Probing Reasoning of Language Models with Inductive In-Context Learning

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Abstract

Despite their recent success, Language Models (LMs) have brought to question whether they statistically repeat data ('stochastic parrots')[Bender *et al.*, 2021] or can learn the underlying *generative process* of the data. Current benchmark used to probe the reasoning ability of LMs can be in-applicable in this setting as they are curated by human annotators and the *generative process* can be ambiguous. Additionally, current work to probe reasoning ability of LMs introduce the same bias as the question they attempt to answer; that is whether the model has learn the statistics of the benchmark data. In this work we introduce a novel evaluation setting that we use with *Inductive In-Context Learning* (IIL) and a dataset, `ReAnalogy`, to probe the reasoning ability of LMs. `ReAnalogy` consists of sequences with positive examples, generated from regular expressions (regex) and contain quasi-natural language. We use regex to evaluate *implicitly* whether a LM can infer 'Rules' (regex) given limited set of examples ('Facts'). We use the LM to generate additional Facts to evaluate whether the generated Facts abide by the Rules. We evaluate a `GPT` model in our setting and compare with the same model where a Rule is injected during training to replace a Fact. We use IIL during evaluation to probe the model to infer the Rule given Facts. We then use the inferred Rule to synthesize an additional Fact. IIL improve 'reasoning' performance by as much as 33%. Our results suggest that LMs can learn more than statistical patterns in the data and we support our findings with ablation studies. We evaluate our dataset with existing benchmark and baselines in inductive programming and find that current state-of-the-art symbolic or neuro-symbolic approaches fail to the complexity of our dataset; while the existing dataset and benchmark in the domain are inapplicable for LMs. Our probing method and dataset are complex enough for LMs and applicable for evaluating the inductive reasoning abilities of LMs, while IIL can improve 'reasoning' of LMs. We open-source our dataset and the code used for our experiments.

1 Introduction

Reasoning capabilities of Language Models (LMs) has been of recent interest in the Machine Learning (ML) community and beyond. Evaluating the ability of LMs to reason is an open problem where current work focus on psychometric tests designed based on human priors that are inapplicable to LMs [Yu *et al.*, 2023; Bubeck *et al.*, 2023]. Principled benchmarks on evaluating reasoning can contain annotator bias due to the ambiguity inherit in natural language. Although the goal is for LMs to perform inductive reasoning comparably to humans a more systematic approach is required to evaluate and compare between models.

Inductive reasoning tasks require a model to infer general rules given observations, where the rule must match the observations such that $\forall x \in$ `Facts` $\rightarrow$ `Rule`. Consider a toy example of an inductive reasoning task in natural language {`Alex`, `Blake`, `Kyle`, ... } $\rightarrow$ `Female Names` can be ambiguous depending on the priors of the annotator i.e. the membership of `Alex` $\in$ `Female / Male Names`. While additional solutions can include `Human Names` or `Names with Two Vowels`. Under ambiguity, abductive inference attempt to find the most probable Rule that describes the data. In ML literature inductive reasoning is an overloaded term and as such we also use the term to refer to the ambiguous setting. In contrast, deductive reasoning would seek to deduce Facts from Rules.

Much of the criticism of language models has been on whether they are 'stochastic parrots' [Bender *et al.*, 2021] as they learn the statistical patterns of the data but not the underlying generative process, i.e. `Human Names`. As such, the benchmarks raise the same questions as they try to answer; whether the model is over-fit to annotator bias as there is no access to a ground-truth underlying generative process of the data, i.e. list of `Male Names`.

Unambiguous evaluation approaches include, Inductive Logic Programming (ILP) where the goal is to deduce the logical rules from a set of examples (positive and negative). For example: $\{$`abcd`, `abzz`, `abeee`, $\ldots\}$ $\rightarrow$ `ab[a-z]*` or 2, 8, 16 $\cdots \rightarrow 2^i$. ILP has been motivated by recent work [Cropper *et al.*, 2022] as a way of evaluating reasoning of LMs. While the field has been stagnated with progress where most benchmark are limited in vocabulary, size or require manual effort in curating and processing examples. While baseline methods are outdated when compared to current LMs.

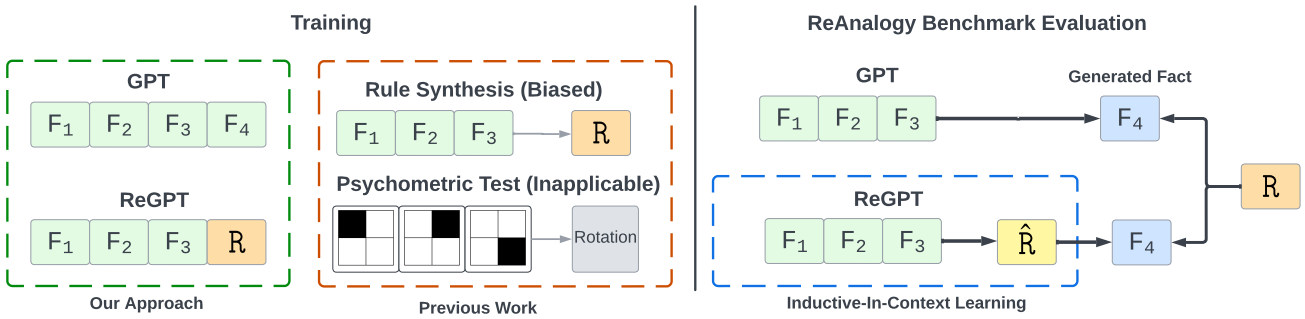

Figure 1: Illustration on the differences between our approach in Training and Evaluating as compared to previous methods. For `ReGPT` we inject a Rule ('R') during training (bottom row of the green outline) as compared to `GPT` that is trained with additional Facts ('F'). Previous approaches directly optimize for a Rule synthesis task (biased) or use tests that can be inapplicable for LMs or neural networks (red outline). During evaluation, we probe `ReGPT` with IIL to infer a Rule ($\hat{R}$) from Facts that we use to generate additional Facts (blue tokens). We evaluate on whether the generated Facts abide by the Rules. Our benchmark can implicitly evaluate on whether a model has learned to infer the underlying Rules as opposed to models that directly maximize for Rule Synthesis. For IIL the soundness and correctness of the Rule are unambiguous and as such it improves performance when compared to a baseline model.

As such, there is a gap between natural language datasets in which there is ambiguity on the ground-truth and synthetic dataset for which we have access to their generative process but are not applicable for modern LMs.

To this end we motivate that to evaluate the inductive reasoning abilities of a LM model we need a dataset to which we have access to `Rules` which we can evaluate unambiguously, *i.e.* the membership of a Fact to a Rule. Similarly, such dataset should be challenging enough to be used with current state-of-the-art models. Additionally, the evaluation protocol should probe the generalization abilities of the model without being subject to the same bias as the question it is trying to answer, for example $P(\texttt{Rule}|\texttt{Facts})$ is an insufficient test when `Facts` are ambiguous. The same protocol is biased when the model is trained such that to maximize $P(\texttt{R}|\texttt{F})$, which can be refer to as *Rule Synthesis*. As such a holistic approach to evaluation can be more effective at probing the reasoning ability of a LM. For reasons of brevity we use `R` and `F` to denote Rules and Facts respectively and use similar notation throughout the text.

Regular Expressions (regex) are Finite Automata and can represent a program (`R`) that can be used to both synthesize and evaluate membership ($\in$) of an example (`F`) in an unambiguous way. Current dataset composed of regex are inapplicable for LMs where, they are too small to train on, or contain random patterns. For this reason we propose `ReAnalogy` benchmark composed of 60,368 regex mined from open-source repositories that contain quasi-natural language. For each expression we generate examples (`F`) and evaluate the model on the ability to generate additional facts from a *limited* set of **only** positive implication examples, Figure 1. Our benchmark is challenging enough for existing LMs, while it is able to evaluate the analogy making ability of a system in an unambiguous manner.

We evaluate LMs and find they perform at 83% accuracy on this benchmark where previous state-of-the-art on ILP synthesis catastrophically fail. As such, we introduce `ReGPT`, where we inject `Rules` in the training sequence to evaluate whether LMs can learn to infer rules. During evaluation we use IIL to prompt a LM to infer Rules that we use to generate Facts and improve performance by as much as 33% compared to a `GPT`. Our approach can be summarized as induction followed by deduction. We evaluate `ReGPT` to find that LMs can improve on inductive reasoning and learn to infer Rules implicitly when probed via IIL. We will make our code and dataset publicly available after the review process concludes. Our contributions can be summarized:

- We introduce `ReAnalogy`, a reasoning benchmark composed of complex quasi-natural language generated from complex regular expressions.
- We introduce `ReGPT` a `GPT` model that 'reason' by performing induction to infer the Rules given a limited set of true Facts via *Inductive In-Context Learning* followed by deduction where it generates novel Facts.
- We ablate and analyze the performance of `ReGPT` and `ReAnalogy` with results that can help to probe and improve the reasoning abilities of LMs.

2 Related Work

We identify three categories of related works most similar to our work. Work that attempt to evaluate and understand the reasoning abilities of LMs; work that introduce evaluation settings of reasoning for LMs; and work that introduce methods for improving reasoning.

2.1 LM probing

Several work evaluate the reasoning abilities of LMs [Yu *et al.*, 2023] by probing to specific tasks, such as inductive, deductive and abductive reasoning.

[Zhang *et al.*, 2022] use a Language Model as a Logic Programmer to reason over knowledge bases. Their dataset contains query, facts, rules where the goal is given the query to find a proof path. Contrary to their method, we evaluate implicitly on how well a model can infer Rules by evaluating the generated Facts. Additionally, IIL deduce rules implicitly,

by first performing inductive reasoning and using the inferred rule to generate facts.

Similarly, [Yang *et al.*, 2022] observe that current inductive reasoning datasets and tasks are superficial in probing the inductive reasoning abilities of LMs. They propose DEER, a dataset which contains a list of short natural language facts accompanied by a rule. The dataset is curated by human evaluators that find facts on the internet given a rule. Contrary to our work, their dataset is not publicly available. We evaluate membership of Facts in Rule in an unambiguous manner where DEER is inapplicable in this setting. Additionally, their benchmark, and similar to [Teru *et al.*, 2020], evaluates the probability of inferring the correct Rule given Facts *i.e.* $P(\mathtt{R}|\mathtt{F})$ and as such can be used auxiliary to our setting.

Other work [Misra *et al.*, 2022] evaluate the ability of LMs to perform property induction, *i.e.* if $\mathtt{F}_i \rightarrow \mathtt{R}_1$ and $\{\mathtt{F}_j, \mathtt{F}_i\} \rightarrow \mathtt{R}_2$ then $\mathtt{F}_i \rightarrow \mathtt{R}_2$. Their evaluation setting is orthogonal to our work, while their dataset is artificially generated from natural language Facts with True / False pairs that are inapplicable to be used for evaluating inductive reasoning.

[Telle *et al.*, 2019] evaluate the minimum number of examples (Facts) required for a model to learn a concept (Rule) using a regex dataset. They find that the total size (in length) of the Facts as opposed to their number is a better indicator of complexity to learn a Rule. Similarly, we evaluate the relationship in learning Rules from Facts to arrive at similar conclusions as we increase the number of facts we use to infer Rules, Table 2. Contrary to their setting we evaluate the inferred Rules by using them to generate additional Facts. Additionally, `ReAnalogy` contain quasi-natural language as opposed terminal 0 and 1 that are constructed from a restrict set of regex.

[Yang *et al.*, 2023; Liu *et al.*, 2022] supplement our analysis where they identify that natural language descriptions alone can be inapplicable to evaluate the reasoning ability of LMs. They combine descriptions (Facts) in natural language with automaton-based representations (Rules) that can be formally verified. Contrary to our work, we evaluate the Facts generated by the Rules inferred via IIL.

Similar to our work, [Min *et al.*, 2022] probe the mechanism behind in-context learning and the aspects of the demonstration that contribute to end-task performance. They find that ground-truth demonstrations (Rules) are not required. Similarly, we find that explicitly learning (Rules) is not required for a LM to learn to generate Facts where a `GPT` model performs with 83% accuracy. Additionally, our method, IIL, improves performance by as much as 33% where Rules are injected during training and the model is probed via IIL.

2.2 Benchmarks

CLUTRR [Sinha *et al.*, 2019] evaluate the ability of a model to provide an answer (Fact) to a question in an in-context learning setting. Contrary to evaluating $P(A|Q)$ we evaluate the ability of a model to generate additional facts. Similar to 'Rule Synthesis' we find that a benchmark that evaluates $P(A|Q)$, can not provide a holistic view. Additionally, their dataset is inapplicable in our setting and the two benchmark are orthogonal and as such can be used auxiliary to each other.

[Bhagavatula *et al.*, 2019] introduce an 'Abductive Reasoning' benchmark where given observations (Facts) they evaluate how well a model can generate (Rules) with natural language pairs. Where the benchmark can seen similar to a Rule Synthesis evaluation protocol such as, $maxP(R|F)$. Work by [Cornelio and Thost, 2021] is similar in that regard as well.

[Yang and Deng, 2021] identify the gap between natural language datasets and first-order logical rules. They propose a dataset that express both natural language as well as formal logic, quasi-natural language. Contrary, `ReAnalogy` use regex to generate the examples where a DFA can generate more complex expressions than first-order logic of limited terminals.

[Mitchell, 2021] propose to evaluate the reasoning ability of AI on psychometric tests designed for humans. Most similar to our evaluation setting, Raven's Progress Matrix, evaluate whether a model can learn to complete a sequence of complex visual patterns. Such setting can be inapplicable to LM as they are not agnostic to prior knowledge *i.e.* identification of shapes, which can be a source of noise during evaluation. For example, it would be hard to diagnose whether the model performs poorly at recognizing rotations as opposed to reasoning. Additionally, such priors can be natural for humans but are known failure points for ML models [Geirhos *et al.*, 2018; Li *et al.*, 2018].

2.3 Reasoning Improvements

[Chen *et al.*, 2020] propose a method to synthesize regular expressions from positive and negative examples and natural language description. The main component of their method is the multimodal approach that combined natural language with AlphaRegex [Lee *et al.*, 2016]; a symbolic search based approach on the regular expressions. Our work are orthogonal, as their method does not work under ambiguity.

Similarly, [Wei *et al.*, 2022] introduce a method for eliciting reasoning via in-context learning of LMs. The reasoning ability of the model increases when the model is demonstrated additional examples. Their analysis supplements our work and to work by [Jaimovitch-Lopez *et al.*, 2021]. Contrary, to [Jaimovitch-Lopez *et al.*, 2021], where they use input output pairs to learn the concepts, we only use input pairs (Facts). Additionally their dataset are artificial sequences of 0s and 1s.

[Rytting and Wingate, 2021] explore a neuro-symbolic approach of using the learned representations of a LM to train symbolic engine. Similarly our approach can also be seen as neuro-symbolic with the use of IIL to infer unambiguous Rules that are used to synthesize Facts. Their analysis is auxiliary to our work, while their method uses a reasoning engine on templates of natural language meant to Synthesize Rules.

3 Preliminaries

3.1 Inductive Reasoning

For Language Models, reasoning is the process of drawing conclusions (Rules) from a set of observations (Facts). In general, it consists of inductive reasoning, deductive reasoning and abductive reasoning. **Inductive reasoning** is to reach generalized conclusions (`Rules`) based on observed specific instances (`Facts`). When there is insufficient evidence to reach

a conclusion abductive inference involves making educated hypotheses from the incomplete observations. Mathematically, the inductive reasoning can be presented in `Facts` $\rightarrow$ `Rules`. On the contrary, **deductive reasoning** uses general principles or rules to make specific conclusions, *i.e.* `Rules` $\rightarrow$ `Facts`.

For example, during inductive reasoning, given the `Facts`: `abcd`, `abzz`, and `abeee`; we can find a $\texttt{Rule}_1$ that they all 'start with `ab`'. Formally, `{abcd, abzz, abeee}` $\rightarrow$ $\texttt{Rule}_1$. While for deductive reasoning, given the `Rule` 'strings that start with `ab`', we can present Facts `{abcd, abzz, abeee}`. `ReGPT` performs inductive reasoning where we infer the `Rules` from a limited set of `Facts` followed by deductive reasoning where we generate Facts from the inferred Rules *i.e.* $P(\texttt{R}|\texttt{F}) \rightarrow P(\texttt{F}_{i+1}|\texttt{R})$. Contrary, `GPT` generates new `Facts` from the previous `Facts`, *i.e.* $P(\texttt{F}_{i+1}|\texttt{F}_i)$.

3.2 Regular Expressions

Regular expressions (regex) are automata and provide a formal representation of a language. Although not equivalent, Formal Languages have been applied in Natural Language in the past. More recent work [Hahn *et al.*, 2022] can find equivalent approximations between the two. While it is no mathematical equivalence between Natural Languages and Formal Languages, we refer to their intersection as *quasi-natural* language.

Regex are algebraic approaches to search for, and manipulate text based on a set of rules. For example, the $\texttt{Rule}_1$ in the previous subsection, 'starting with `ab`', can be represented as `ab.*`. The syntax of regular expressions include operators such as ('.','^', '$','|','[ ]','[^]','*','+','$\{m,n\}$'), that provide control for iteration (*i.e.* $*$ matches any of the previous characters any number of times), logic (*i.e.* $|$ can be used as an OR operator), while other characters are considered terminals. Rules are combined to form complex relationships with terminal character symbols such as letter or strings. For example "[dog,]." would match "dog,dog,dog. . . ". Complex strings can be matched with sufficiently complex expressions.

Finite Automata (FA) contain transitions between states. For example, the expression "[a-z]*" defines all strings that contain any number of lower case characters from the English alphabet. The states would be characters 'a' through 'z' and a terminal state. Nondeterministic FA (NFA) define probability of transition between states, *i.e.* probability of 'a' followed by 'a'. We can interpret regex in this work as NFA where at each state we randomly sample a transition at an even probability. `ReAnalogy` is the only dataset that supports the full regex functionality and for unicode character-set; an artifact of our efficient implementation in generating regexes.

3.3 In-Context Learning

In-Context Learning (ICL) [Brown *et al.*, 2020] is a paradigm that trains LMs with a limited set of demonstrations. Different from fine-tuning, which updates parameters of pre-trained networks by training on datasets of new tasks, ICL requires no parameter optimization for a downstream tasks. Instead, ICL uses as input a 'prompt' that can guide the model to perform a task. Given that a LM has been pre-trained on large-scale corpus, it can perform tasks that were not contained in the training data such as generating new data via *generative sampling*. We find that ICL is the correct setting to evaluate the generalization ability of a LM as it is not biased in training the model for a specific task, *i.e.* Rule Synthesis. While the prompt can be used to guide a the pre-trained model via next-token prediction on Facts or Rules and generate additional Rules or Facts.

Generative Sampling uses a scoring function to calculate the probability of a sequence token given all previous tokens (*i.e.* the prompt). There are several ways to perform sampling, such as beam-search over all candidate options. To avoid the bias introduced by the sampling method we evaluate the generative ability of a model using 'top-k' sampling, where we randomly sample from the top-k most probable tokens.

4 Method

4.1 `ReAnalogy`

As the current datasets with regex lack sufficient complexity to effectively evaluate the performance of LMs and probe their limitations, we contribute to a new dataset `ReAnalogy`, which consists of 60,368 regex acquired from open-source repositories and augmented with 43,896 python regex from [Davis *et al.*, 2019].

Similarly to [Davis *et al.*, 2019], we find that majority of the expressions are not compilable, contain bugs, vulnerabilities, are too large, or lack complexity *i.e.* when a large portion of the expression is a fixed pattern, usually code. We mine additional examples from open-source repositories using the GitHub API and search for code that utilizes the 're' library and we extract the string literals used for those expressions. We find that a significant portion of the expressions we mine contain natural language and code *i.e.*

```
expr1="While linting files .*"
```

We use the expressions to model `Rules`, that describe `Facts`, such as text that match the `Rule` pattern. There are several avenues in finding `Facts` that match the `Rule`. The expression has a specific use-case and it can be difficult to find multiple string examples *in-the-wild* that provide an exact match at scale *i.e.* matching *expr1* with a natural language corpus. We attempt to use expressions directly to large corpus on dataset like Common Crawl[1] and Github repositories. Based on sub-sample of our results it is computationally expensive to perform exhaustive search and did not lead to sufficient matches from a limited set of examples we sampled.

As such, we generate `Facts` by randomly traversing the regex execution graph where we modify XEGER[2]. We perform modifications to set a stop condition for some operations *i.e.* '*' that limit the number of terminal repetitions to 10. Additionally, the existing library provide poor support for specific binary terminals *i.e.* unicode characters. The vocabulary of `ReAnalogy` is composed of 242 terminals. While our implementation is performant enough that the generation can be performed on-the-fly and is not the bottleneck in training a LM.

[1] https://commoncrawl.org/

[2] https://github.com/crdoconnor/xeger

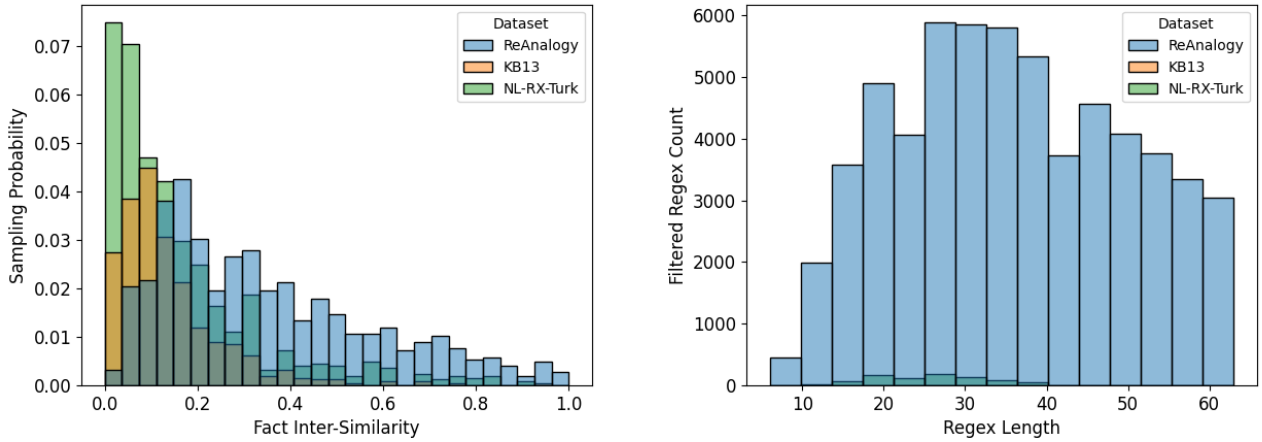

Figure 2: Quantitative evaluation of `ReAnalogy` **Left** Probability of sampling examples by a regex for a given inter-similarity range and as measured by the Jaccard index. A higher inter-similarity signifies larger distance between generated Facts where 0 would be as good as random *i.e.* generated using the expression ".*". On the contrary, Facts with high inter-similarity would be generated by fixed regex *i.e.* "hello world". For `ReAnalogy` the Fact inter-similarity is evenly distributed signifying diversity and complexity of generated expressions, with a median of 0.70. Contrary KB13 and Deep-Regex that have 0.90 and 0.88 respectively. **Right** Distribution of the filtered regex by length for each dataset. `ReAnalogy` is significantly larger than KB13 [Kushman and Barzilay, 2013] and Deep-Regex [Locascio *et al.*, 2016] where the latter dataset are dwarfed by the size of `ReAnalogy` distribution. Additionally, `ReAnalogy` has diversity on the size of expressions that can be observed by the even distribution in the histogram.

Filtering To enhance the diversity and complexity of the generated data, we filter for regex that can not produce more than 10 unique examples over multiple random repetitions. The goal of filtering is to avoid evaluating or training on very simple examples, such as `[a,a,...]` $\rightarrow$ `a`. For example, regex for fixed strings such as 'GRID_PPEM' were removed from the dataset. While there were patterns that were not compilable in Python such as "\\G . . . " due to lack of support of operator \\G. Additionally, operators such as "\b" (backspace) are not applicable in our generative setting.

4.2 Fact Score

We define a metric to evaluate the membership of generated Facts implicitly by a Ground Truth Rule under ambiguity of reasoning. We find that directly predicting and evaluating the Rule is biased, as a model can learn to synthesize a Rule from Facts but does not answer whether the model can learn to do the same under more general conditions. Throughout this work we use the term $P(\mathtt{R}|\mathtt{F})$ to describe this setting.

As such, we propose to evaluate generated Fact with the ground-truth Rule ($\mathtt{R}^*$) as opposed to performing equivalence comparison of an inferred Rule ($\hat{\mathtt{R}}$) and $\mathtt{R}^*$. First, two Rules can be equivalent but expressed differently. In the context of regular expressions it can be trivial to evaluate *i.e.* `[a-zA-Z0-9_] == \w`, but in natural language rule equivalence can ambiguous. Second, more than one Rule can interpret the Facts, *i.e.* $\hat{\mathtt{R}} \rightarrow \{F\}$, $\mathtt{R}^* \rightarrow \{F\}$ and $\hat{\mathtt{R}} \neq \mathtt{R}^*$. Under an ambiguous setting, $\hat{\mathtt{R}}$ can be considered correct in the context of probing the ability of a model to reason. For example `F = {ab, ac, ad}` $\rightarrow$ $\hat{\mathtt{R}}$ has many ambiguous and correct interpretations $\hat{\mathtt{R}} \in \{\mathtt{a*}, \ldots, \mathtt{a[bcde]}\}$.

We disambiguate the evaluation and contrary to comparing between Rules, we propose to compare if the generated Facts satisfy $\mathtt{R}^*$; *i.e.* whether $\mathtt{R}^* \rightarrow \mathtt{ae}$. The Fact Score (FS) is computed as the mean accuracy score of $\mathtt{R}^* \rightarrow \mathtt{F}_{i+1}$ and calculated as:

$$\mathtt{FS} = P(\mathtt{R}^* \rightarrow \mathtt{F}_{i+1} | \{\mathtt{F}_1, .., \mathtt{F}_i\}); \forall \mathtt{R}^* \in \mathtt{ReAnalogy} \quad (1)$$

Fact Score can be extended beyond regex to natural language whereby $\mathtt{R}^* \rightarrow \mathtt{F}_{i+1}$ can be evaluated by an auxiliary model or human feedback. However, we limit our work to regex as a proving ground for our evaluation protocol, as a regex can be unambiguously evaluated without significant manual effort in curating a dataset.

4.3 `ReGPT`

We use `ReGPT` to implement Inductive In-Context Learning (IIL) and evaluate on whether LMs can learn to infer Rules given Facts. The goal of `ReGPT` is to probe the reasoning ability of LMs and not to introduce a novel architecture. As such, we compare with a baseline `GPT`. Both models are trained on a set of Facts. The difference for `ReGPT` is that we replace a Fact with a Rule injected in the training sequence but without direct or explicit supervision to associate the Rule with the Facts, Figure 1. To ablate the influence of model structure, `ReGPT` uses the same backbone as `GPT`, *i.e.* Transformer. `ReGPT` is used to evaluate two hypothesis:

1. Do LMs learn to infer Rules from the data?
2. Do LMs learn to associate Facts and Rules implicitly?

We evaluate 1. with `GPT` probed with `ReAnalogy` to generate Facts and evaluated using FS, Section 4.2 with 83% performance. We evaluate 2. with Inductive In-Context Learning (IIL) and probe `ReGPT` to generate rules and evaluate

those rules by using them to generate facts. There are two aspects of our approach that can extend beyond our work. Firstly, generating Rules is unambiguous and even in natural language. *i.e*. inferring a rule such that "dogs are mammals" or "octopuses are mammals" are both unambiguous but not necessarily correct. Secondly, evaluating whether the inferred rule is *correct* (given the facts) is evaluated by the ability of the model to produce names of mammals given a list of name of animal *i.e*. [bird, ..., horse, $\texttt{Fact}_{i+1}$] $\rightarrow$ $\texttt{Fact}_{i+1}$ = "octopus" or "dog". The evaluation is performed on whether the generated fact is correct. *i.e*. "mammal" $\rightarrow$ "octopus".

The advantage of this approach is that the evaluation metric used from Section 4.2, can account for ambiguity of the probing facts. We find that the performance of a model is influenced both by the quality and quantity of facts, Section 5.2. Inductive In-Context Learning can probe on whether the model has successfully learned associations between concepts and whether it can reason on this association and extrapolate beyond the evidence present in the prompt.

Our approach has similarities to *Neuro-Symbolic* methods, where we evaluate the correctness of the inferred Rule in an unambiguous manner, for example on whether an expression can be compiled or a natural language syllogism follows a template. While generating facts can be seen as deductive reasoning. As such our approach can be summarized as induction (inferring sound Rules from Facts) followed by deduction (inferring Facts from Rules).

5 Experiments

We use `ReAnalogy` composed of 60,368 expressions that we use to synthesize examples on the fly during training of both `GPT` and `ReGPT`. Based on the insights from [Telle *et al.*, 2019] and to avoid the learning bias from the size of examples and regular expressions we limit their length to 64 characters. The total sequence length can reach 1024 characters which is identical to [Radford *et al.*, 2019]. In Section 5.1 we evaluate and compare `ReAnalogy` with a baseline method Deep-Regex[Locascio *et al.*, 2016]. We evaluate on their dataset NL-RX-Turk and KB13 [Kushman and Barzilay, 2013] that are regex dataset used in literature and show both the difference in complexity, size as well as applicability of each dataset to LMs. Next in Section 5.2 we evaluate and compare `ReGPT` with the baseline `GPT` to evaluate the effects of Inductive In-Context Learning using Fact Score from Section 4.2. We use Adam with 0.001 LR for all our experiments. For reasons of brevity on hyper-parameters and implementation details we refer the reader to our open-source repository that is well-documented.

5.1 ReAnalogy

For the experiments in this section we train 8 `ReGPT` of different sizes [2,6,24] layers on Datasets Deep-Regex and KB13 as a way to evaluate the applicability of LMs on the respective datasets. We found that smaller model worked better for the smaller dataset, and as such we report the best performance from the range. We compare with a `ReGPT` model of 24 layers pre-trained on `ReAnalogy` and evaluated on each respective dataset. We find that both NL-RX-Turk and KB13 are too small to be used with a LM where their in-dataset evaluation performance correlates with their size 824 for KB13 and 10000 for Deep-Regex with 0.20 and 0.66 FS respectively, Table 1. On the contrary, evaluating the same dataset using a pre-trained model leads to contrary results where `ReGPT` performs better for a simpler dataset KB13. We conclude that both datasets are inapplicable to use with a LM. Additionally, pre-trained `ReGPT` outperforms a baseline method on the datasets, where Deep-Regex also suffers in performance on KB-13 due to the small size.

Model	Train Dataset	Eval. Dataset	FS
Deep-Regex	NL-RX-Turk	NL-RX-Turk	0.58
`ReGPT`	NL-RX-Turk	NL-RX-Turk	0.66
`ReGPT`	`ReAnalogy`	NL-RX-Turk	**0.80**
Deep-Regex	KB13	KB13	0.65
`ReGPT`	KB13	KB13	0.20
`ReGPT`	`ReAnalogy`	KB13	**0.85**

Table 1: Cross-Evaluation and comparison of `ReGPT` when trained and evaluated using existing dataset from literature. NL-RX-Turk and KB13 are too small and not complex enough to be used with LMs, Figure 2. When trained and evaluated on the same dataset `ReGPT` performs poorly compared to baseline Deep-Regex. `ReGPT` performance is proportional to the complexity and size of the dataset where KB13$\ll$NL-RX-Turk. When `ReGPT` is pre-trained on `ReAnalogy` it outperforms the baseline. Our results highlight the in-applicability of existing dataset in training LMs, as well as the effectiveness of `ReGPT` when compared to a baseline.

Next we evaluate the characteristics of the two datasets qualitatively and quantitatively. Figure 2 shows the distribution of inter-similarity between facts and the distribution of the length for the expressions present in the filtered dataset. The Facts sampled by `ReAnalogy` have relative evenly distributed inter-similarity, while those in KB13 and Deep-Regex are concentrated in intervals with low inter-similarity. Therefore, `ReAnalogy` is the only that has a large number of diverse expressions with complex characteristics. During our filtering process described in Section 4.1 we find 5% of 'fixed' string regex in `ReAnalogy` compared to 0.5% for KB13 and 0.3% Deep-Regex. The observation is an artifact of expressions that contain natural language, *i.e*. Figure 3.

5.2 Inductive In-Context Learning

We compare and evaluate a `GPT` model with 24 layers and we ablate the importance of the number of facts as well as the sampling 'freedom' to generate facts ('top-k'). We train 8 different models for the experiments in this section until the validation loss plateaus (which differs for different 'N. Facts').

Our experiments on the number of facts provided to the model provide two insights. First, the performance of `GPT` plateaus and decreases for 'N. Facts' > 5 and 'Top-K'$= 2$ sampling; first row of Table 2. Our observation could be an artifact of the inability of the model to draw associations between Facts as complexity from additional facts arises.

Contrary `ReGPT` with IIL outperforms a `GPT` under equivalent settings. When increasing 'N. Facts', the performance of `ReGPT` improves, as shown in the second row of Table 2. Ad-

Listing 1: KB13 Examples

```
(.*[0-9].*){5,}
1.  Er9?=0mQL92:?$)\\BzG 1 ...
2.  xL`\2r{IngtO\f6Kd<R<1JM...
3.  2fYz6FX\XW6WH#a?=\rv0>...
```

Listing 2: NL-RX-Turk Examples

```
.*([0-9])|(dog)|(.).*
1. 23j
2. XrK%0
3. dog
```

Listing 3: ReAnalogy Examples

```
applying.*?jquery.*?script
1. applying+WjqueryaK-6py|w9$script
2. applyingV\f>\/Rk{qkTjqueryscript
3. applyings\tH'jquery#script
```

Figure 3: Qualitative evaluation on samples between KB13, NL-RX-Turk and `ReAnalogy` datasets. Compared to `ReAnalogy`, other dataset contain mostly random expressions for which the generated examples do not contain meaningful semantics. NL-RX-Turk contain expressions that contain natural language, while `ReAnalogy` is the only that contains complex quasi-natural language expressions.

Top-k	N. Facts (FS)	2	5	8	12
2	GPT	0.54	0.71	0.71	0.70
	ReGPT	**0.71**	**0.80**	**0.83**	**0.86**
6	GPT	0.55	0.74	0.75	0.77
	ReGPT	0.69	0.78	0.80	0.85
10	GPT	0.58	0.75	0.79	0.81
	ReGPT	0.66	0.79	0.79	0.84
14	GPT	0.60	0.76	0.80	0.83
	ReGPT	0.66	0.78	0.80	0.84
18	GPT	0.61	0.77	0.82	0.83
	ReGPT	0.65	0.79	0.79	0.83

Table 2: Comparison of `GPT` and `ReGPT` evaluated with IIL using FS from Section 4.2, where we evaluate a generated Fact by the ground-truth Rule. We generate Facts using Top-K sampling with different hyper-parameters. We find that increasing Top-K can increase the performance of `GPT`, but harms the performance of `ReGPT`. Although `ReGPT` consistently outperforms `GPT`, we find this to be an artifact of generating more improbable Rules. Using IIL improve the Fact-Score accuracy by a significant margin 9% on average and as much as by 33%.

ditionally, `ReGPT` is *example efficient* where for fewer N.Facts the performance gap is larger between `GPT` and `ReGPT` with 0.54 and 0.71 FS respectively.

Top-K sampling used to randomly sample from the K most likely terminals. Larger K result in samples with higher diversity. `GPT` benefits from the diversity of the generated facts where larger K improve FS, while the contrary can be observed for `ReGPT` where larger K can harm performance. Our results suggest that there is a *risk-creativity* trade-off where higher creativity can be exhibited by LMs at the risk of improbable Rules. This is an artifact that diversity in generation of rules leads to improbable outcomes and we hypothesize it could also explain hallucinations [Ji *et al.*, 2023], where on the one hand LMs display 'creativity' in generating impressive content and often catastrophically fail by producing improbable text. We conclude that hallucinations can be explained by the poor inference of the underlying structure of the data (the inferred Rules).

6 Discussion

Understanding the performance of LMs requires principled methods of evaluation that can disambiguate the inherit ambiguity of realistic reasoning tasks. One can argue that a limitation of our work is the quasi-natural language while we find that to be necessary and a novel aspect of our work. Quasi-natural language does not contain ambiguous reasoning arguments while being close enough to a natural language for a principled but challenging evaluation setting. We motivate that there should be a diverse set of benchmarks and datasets for evaluation of LMs. We introduce and evaluate IIL on a quasi-natural language setting as a starting point and find that additional work and open problems in this area would need to be addressed before applying Inductive In-Context Learning directly in natural language. Additional manual effort will be required for creating and curating a natural language dataset specific to IIL. We motivate that such benchmarks and dataset should be open and accessible to everyone by open-sourcing `ReAnalogy` and urge for other researchers to do the same. Our work is in the intersection of LMs and ILP and bares similarities to neuro-symbolic approaches. While a lot of work is emerging in this domain it is still unclear how to best compare between methods in a principled manner.

7 Conclusion

We find that previous benchmark and evaluation settings can not provide a holistic view of the reasoning ability of a LM. Previous work explicitly optimizing the performance of a LM for the reasoning task and can be inapplicable in evaluating on whether reasoning emerges in LMs. We propose an evaluation setting using Inductive In-Context Learning where we probe the reasoning abilities of a LM by evaluating the generated Facts as opposed to directly evaluating the Rules it generates. We find that LM can perform at our benchmark without explicit supervision on the reasoning task. We use IIL with `ReGPT` and evaluate whether a LM can implicitly learn to associate Facts with Rules where we improve 'reasoning' performance of a `GPT` by 33%.

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