsilon_f^g+C_2\epsilon_f\right) \tag{58}$$

$$=\frac{1}{1-\gamma}\left(L_1(\epsilon_r^g+L_rC_0\epsilon_f)\right)+\frac{\gamma}{(1-\gamma)^2}L_r\epsilon_f^gL_1^2+\frac{\gamma}{(1-\gamma)^2}C_0L_fL_r\epsilon_f \tag{59}$$

$$=C_3\epsilon_r^g+C_4\epsilon_f^g+C_5\epsilon_f. \tag{60}$$

□

Proof of the Lemma 5.

$$\sum_{t=1}^{T}\alpha_t\left\langle g(\phi_t),G(\phi_t,\xi_t)-g(\phi_t)\right\rangle \tag{61}$$

$$=\sum_{t=1}^{T}\alpha_t\left(\left\langle g(\phi_t),G(\phi_t,\xi_t)-G(\phi_t,\xi_{t+\tau})\right\rangle+\left\langle g(\phi_t),G(\phi_t,\xi_{t+\tau})-g(\phi_t)\right\rangle\right). \tag{62}$$

We first bound the first term in the right hand side.

$$\|\sum_{t=1}^{T}\alpha_t\left(\langle g(\phi_t), G(\phi_t,\xi_t)-G(\phi_t,\xi_{t+\tau})\rangle\right)\| \tag{63}$$

$$\leq\|\sum_{t=1}^{\tau}\alpha_t\langle g(\phi_t), G(\phi_t,\xi_t)\rangle\|+\|\sum_{t=T-\tau+1}^{T}\alpha_t\langle g(\phi_t), G(\phi_t,\xi_{t+\tau})\rangle\| \tag{64}$$

$$+\|\sum_{t=\tau+1}^{T}\alpha_t\langle g(\phi_t), G(\phi_t,\xi_t)-G(\phi_{t+\tau},\xi_t)\rangle\| \tag{65}$$

$$\leq 2\tau\alpha_0 L_1^2+\|\sum_{t=\tau+1}^{T}\alpha_t\langle g(\phi_t), G(\phi_t,\xi_t)-G(\phi_{t+\tau},\xi_t)\rangle\| \tag{66}$$

$$\leq 2\tau\alpha_0 L_1^2+\sum_{t=\tau+1}^{T}\alpha_t\|\sum_{k=t}^{t+\tau-1}\langle g(\phi_t), G(\phi_k,\xi_t)-G(\phi_{k+1},\xi_t)\rangle\| \tag{67}$$

$$\leq 2\tau\alpha_0 L_1^2+\sum_{t=\tau+1}^{T}\alpha_t L_2\|\sum_{k=t}^{t+\tau-1}\langle g(\phi_t), \phi_k-\phi_{k+1}\rangle\| \tag{68}$$

$$\leq 2\tau\alpha_0 L_1^2+\sum_{t=\tau+1}^{T}\alpha_t^2 L_1^2 L_2\tau. \tag{69}$$

$$\tag{70}$$

Now, we will bound the second term in equation in expectation.

$$\|\mathbb{E}\left(\sum_{t=1}^{T}\alpha_t\left(\langle g(\phi_t), G(\phi_t,\xi_{t+\tau})-g(\phi_t)\rangle\right)\right)\| \tag{71}$$

$$=\sum_{t=1}^{T}\alpha_t\left(\langle g(\phi_t), \mathbb{E}\left(G(\phi_t,\xi_{t+\tau})-g(\phi_t)|\mathcal{F}_t\right)\rangle\right) \tag{72}$$

$$\leq\sum_{t=1}^{T}\alpha_t L_1\|\int G(\phi_t,\xi)\left(P(\xi_{t+\tau}\in\xi|\mathcal{F}_t)-\mu(\xi)\right)d\xi\| \tag{73}$$

$$\leq\sum_{t=1}^{T}\alpha_t L_1 d_{TV}(P(\xi_{t+\tau}\in\cdot|\mathcal{F}_t),\mu(\cdot)) \tag{74}$$

$$\leq\sum_{t=1}^{T}\alpha_t L_1 m\rho^{\tau}. \tag{75}$$

□

Proof of the Lemma 6.

$$J=\mathbb{E}\sum_{t=1}^{\infty}\gamma^t r_t\leq\frac{r_{max}}{1-\gamma}=U_J. \tag{76}$$

□

Proof of the Lemma 7. The first inequality comes from Lemma B.4 in Wu et al. (2020). The second inequality comes from Lemma 3.2 in Zhang et al. (2020a).

□

Proof of the Lemma 8.

$$\left\|\mathbb{E}\left(\frac{dR^M}{ds}\right)\right\| \tag{77}$$

$$=\left\|\mathbb{E}\sum_{t=0}^{\infty}\gamma^t\frac{dr_t}{ds_t}\frac{ds_t}{s_0}\right\| \tag{78}$$

$$\leq\mathbb{E}\sum_{t=0}^{\infty}\gamma^t\left\|\frac{dr_t}{ds_t}\frac{ds_t}{s_0}\right\| \tag{79}$$

$$\leq\sum_{t=0}^{\infty}\gamma^t L_1 L_r \tag{80}$$

$$=\frac{1}{1-\gamma}L_1 L_r. \tag{81}$$

$$\left\|\mathbb{E}\frac{dR^M}{d\phi}\right\| \tag{82}$$

$$=\left\|\mathbb{E}\left(\sum_{t=0}^{\infty}\gamma^t\left(\frac{dr_t}{d\phi}\right)+\gamma\sum_{t=0}^{\infty}\gamma^t\left(\frac{dR}{ds_{t+1}}\frac{ds_{t+1}}{da_t}\frac{dat}{d\phi}\right)\right)\right\| \tag{83}$$

$$\leq\frac{1}{1-\gamma}L_r L_\pi+\left\|\mathbb{E}\gamma\sum_{t=0}^{\infty}\gamma^t\left(\frac{dR}{ds_{t+1}}\frac{ds_{t+1}}{da_t}\frac{da_t}{d\phi}\right)\right\| \tag{84}$$

$$\leq\frac{1}{1-\gamma}L_r L_\pi+\frac{\gamma}{(1-\gamma)^2}L_1 L_\pi L_f L_r \tag{85}$$

$$=L_R. \tag{86}$$

□

Proof of the Lemma 9. The conclusion comes from the Equation 4.4.9 in Nesterov et al. (2018). □

Proof of the Lemma 10. Since $\sqrt{x}$ and $\sqrt{y}+\sqrt{z}$ are both positive, then we have $x\leq(\sqrt{y}+\sqrt{z})^2$, i.e., $x\leq y+z+2\sqrt{yz}$, note that $2\sqrt{yz}\leq y+z$, then we have $x\leq 2y+2z$. □