# A Constrained Multi-Objective Reinforcement Learning Framework

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**Abstract:** Many real-world problems, especially in robotics, require that reinforcement learning (RL) agents learn policies that not only maximize an environment reward, but also satisfy constraints. We propose a high-level framework for solving such problems, that treats the environment reward and costs as separate objectives, and learns what preference over objectives the policy should optimize for in order to meet the constraints. We call this Learning Preferences and Policies in Parallel (LP3). By making different choices for how to learn the preference and how to optimize for the policy given the preference, we can obtain existing approaches (e.g., Lagrangian relaxation) and derive novel approaches that lead to better performance. One of these is an algorithm that learns a *set* of constraint-satisfying policies, useful for when we do not know the exact constraint a priori.

**Keywords:** constrained RL, multi-objective RL, deep RL

## 1 Introduction

Many problems, especially in the real world and in robot applications, require that policies meet constraints [1]. For instance, we might want a factory robot to optimize task throughput while keeping actuator forces below a threshold, to limit wear-and-tear. Or, we might want to minimize energy usage for cooling a data center while ensuring temperatures remain below some level [2].

Existing approaches for constrained RL typically formulate the problem as a constrained Markov Decision Process (CMDP) [3], and many rely on Lagrangian relaxation [4] to solve it. Lagrangian relaxation transforms the constrained optimization into an unconstrained one, where the objective is a sum of the expected task return and weighted constraint functions. The weights (i.e., Lagrange multipliers) are updated to ensure constraint satisfaction.

However, some constrained RL problems do not fit the CMDP formulation. For instance, when we do not have an exact desired constraint threshold in mind, we may want to choose from a set of policies that have costs within a viable range. Or, in some settings we have *soft constraints* [5], where violating the constraint is undesirable but not catastrophic. These situations arise because of the inherent trade-off between maximizing task reward and satisfying constraints.

Our key insight is to view constrained RL from a multi-objective perspective, where the task reward and each constraint are separate objectives. To train a policy, we must tell it what preference over objectives to optimize for—i.e., how much the policy should prioritize one objective over another. At the extremes, we could tell the policy to either focus entirely on maximizing task reward while disregarding constraint satisfaction, or focus entirely on satisfying the constraints without caring about the task. Some preference settings will lead to policies that satisfy the constraints, but we do not know in advance which ones these are.

Thus we propose to learn which preferences lead to constraint-satisfying policies, while simultaneously training policies for those preferences. We call this framework ***Learning Preferences and Policies in Parallel*** **(LP3)**, shown in Fig. 1. In LP3 there are two main design choices: 1) which multi-objective RL algorithm is used to train the policy, and 2) whether to learn a single preference or a distribution over preferences. Lagrangian relaxation is one possible instantiation—it uses linear scalarization as the multi-objective RL algorithm and learns a single preference (Sec. 4.2).

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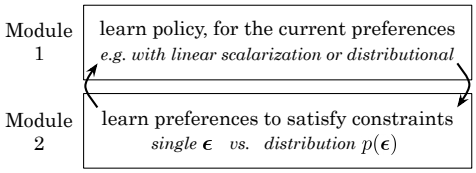


| Algorithm | Module 1 | Module 2 |
|---|---|---|
| Lagrangian relaxation | linear scalarization | single |
| LP3 [MO-MPO] | distributional (MO-MPO) | single |
| LP3 [MO-MPO-D] | distributional (MO-MPO) | distribution |

Figure 1: We use our proposed framework (left), Learning Preferences and Policies in Parallel (LP3), to derive two novel algorithms for constrained RL by making different design choices (in italics).

We explore two other instantiations of our framework. In the first, to train the policy we use a state-of-the-art multi-objective RL algorithm, Multi-Objective Maximum a Posteriori Optimization (MO-MPO) [6]. In the second, we learn a probability distribution over preferences, based on how likely they are to lead to constraint-satisfying policies. This enables finding a *set* of constraint-satisfying policies in a single training run, that differ in terms of task return and constraint performance.

Our main contributions, among others, are:

- We propose a multi-objective framework for constrained RL, called LP3 (Sec. 4). This is a general framework, that includes Lagrangian relaxation as one possible instantiation. Although the connection between constrained and multi-objective problems is intuitive [7, 8], our work is the first to leverage this to create novel algorithms for constrained RL.
- Based on this framework, we derive two novel algorithms for constrained RL. One of these is the first algorithm, to the best of our knowledge, for finding a set of constraint-satisfying policies—this is important when the constraint is soft or not exactly known a priori.
- We run extensive experiments that compare these two new approaches with Lagrangian-based approaches on continuous control tasks across three task suites, including high-dimensional locomotion tasks. Videos are at `http://sites.google.com/view/cmorl`.

## 2 Related Work

**Constrained reinforcement learning.** Constrained RL algorithms learn policies that satisfy constraints while maximizing task return. Many approaches formulate this problem as a CMDP and use Lagrangian relaxation [4]. Recent works claim this converges to constraint-satisfying [9] or optimal [10] solutions. Other works seek to stabilize this optimization by incorporating convex relaxations [11] or derivatives of the constraint function [12]. Others have applied Lagrangian relaxation to mean-value constraints [9], convex set constraints [13], and local constraint satisfaction [14].

For soft constraints, MetaL [5] uses meta-gradients to adjust the Lagrangian learning rate, to find a single policy that achieves high task reward without overly violating the constraints. However, this particular trade-off between the reward and cost may not be desirable. Instead, we propose an approach that returns a *set* of policies to choose from, that make different trade-offs.

**Safe reinforcement learning.** In safe RL, the aim is to achieve constraint satisfaction not only at deployment, but also during learning [15]. Recent works for deep RL modify the policy improvement step to guarantee that the policy will never violate constraints during training [16, 17, 18, 19, 20, 21]. These approaches require, however, that the initial policy (nearly) satisfies the constraints; otherwise, performance may decrease significantly [22]. We focus on constrained RL, rather than safe RL, but we discuss how to extend our approach to reduce constraint violation during training (Sec. 6).

**Multi-objective reinforcement learning.** In our framework, the policy is trained with multi-objective RL (MORL). MORL consists of single-policy and multi-policy approaches. Single policy methods learn a policy that is optimal for a given trade-off. Most rely on linear scalarization [23], which restricts solutions to the convex portions of the Pareto front and can be sensitive to reward scales. Non-linear scalarizations exist [24, 25, 26], but these are harder to combine with value-based RL and have seen limited use in deep RL. Recently, Abdolmaleki et al. [6] introduced MO-MPO, a single-policy approach that does not rely on scalarization and is invariant to reward scales.

Multi-policy MORL aims to find a *set* of policies that covers the whole Pareto front. Recent works learn a manifold in parameter space [27, 28] or combine single policy approaches with a general objective [29], to directly optimize the hypervolume of the Pareto front.

# 3 Background and Notation

## 3.1 Markov Decision Processes

A Markov Decision Process (MDP) is defined by $\{\mathcal{S}, \mathcal{A}, p, r_0, \gamma\}$: the state space $\mathcal{S}$, action space $\mathcal{A}$, transition function $p(s'|s,a)$, bounded task reward function $r_0(s,a)$, and discount factor $\gamma \in [0,1]$.

**Constrained MDP.** Constrained RL problems are typically formulated as constrained MDPs (CMDP). A CMDP is defined by $\{\mathcal{S}, \mathcal{A}, p, r_{0:K}, \beta_{1:K}, \gamma\}$. Compared to a regular MDP, a CMDP includes additional constraint-specific reward functions $\{r_k(s,a)\}_{k=1}^K$ with corresponding thresholds $\{\beta_k\}_{k=1}^K$. All reward functions $r_k$ return scalar rewards; we will refer to these as separate *objectives*.

A policy $\pi(a|s)$ maps from a state to a distribution over actions. Let $J_k^\pi = \mathbb{E}_\pi[\sum_t \gamma^t r_k(s_t, a_t)]$ denote the expected return for objective $k$ when following policy $\pi$, given a fixed initial state distribution. The action-value function $Q_k^\pi(s,a)$ is the expected return for objective $k$ after taking action $a$ in state $s$, and then acting based on policy $\pi$.

The optimal solution to a CMDP is a policy that maximizes expected task return, while ensuring expected returns for all constraint-specific rewards satisfy their thresholds:

$$\max_\pi \; J_0^\pi \quad \text{s.t. } J_k^\pi \geq \beta_k \;\; \forall k = 1, \ldots, K\,. \tag{1}$$

Many approaches for solving CMDPs are based on Lagrangian relaxation, which turns the constrained problem into the following unconstrained optimization problem:

$$\min_{\lambda \geq 0} \max_\pi \; J_0^\pi \; + \sum_{k=1}^K \lambda_k \Big( J_k^\pi - \beta_k \Big)\,. \tag{2}$$

These approaches alternate between optimizing the Lagrange multipliers $\lambda_k$ and the policy $\pi$.

**Multi-Objective MDP.** A multi-objective MDP (MO-MDP) is defined by $\{\mathcal{S}, \mathcal{A}, p, r_{0:K}, \gamma\}$. In contrast to an (C)MDP, there are multiple rewards to optimize for, and no constraints. The optimal policy depends on the desired preference $\boldsymbol{\epsilon}$ across objectives. In *linear scalarization*, the preference $\boldsymbol{\epsilon}$ defines a weighting across objectives; the optimal policy is the one that maximizes $\sum_{k=0}^K \epsilon_k J_k^\pi$.

A policy is *non-dominated* if there is no other policy that improves its return for one objective without decreasing return for another. The *Pareto front* is the set of all non-dominated policies.

## 3.2 Multi-Objective MPO (MO-MPO)

Whereas most multi-objective RL algorithms use linear scalarization, MO-MPO [6] takes a distributional approach to combining objectives. MO-MPO extends MPO [30] to the multi-objective setting. MO-MPO is a policy iteration algorithm with two steps: policy evaluation and policy improvement.

**Policy evaluation.** In this step, MO-MPO evaluates the current policy $\pi_{\text{old}}(a|s)$ by training a Q-function per objective, as according to Q-decomposition [31].

**Policy improvement.** Given the previous policy $\pi_{\text{old}}(a|s)$ and Q-functions $\{Q_k^{\text{old}}(s,a)\}_{k=0}^K$, in this step MO-MPO improves the policy for a given state visitation distribution $\mu(s)$. There are two sub-steps: finding per-objective improved action distributions, and distilling these into a new policy.

*Finding per-objective distributions:* For each objective $k$, MO-MPO computes a nonparametric policy $q_k(a|s)$ that improves the old policy with respect to that objective, subject to a non-negative constraint $\epsilon_k$ on the KL-divergence between the improved and old policies:

$$\max_{q_k} \; \mathbb{E}_{\mu(s)} \Big[ \int_a q_k(a|s)\, Q_k^{\text{old}}(s,a)\, \mathrm{d}a \Big] \quad \text{s.t. } \mathbb{E}_{\mu(s)} \Big[ \mathrm{KL}(q_k(\cdot|s) \,\|\, \pi_{\text{old}}(\cdot|s)) \Big] < \epsilon_k\,. \tag{3}$$

*Learning a new parameterized policy:* MO-MPO then updates the policy via supervised learning on these nonparametric policies, subject to a trust region with bound $\beta > 0$ for more stable learning:

$$\min_\theta \sum_{k=0}^N \mathbb{E}_{\mu(s)} \Big[ \mathrm{KL}(q_k(\cdot|s) \,\|\, \pi_\theta(\cdot|s)) \Big] \quad \text{s.t. } \mathbb{E}_{\mu(s)} \Big[ \mathrm{KL}(\pi_{\text{old}}(\cdot|s) \,\|\, \pi_\theta(\cdot|s)) \Big] < \beta\,. \tag{4}$$

The preference is expressed via the KL-constraints $\{\epsilon_k\}_{k=0}^K$: the larger $\epsilon_k$ is, the more influence objective $k$ has.

# 4 Learning Preferences and Policies in Parallel (LP3)

In this section we introduce LP3, our constrained multi-objective RL framework. We first formulate the constrained RL problem as a Constrained Multi-Objective MDP (CMO-MDP). Following this, we describe how LP3 solves this MDP, and detail how Lagrangian relaxation algorithms are an instantiation of this framework. We then derive two novel constrained RL algorithms based on LP3.

## 4.1 Constrained Multi-Objective MDP (CMO-MDP)

The CMDP formulation implies a single optimal solution. But in real-world settings, one may be considering a *range* of viable constraint thresholds, which correspond to different trade-offs between the task reward and constraints. This connects to the multi-objective setting.

We define a constrained multi-objective MDP (CMO-MDP) as $\{\mathcal{S}, \mathcal{A}, p, r_{0:K}, \beta_{N:K}, \gamma\}$, where $N \geq 1$. In a CMO-MDP, the optimization is over $K$ reward functions, where $N$ of these are task rewards, and the remaining $K - N$ rewards have constraints on them. Thus a CMO-MDP differs from a CMDP in two key ways: 1) the optimization objective is over *all* rewards, including the constraint-specific ones; and 2) there may be multiple task rewards $r_{0:N}$.[1]

A CMO-MDP compares constraint-satisfying policies in terms of their returns for the task reward *and* constraint functions—its solution is the set of all constraint-satisfying Pareto optimal policies. In contrast, a CMDP ranks policies according to only task return—its solution is the single constraint-satisfying policy with the highest task return.

## 4.2 The LP3 Framework for Solving CMO-MDPs

We now describe LP3, a unifying framework (Fig. 1) for deriving algorithms that solve CMDPs and CMO-MDPs. This framework has two modules: learning the policy, and learning which preference(s) (over the task and constraints) the policy should optimize for to meet the constraints.

For the first module, the key choice is which multi-objective RL algorithm (such as linear scalarization or MO-MPO) is used to train the policy $\pi_\theta$ to optimize for a given preference.

For the second module, the key choice is whether to learn a single preference $\boldsymbol{\epsilon}$ or a preference distribution $p(\boldsymbol{\epsilon})$. The former corresponds to learning a policy $\pi_\theta(a|s)$. Learning a distribution over preferences, however, requires training a preference-conditioned policy $\pi_\theta(a|s, \boldsymbol{\epsilon})$, that learns to optimize for different preference settings. This enables finding a set of constraint-satisfying policies, produced by sampling preferences from the final $p(\boldsymbol{\epsilon})$ and rolling out the fully-trained $\pi_\theta(a|s, \boldsymbol{\epsilon})$, conditioned on that preference. In practice, using a preference distribution can also improve learning, because training over multiple preferences enables more diverse data collection and exploration.

Note that we can use different time scales for training the two modules in this framework. In our experiments, we alternate between taking one learning step on each.

**Connection to Lagrangian Relaxation.** As mentioned, Lagrangian relaxation methods are equivalent to LP3 with linear scalarization and a single preference optimized using gradient descent. To show this, define the preference $\boldsymbol{\epsilon} = [1, \lambda_1, \lambda_2, \ldots, \lambda_K]^\top$, where $\lambda_{1:K}$ are the Lagrange multipliers.

In the first module, we train the policy to maximize a linear combination of Q-values, weighted by the current preferences $\boldsymbol{\epsilon}^{\text{old}}$: $\sum_{k=0}^{K} \epsilon_k^{\text{old}} Q_k(a, s)$. We can use any standard RL algorithm for this.

In the second module, given the current policy $\pi_{\text{old}}(a|s)$, we use gradient descent to optimize for a single preference setting $\boldsymbol{\epsilon}$ with respect to the loss $\mathcal{L}(\boldsymbol{\epsilon} > 0) = \sum_{k=1}^{K} \epsilon_k f_k$. Here, $f_k$ is the fitness score corresponding to whether the policy respects threshold $\beta_k$:

$$f_k = \mathbb{E}_{\mu(s)}\Big[Q_k^{\text{old}}(s, \mathbb{E}_{\pi_{\text{old}}(a|s)}[a])\Big] - \beta_k \,. \tag{5}$$

[1] Notice that if there is a single task reward ($N = 1$), then the solution of a CMO-MDP includes that of the corresponding CMDP (that does not optimize for constraint-specific rewards).

### 4.3 Novel Instantiations of LP3

We will now investigate different design choices in the LP3 framework, compared to what Lagrangian relaxation uses, that lead to two novel algorithms for constrained RL. Our first algorithm replaces linear scalarization with a state-of-the-art multi-objective RL algorithm, MO-MPO. The motivation is that MO-MPO may be better at training policies for a given preference—it has been shown to outperform linear scalarization on a variety of control problems [6]. We call this algorithm ***LP3 [MO-MPO]***. It uses the same second module (for learning the preference) as Lagrangian relaxation. Our second algorithm, in addition to using MO-MPO, learns a distribution over preferences. We call this algorithm ***LP3 [MO-MPO-D]*** and describe it below; full details are in the appendix.

#### 4.3.1 Module 1: Preference-Conditioned MO-MPO

Learning a distribution over preferences requires training a preference-conditioned policy $\pi_\theta(a|s,\boldsymbol{\epsilon})$, but MO-MPO can only train a policy $\pi_\theta(a|s)$ for a single preference $\boldsymbol{\epsilon}$. We extend MO-MPO to train a *single* policy $\pi_\theta(a|s,\boldsymbol{\epsilon})$ conditioned on preferences $\boldsymbol{\epsilon} \sim p_{\text{old}}(\boldsymbol{\epsilon})$, by making both the policy evaluation and policy improvement steps preference-conditioned as follows.

**Preference-conditioned policy evaluation.** We evaluate the current policy $p_{\text{old}}(\boldsymbol{\epsilon})\pi_{\text{old}}(a|s,\boldsymbol{\epsilon})$ via off-policy learning of per-objective Q-functions $Q_k^\pi(a,s,\boldsymbol{\epsilon})$. Note that these Q-functions must be preference-conditioned because the policy is. For each learning step we sample $L$ transitions from the replay buffer $\{s_i, a_i, \{r_i^k\}_k^K, s_i'\}_i^L$. To stabilize off-policy learning and improve sample efficiency, we use relabeling to augment the states with preferences sampled from the current preference distribution $\boldsymbol{\epsilon}_i \sim p_{\text{old}}(\boldsymbol{\epsilon})$, resulting in transitions $\{[s_i,\boldsymbol{\epsilon}_i], a_i, \{r_i^k\}_k^K, [s_i',\boldsymbol{\epsilon}_i]\}_i^L$. Any policy evaluation algorithm can be used here. We use distributional policy evaluation with 5-step return [32].

**Preference-conditioned policy improvement.** This step improves the current policy $\pi_{\text{old}}(a|s,\boldsymbol{\epsilon})$.

*Finding per-objective distributions:* To find preference-conditioned per-objective improved action distributions $q_k(a|s,\boldsymbol{\epsilon})$, we optimize the following policy optimization problem for each objective:

$$\begin{aligned} \max_{q_k} \quad & \mathbb{E}_{p_{\text{old}}(\boldsymbol{\epsilon})\mu(s)}\Big[\int_a q_k(a|s,\boldsymbol{\epsilon})\, Q_k^{\text{old}}(s,a,\boldsymbol{\epsilon})\,\mathrm{d}a\Big] \\ \text{s.t.} \quad & \mathbb{E}_{\mu(s)}\Big[\mathrm{KL}(q_k(\cdot|s,\boldsymbol{\epsilon})\|\pi_{\text{old}}(\cdot|s,\boldsymbol{\epsilon}))\Big] < \epsilon_k \quad \forall\, \boldsymbol{\epsilon} \sim p_{\text{old}}(\boldsymbol{\epsilon})\,. \end{aligned} \tag{6}$$

This optimization problem generalizes (3) from MO-MPO, which assumes fixed KL-constraints $\boldsymbol{\epsilon}$, to a distribution $p_{\text{old}}(\boldsymbol{\epsilon})$ over KL-constraints. We use a closed-form solution for $q_k$ (see appendix).

*Learning a new parameterized policy:* After obtaining per-objective improved policies, we use supervised learning to distill these into a new parameterized preference-conditioned policy:

$$\begin{aligned} \min_{\theta} \quad & \sum_{k=0}^{K} \mathbb{E}_{p_{\text{old}}(\boldsymbol{\epsilon})\mu(s)}\big[\mathrm{KL}(q_k(\cdot|s,\boldsymbol{\epsilon})\|\pi_\theta(\cdot|s,\boldsymbol{\epsilon}))\big] \\ \text{s.t.} \quad & \mathbb{E}_{p_{\text{old}}(\boldsymbol{\epsilon})\mu(s)}\big[\mathrm{KL}(\pi_{\text{old}}(\cdot|s,\boldsymbol{\epsilon})\|\pi_\theta(\cdot|s,\boldsymbol{\epsilon}))\big] < \beta. \end{aligned} \tag{7}$$

#### 4.3.2 Module 2: Learning Preferences

The second module optimizes the preference $\boldsymbol{\epsilon}$ or preference distribution $p(\boldsymbol{\epsilon})$ to lead to better constraint satisfaction. This assumes fitness functions $f_k$ that evaluate satisfaction of the constraint threshold $c_k$, given the current policy $\pi_{\text{old}}$ and Q-function $Q_k^{\text{old}}$.[2] We can use any gradient-based algorithm to optimize the preferences, initialized by $\boldsymbol{\epsilon}_{\text{old}}$ or $p_{\text{old}}(\boldsymbol{\epsilon})$, to maximize the fitness.

In our empirical evaluation, we use fitness functions that give a score of zero to preferences that lead to constraint satisfaction, and negative scores to all others. For equality constraints we use a fitness function that penalizes the difference between the expected Q-values and constraint threshold, and for inequality constraints it penalizes the amount the expected Q-values violate the threshold:

$$f_k^{\text{eq}}(\boldsymbol{\epsilon}) = -\Big|\mathbb{E}_{\mu(s)}\Big[Q_k^{\text{old}}(s,\mathbb{E}_{\pi_{\text{old}}}[a],\boldsymbol{\epsilon})\Big] - \beta_k\Big| \tag{8}$$

$$f_k^{\text{in}}(\boldsymbol{\epsilon}) = \min\Big(0,\ \mathbb{E}_{\mu(s)}\Big[Q_k^{\text{old}}(s,\mathbb{E}_{\pi_{\text{old}}}[a],\boldsymbol{\epsilon})\Big] - \beta_k\Big). \tag{9}$$

[2] The flexibility in choosing the fitness function enables specifying a variety of constraints: e.g., expected return equal to a threshold, within an interval of thresholds, or within the union of disjoint threshold intervals.

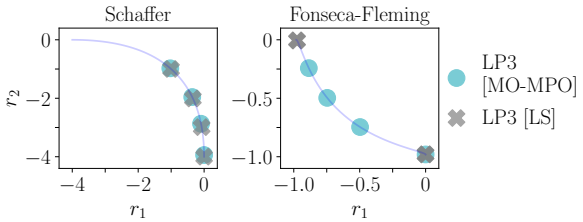


Figure 2: The curve denotes the function. For a convex Pareto front (left), both approaches find optimal solutions for all constraints on $r_2$. However, for a concave Pareto front, only LP3 [MO-MPO] finds optimal solutions for the constraints of $-0.25$, $-0.5$, and $-0.75$.

# 5 Experiments

To recap, in the LP3 framework, Lagrangian relaxation corresponds to using linear scalarization and learning a single preference (Fig. 1). Lagrangian relaxation is a widely-used approach for constrained RL because of its simplicity and effectiveness [22, 12]. We proposed two novel changes: 1) using MO-MPO instead of linear scalarization in Module 1, and 2) learning a distribution over preferences rather than a single preference in Module 2. In our empirical evaluation, we first run ablation studies that study the impact of each of these changes. Then, we run further experiments comparing LP3 [MO-MPO-D], that combines both improvements, against Lagrangian relaxation.

The main baseline we compare against is ***LP3 [LS]***, which uses MPO [30] to optimize the Lagrangian dual objective (2); LS stands for linear scalarization. This is a strong Lagrangian-based baseline that obtains state-of-the-art results on benchmark constrained RL tasks (Appendix C.2).

**Implementation details.** We train Gaussian policies parameterized by neural networks. Our implementations are based on the open-sourced MPO and MO-MPO in Acme [33]. For LP3 [MO-MPO-D], the preference distribution is initialized to uniform and is a discrete distribution over 100 linearly-spaced values from $10^{-5}$ to $\epsilon_{\max}$. Unless otherwise mentioned, we use (9) as the fitness function for LP3 [MO-MPO] and use (8) for LP3 [MO-MPO-D]—so that it converges to a single policy, rather than a set of policies, for easier comparison with the single-preference approaches. We sweep over hyperparameters for each approach; details and evaluation procedure are in the appendix.

## 5.1 Module 1: Linear Scalarization vs. MO-MPO

To first assess the impact of using MO-MPO instead of linear scalarization, we compare LP3 [MO-MPO] against LP3 [LS] across several domains. LP3 [LS] is an ablation of LP3 [MO-MPO] with respect to Module 1; both learn a single preference in the same way for Module 2.

**Toy Domain.** First, we consider a toy domain with a continuous action ($a \in \mathbf{R}$) and two objectives ($r(a) \in \mathbf{R}^2$). The constraint is on $r_2(a)$. For the reward, we use either the Schaffer or Fonseca-Fleming function, which are standard test functions in multi-objective optimization [34].

For each function, we train policies for several constraint thresholds. For Schaffer, both approaches find the optimal solutions. However, for Fonseca-Fleming, which has a concave Pareto front, only LP3 [MO-MPO] can find all optimal solutions (Fig. 2, right). LP3 [LS] only finds solutions at the two extremes, even after tuning the learning rate. This is likely because linear scalarization cannot find solutions on the concave portions of a Pareto front [35]. Even in this simple setting, we can see the potential advantages of using MO-MPO over linear scalarization, in Module 1 of LP3.

**Humanoid.** We next consider two high-dimensional locomotion tasks, humanoid run and walk from the DeepMind Control Suite [36]. The constraint is imposed on the expected negative control norm (i.e., $-\|a\|_2$), which roughly captures the "energy" expended by the agent.

We train policies using LP3 [LS] and LP3 [MO-MPO] for a variety of different constraint thresholds. We plot the average task reward and cost for the trained policies in Fig. 3, where for both axes, higher values are better—a policy is non-dominated if no other point lies both above and to the right of it. LP3 [MO-MPO] finds higher-quality solutions for both tasks, especially for humanoid walk. This may be because the ground-truth Pareto front is concave, and as seen in the toy domain, Lagrangian-based approaches struggle to find solutions on concave Pareto fronts.

LP3 [MO-MPO] is also more sample efficient: it learns policies that reach moderate task reward mid-way through training for humanoid run, whereas the majority of LP3 [LS] policies obtain zero task reward at this point (Fig. 3, "run, mid-training"). This could be because with two conflicting objectives, LP3 [MO-MPO] can optimize for both at once, whereas LP3 [LS] cannot (Fig. 4, left).

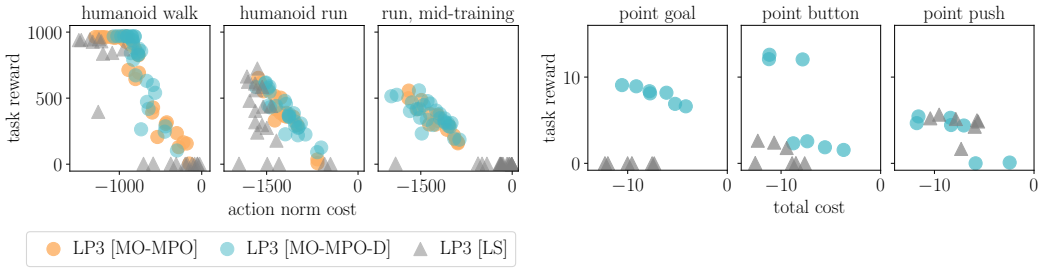


Figure 3: For hard-to-satisfy constraints, the two MO-MPO-based approaches consistently find higher-quality policies than LP3 [LS]. In these plots, above (i.e., higher task reward) and to the right (i.e., lower cost) is better. LP3 [LS] cannot find solutions on a concave Pareto front (humanoid walk) and is less sample-efficient—it takes longer to obtain task reward (run, mid-training). For point push and easy-to-satisfy constraints, the approaches perform on par (see appendix).

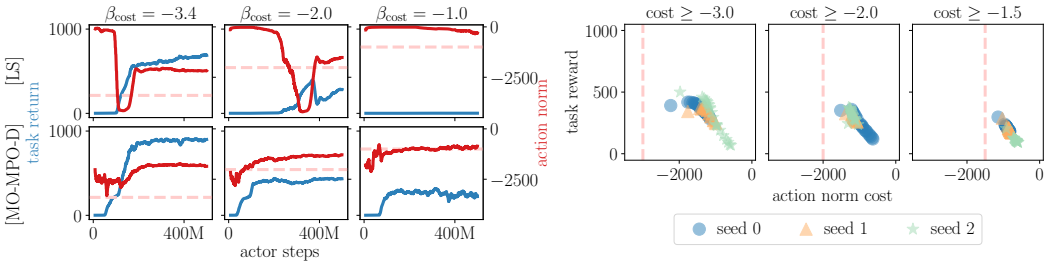


Figure 4: Both plots are for humanoid run; the dashed lines indicate the constraint threshold. **Left:** LP3 [MO-MPO-D] (bottom) is able to improve the policy's performance on both objectives at once, whereas LP3 [LS] (top) alternates between the two. **Right:** LP3 [MO-MPO-D] can find a *set* of constraint-satisfying nondominated policies, i.e., a portion of the Pareto front..

### 5.2 Module 2: Single Preference vs. Distribution over Preferences

We combine MO-MPO, because it outperforms linear scalarization, with learning a distribution over preferences to obtain LP3 [MO-MPO-D]. LP3 [MO-MPO] is an ablation of LP3 [MO-MPO-D] with respect to Module 2. For humanoid, both approaches perform similarly (Fig. 3).

A key advantage of using LP3 [MO-MPO-D], as opposed to single-preference approaches, is the ability to solve a CMO-MDP and discover a *set* of constraint-satisfying non-dominated solutions, rather than finding a single solution. This is useful because in practice, we may not know the true constraint ahead of time, and would thus prefer to see several options along the Pareto front. For humanoid run, we run LP3 [MO-MPO-D] with (9) as the fitness function. This indeed learns a portion of the Pareto front that satisfies the constraint (Fig. 4, right). For these plots, we evaluated the policy conditioned on preferences at evenly-spaced percentiles of the learned distribution.

### 5.3 Evaluation of Full Algorithm

Finally, we focus on comparing LP3 [MO-MPO-D], which combines both improvements, against existing approaches in the literature, as well as our LP3 [LS], a strong Lagrangian-based approach.

**Domains.** We evaluate on two benchmark suites for constrained RL: Real World RL (RWRL) [37] and OpenAI Safety Gym [22]. From RWRL, we use the same four tasks and corresponding constraints as used by Calian et al. [5]. The cost is binary and depends on the task, e.g., for humanoid walk, the cost is $-1$ if the joint angles are outside of a specified range, and 0 otherwise.

From Safety Gym, we use the Level 2 point mass tasks: goal, button, and push. There is a sparse reward for either reaching a goal location or pushing a box to a goal. There is a cost for running into or over objects. We also created an analagous simulated robot manipulation task with the Sawyer arm, where there is a sparse reward for reaching a green block and pushing it to a target position. There is a cost for either the gripper or green block colliding with the yellow block (Fig. 5, left).

**Results.** For the RWRL tasks, we compare against the algorithms evaluated in Calian et al. [5] (described in the appendix). For each task, we used LP3 [MO-MPO-D] and LP3 [LS] to train five

| Task | Algorithm | Task | Cost |
|---|---|---|---|
| cartpole swingup | MetaL | $850 \pm 1$* | $-342 \pm 1$* |
| | RS-0.1 | $851 \pm 1$* | $-358 \pm 1$* |
| | D4PG-Lag | $376 \pm 41$* | $-199 \pm 29$* |
| | LP3 [LS] | $166 \pm 20$* | $-293 \pm 108$* |
| | LP3 [MO-MPO-D] | $\mathbf{871 \pm 4}$ | $\mathbf{-80 \pm 4}$ |
| humanoid walk | MetaL | $659 \pm 54$* | $-368.3 \pm 26$* |
| | RS-0.1 | $317 \pm 67$* | $-295.6 \pm 68$* |
| | D4PG-Lag | $571 \pm 53$* | $-361.3 \pm 19$* |
| | LP3 [LS] | $668 \pm 149$ | $-104.3 \pm 41$ |
| | LP3 [MO-MPO-D] | $\mathbf{828 \pm 28}$ | $\mathbf{-32 \pm 3}$ |

| Task | Algorithm | Task | Cost |
|---|---|---|---|
| quadruped walk | MetaL | $999 \pm 0$* | $-615 \pm 15$* |
| | RS-0.1 | $999 \pm 0$* | $-28 \pm 25$ |
| | D4PG-Lag | $999 \pm 0$* | $-632 \pm 14$* |
| | LP3 [LS] | $\mathbf{1000 \pm 0}$ | $-33 \pm 8$* |
| | LP3 [MO-MPO-D] | $\mathbf{1000 \pm 0}$ | $\mathbf{0 \pm 0}$ |
| walker walk | MetaL | $966 \pm 0$* | $-164.1 \pm 9$* |
| | RS-0.1 | $978 \pm 0$* | $-221.0 \pm 4$* |
| | D4PG-Lag | $905 \pm 11$* | $\mathbf{-76.8 \pm 3}$ |
| | LP3 [LS] | $\mathbf{983 \pm 1}$ | $\mathbf{-58 \pm 5}$ |
| | LP3 [MO-MPO-D] | $\mathbf{983 \pm 0}$ | $\mathbf{-61 \pm 28}$ |

Table 1: On RWRL, LP3 [MO-MPO-D] achieves better task reward *and* cost than other algorithms. The bounds are on standard error. Results for MetaL, D4PG-Lag, and RS-0.1 are from [5]; these all rely on linear scalarization. * indicates $p \leq 0.05$, compared to LP3 [MO-MPO-D] (Welch's t-test).

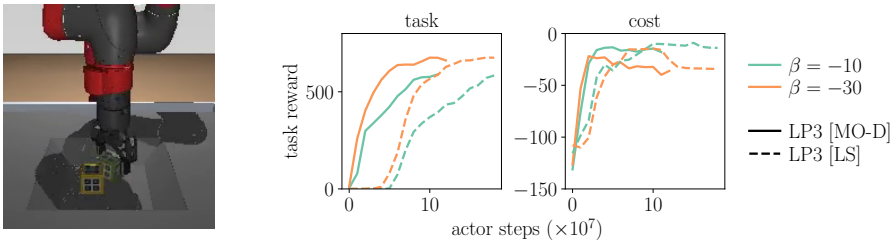


Figure 5: Learning curves for the robot push task. $\beta$ denotes the constraint threshold.

policies with different random initializations. LP3[MO-MPO-D] performs best on three out of four tasks, incurring less cost while achieving on-par or better task reward (Table 1).

For the point tasks, on two out of the three tasks, LP3 [MO-MPO-D] finds better solutions than LP3 [LS], on harder-to-satisfy constraint thresholds (Fig. 3). For point push and for easier-to-satisfy constraint thresholds (see appendix), LP3 [MO-MPO-D] does not give an additional advantage over LP3 [LS]. We hypothesize this is because when meeting the constraint does not conflict much with the task objective, it is easier to find high-task-reward solutions that incur small cost.

On the robot manipulation task, LP3 [MO-MPO-D] and LP3 [LS] converge to similar task performance and constraint satisfaction, but LP3 [LS] takes about twice as many actor steps to do so (Fig. 5). This is likely due to the use of MO-MPO in Module 1—recall that we saw in the first ablation study that this improves sample efficiency in humanoid run (Sec. 5.1),

**Additional Analysis.** LP3 [MO-MPO-D] is also better able to find policies that satisfy the constraints, compared to LP3 [LS] (Appendix C.5). In addition, LP3 [MO-MPO-D] scales better to multiple constraints than LP3 [LS] (Appendix C.6). We also ran experiments with multiple random seeds per constraint threshold; these support the conclusions drawn above (Appendix C.7).

## 6 Conclusion and Future Work

In this paper, we proposed to formulate constrained RL as a constrained multi-objective MDP. Based on this multi-objective perspective, we described a general framework, LP3, for solving constrained RL problems. We used this perspective to introduce two new algorithms: 1) using a better multi-objective RL algorithm (in our case, MO-MPO) instead of linear scalarization, and 2) learning a distribution over preferences instead of a single preference, to find a set of constraint-satisfying non-dominated policies. To apply LP3 [MO-MPO-D] for safe RL, it could be initialized with a conservative distribution over preferences that prioritizes minimizing cost.

A promising direction of future work is to apply recent advances in improving the stability and performance of Lagrangian relaxation [e.g. 14, 5] toward single-preference approaches from our framework. Another promising area of future work is to run LP3 [MO-MPO(-D)] on a real robot platform. MPO with Lagrangian relaxation (as in LP3 [LS]) has been successfully run on robots [e.g. 14], and our ablation study suggests LP3 [MO-MPO] should perform at least as well.

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