A SELF-EXPLANATORY METHOD FOR THE BLACK BOX PROBLEM ON DISCRIMINATION PART OF CNN

Anonymous authors
Paper under double-blind review

ABSTRACT

Convolution neural networks (CNNs) have surpassed human's abilities in some specific tasks. Recently, the black box problem of the discrimination part of CNN, which is composed of the fully connected layers of the CNN, has been studied by different scientific communities. Many methods were proposed for extracting an interpretable model from the discrimination part, which can explain the prediction of the part. However, it is hard for the interpretable models to approximate the discrimination part. We think that the problem could be solved by shrinking the distance between the interpretable model and the discrimination part. This paper firstly introduces a Markov random field model (MRF), namely the Deep Cognitive Learning Model(DCLM), which can express the causal relationship implicated in the discrimination part of CNN. The DCLM is initiatively extracted from the discrimination part by solving a Maximum Satisfiability(MAX-SAT) problem. And then, a game process is implemented for shrinking an interpretation distance which can evaluate how close the discrimination part is to the DCLM. Finally, the proposed self-explanatory method was evaluated by some contrastive experiments with certain baseline methods on some standard image processing benchmarks. These experiments indicate that the proposed method can reduce the distance between the discrimination part and the DCLM without largely reducing its generalization performance. Moreover, the generalization performance of the DCLM also can be improved and the discrimination part can be explained by the DCLM at every epoch during the training process after the interpretation distance converges.

1 INTRODUCTION

Convolution neural network(CNN) has surpassed human abilities in some specific tasks such as computer game and computer vision etc. However, they are considered difficult to understand and explain(Brandon, 2017), which leads to many problems in aspects of privacy leaking, reliability and robustness. Explanation technology is of immense help for companies to create safer, more trustable products, and to better manage any possible liability of them (Riccardo et al., 2018). Recently, for finding inherent causality in CNN, which can conclude the reason for its prediction results, the unexplainable problem of CNN, especially concerning the discrimination part which is composed of the fully connected layers of the CNN, has been studied by different scientific communities. Many methods were proposed for extracting an interpretable model from the optimal discrimination part for expressing causal relationships between inputs and outputs of the part. However, because of data bias and noisy data in the training data set, it is difficult for the interpretable model to approximate the optimal discrimination part. We think that the problem could be solved by the following procedure. Firstly, based on logical relationships between activated states of the neurons in its first layer and its output layer of the discrimination part, extracting an interpretable model of the part in training process. Then, iteratively shrinking a distance between the model and the part, and amending them. Finally, an optimal interpretable model can be obtained for providing a well interpretation for the predication of the part. Moreover, based on the procedure, it is also possible to monitor the evolution of logical relationships implied in the part in the process.

Main contributions of this paper can be summarized as follows:

- An interpretable model, Deep Cognitive Learning Model(DCLM), is proposed to express the causal relationship in the discrimination part, and a greedy method is given for initiatively extracting the DCLM from the discrimination part by solving its Maximum Satisfiability(MAX-SAT) Problem.
- A new game method is proposed to improve the interpretability performance of the discrimination part without largely reducing its generalization performance by iteratively shrinking a distance between DCLM and the discrimination part and amending the two models.
- Interpretable distance is proposed to evaluate a distance between the discrimination part and the interpretable model on the unexplainable problem.

2 RELATED WORK

There are usually two types of methods for the unexplainable problem of the discrimination part, such as post-hoc method and ante-hoc method (Holzinger et al., 2019). However, because ante-hoc method is a transparent modeling method(Arrietaa et al., 2020), it can not obtain an explanation about the discrimination part. So, the post-hoc method will be reviewed.

Early post-hoc method can obtain global explanations for a neural network by extracting an interpretable model. Some references(Craven & Shavlik, 1999; Krishnan et al., 1999; Boz, 2002; Johansson & Niklasson, 2009) proposed a few methods that can find a decision tree for explaining a neural network by maximizing the gain ratio and an estimation of the current model fidelity. Other references (Craven & Shavlik, 1994; Johansson & Niklasson, 2003; Augasta & Kathirvalavakumar, 2012; Sebastian et al., 2015; Zilke et al., 2016) proposed rule extraction methods for searching optimal interpretable rules from a neural network.

Recently, some feature relevance methods have become progressively more popular. Montavon et al.(Montavon et al., 2017) proposed a decomposition method from a network classification decision into contributions of its input elements based on deep Taylor decomposition. Shrikumar et al.(Shrikumar et al., 2016) proposed DeepLIFT which can compute importance scores in a multilayer neural network by explaining the difference of the output from some reference output in terms of differences of the inputs from their reference inputs.

Some other works make complex black box model simpler. Che et al.(Che et al., 2017) proposed a simple distillation method called Interpretable Mimic Learning for extracting an interpretable simple model by gradient boosting trees. Thiagarajan et al.(Thiagarajan et al., 2016) build a Treeview representation of the complex model by hierarchical partitioning of the feature space. In addition, some references (Hinton et al., 2015; Bucila et al., 2006; Frosst & Hinton, 2017; Traore et al., 2019) proposed the distillation method of knowledge from an ensemble of models into a single model. Wu et al.(M. Wu, 2018) proposed a tree regularization method via knowledge distillation to represent the output feature space of a RNN based on a Multilayered perception. However, these methods can only solve the unexplainable problem of trained neural network or trained deep neural networks with explicit input characteristics. Wan et al.(Wan et al., 2020) constructed a decision tree using the last fully connection layer of the discrimination part of a CNN based on a prior structure.

In the paper, our goal is to improve the interpretability performance of multi fully connected layers of CNN without hurting its generalization performance by initiatively extracting its logic relationships with no prior structure and finally obtain its explanation by these logic relationships.

3 DEEP COGNITIVE LEARNING MODEL

For expressing the causal relationship between these neurons in the discrimination part, a new interpretable model is designed in the section. As we all known, a CNN includes a feature extractor and a discrimination part. The feature extractor composes of some convolution layers and some pooling layers. The outputs from the feature extractor are the inputs of the discrimination part of the CNN, namely feature maps, $\tau_1, \tau_2, ..., \tau_k$ where k is the number of feature maps. All these feature maps form a feature set Γ.

We suppose that the discrimination part should better be explained by the logic relationships of the activated states of the neurons in its first layer and its output layer. This is because the relationships

are the inherent property of the part. In order to express the relationships, a deep cognitive learning model (DCLM)is proposed, shown in Fig.1(b).

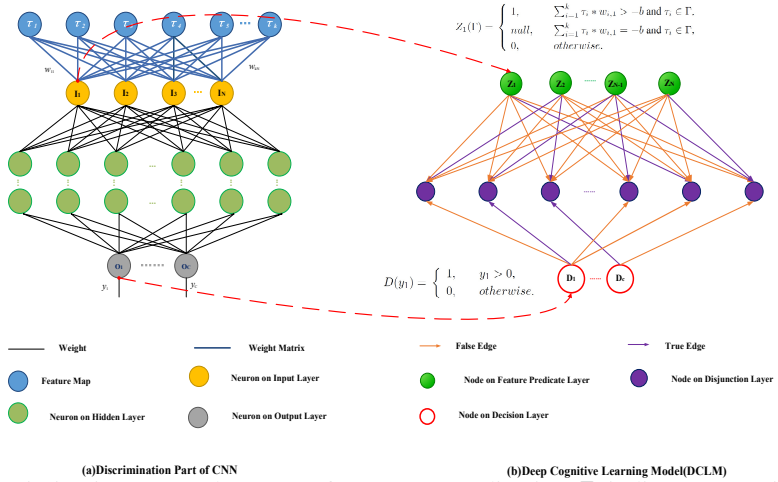

Figure 1: Discrimination part and DCLM of a CNN(A predication Z_i indicates an activated state of a neuron I_i of the discrimination part of the CNN and a predication D_i indicates an activated state of a neuron O_i)

The DCLM consists of three layers:feature predicate layer, disjunction layer, and decision layer. The top layer is feature predicate layer which consists of many nodes. Every node has a predicate $Z_j(\Gamma)$ that expresses a positive action or negative action of features which the jth neuron in the first fully connected layer of the discrimination part captures. The predicate $Z_j(\Gamma)$ is defined as follows:

$$Z_1(\Gamma) = \begin{cases} 1, & \sum_{i=1}^{k} \tau_i * w_{i,1} > -b \text{ and } \tau_i \in \Gamma, \quad (1) \\ null, & \sum_{i=1}^{k} \tau_i * w_{i,1} = -b \text{ and } \tau_i \in \Gamma, \quad (1') \\ 0, & otherwise. \quad (1'') \end{cases}$$

where $j \in 1, 2, ..., N$, N is the number of the input neurons of the first fully connection layer of the discrimination part of the CNN. $w_{i,j}$ is a weight vector between the ith feature map and the jth neuron, b_j is the bias of the jth neuron, and "$*$" is a convolution operation. "1" and "0" denote a positively activated state and a negatively activated state of the neuron respectively. "null" denotes an inactivated state.

The bottom layer is a decision layer which includes all nodes used for decision. Every node has a predicate which expresses an activated state of an output neuron of the discrimination part. It is defined as follows:

$$D(y_1) = \begin{cases} 1, & y_1 > 0, \quad (2) \\ 0, & otherwise. \quad (2') \end{cases}$$

where $i \in 1, 2, ..., C$,C is number of the output neurons of the CNN, y_i is the output value of the ith output neuron of the discrimination part. All nodes on the feature predicate layer and every node on the decision layer are connected to one or more nodes on the middle layer, namely as disjunction layer, with true or false edges. Every node represents a disjunction relation, which is expressed by a disjunctive normal form. It is worth mentioning: if a node is connected to a node on the disjunction layer by a false edge, its predicate follows after a non-operator in the disjunctive normal form.

The potential function of a disjunctive normal form can be obtained by using the Lukasiewicz method(Giles, 1975).

$$\phi_c(y_i) = \min(1, T(\Gamma, y_i)) \quad (3)$$

where $T(\Gamma, y_i) = \sum_{j=1}^{N}\{a_j[1 - Z_j(\Gamma)] + (1 - a_j)Z_j(\Gamma)\} + (a_N + 1)D(y_i)$ and N is the number of the nodes on the feature predicate layer. If $a_j = 1$, there is a false edge. Otherwise, there is a true edge.

The conditional probability distribution that a ground DCLM including all disjunctive normal forms is true is

$$p(\mathbf{y}, \Gamma) = \frac{1}{\Xi}\exp(\frac{\sum_{i=1}^{G}\lambda_i\phi_{ci}(y_i)}{\sum_{i=1}^{G}\lambda_i}) \tag{4}$$

where G is the number of all ground formulas, $\Xi = \sum_{\Gamma\in\mathbb{F}}\exp(\frac{\sum_{i=1}^{G}\lambda_i\phi_{ci}(y_i)}{\sum_{i=1}^{G}\lambda_i})$ is a partition function, $\mathbf{y} = (y_1, y_2, ..., y_G)$, y_i is an output value of the CNN and λ_i is a weighted value of the ith ground formula.

By maximizing its likelihood function, the optimal a_i and λ_i in the DCLM can be obtained.

$$C(\Gamma) = arg\max_{a_i,\lambda_i}[\log p(\mathbf{y}, \Gamma)] = arg\max_{a_i,\lambda_i}(\frac{\sum_i \lambda_i\phi_{ci}(y_i)}{\sum_i \lambda_i} - \log(\Xi)) \tag{5}$$

For extracting a optimal DCLM, a Maximum A Posterior(MAP) algorithm on the Maximum Satisfiability Problem (MAX-SAT) was designed. Using the disjunction normal form with the greatest weighted value in the optimal DCLM, a prediction of an input image can be obtained.

4 Evaluation of interpretability performance

We consider that if the discrimination part of a CNN has a similar shape of function curve with its optimal interpretable model, the former can be easily explained by the latter. Therefore, the interpretable performance of the discrimination part can be measured by the shape similarity between it and its optimal interpretable model. We posit that given the same input data set, the similarity may be measured by variance of differences between outputs of the both models. It can be named interpretation distance. It is easily proved that the smaller the interpretation distance is, the more similar their shapes are, and the better the interpretability performance of the discrimination part would be.

Definition 1 If X is a compact metric space and ν is a Borel measure in X, such as Lebesgue measure or marginal measures, in $\mathcal{L}_\nu^2(X)$, a square integrable function space on X, the interpretation distance, $\phi_d(P^*, f)$, between a discrimination part $f(x)$ and its optimal DCLM $P^*(x)$ is

$$\phi_d(P^*, f) = \int_Z (f(x) - P^*(x) - \mu^{P^*}(f))^2 d\nu \tag{6}$$

where

$$\mu^{P^*}(f) = \int_Z (f(x) - P^*(x))d\nu \tag{7}$$

5 Game between a DCLM and the discrimination part of a CNN

As discussed above,when the shapes of the discrimination part of a CNN and its optimal interpretable model are enough similar, the discrimination part has well interpretability performance. However, its generalization performance will tend to decrease. This is mainly attributed to the fact that because of data bias and noisy data in training data set, the sufficient and necessary condition for the consistent convergence of the two performances, $\phi_d(P^*, f^*) = 0$(f^* is the optimal prediction model), is difficult to be guaranteed. Therefore, for the tradeoff problem, in training process, extracting an DCLM from the discrimination part and then iteratively reducing the interpretation distance between the two models may be a feasible solution. A detailed discussion about the problem can be found in the appendix.

To avoid reducing the generalization performance, the maximum probability $p(\mathbf{w} \mid X, \mathbf{y}_t)$ should be guaranteed, where X is a training sample, $\mathbf{w}$ is the parameter set of the CNN and $\mathbf{y}_t$ is the target vector of X.

$$p(\mathbf{w} \mid X, \mathbf{y}_t) = \frac{p(\mathbf{w} \mid X)p(\mathbf{y}_t \mid \mathbf{w}, X)}{p(\mathbf{y}_t \mid X)} \propto p(\mathbf{w})p(\mathbf{y}_t \mid w, X) \tag{8}$$

Algorithm 1 Game between DCLM and the discrimination part of a CNN(Its time complexity is $O(N+M)$ where $O(N)$ is a time complexity of training CNN,$O(M)$ is a time complexity of construction of Logic Net.)

Input: $Data_set$
Output: $DCLM$
Repeat
$CNN = CNN_Train(Data_set, Adam, loss = CrossEntropy)$
for data,label **in** Data set **do**
  $Feature_map = CNN_feature_extractor(CNN)$.
  $disjunctive_normal_form = Disjunction(Feature_map, Fnn, W_{i,j})$.
  To predicate value of neuron according to Eq.1
  Use the disjunction forms of data obtained from the all predicate combinations to update $DCLM$.
end for
for $i = 1$ **to** n **do**
  for data,label **in** Data set **do**
    $Feature_map = CNN_feature_extractor(CNN)$
    $ym = DCLM(Feature_map)$
    $CNN_DCLM(data, ym, loss = Eq.14)$
  end for
end for
Until the interpretation distance and accuracy converge

where $p(\mathbf{y}_t \mid \mathbf{w}, X) = \int p(\mathbf{y}_t \mid f, \mathbf{w}, X) \int p(f \mid \mathbf{y}_{dclm}, \mathbf{w}, X) p(\mathbf{y}_{dclm} \mid \mathbf{w}, X) d\mathbf{y}_{dclm} df$ and $\mathbf{y}_{dclm}$ is a prediction of DCLM.

When the DCLM is known, $\mathbf{y}^*_{dclm}$ is its optimal prediction and $p(\mathbf{y}^*_{dclm} \mid \mathbf{w}, X) = 1$. Then

$$\int p(f \mid \mathbf{y}_{dclm}, \mathbf{w}, X) p(\mathbf{y}_{dclm} \mid w, X) d\mathbf{y}_{dclm} = p(f \mid \mathbf{y}^*_{dclm}, \mathbf{w}, X) \tag{9}$$

Similarly, known the input X and $\mathbf{w}$, f_{nn} is the optimal solution of the CNN.

$$p(\mathbf{y}_t \mid \mathbf{w}, X) = p(\mathbf{y}_t \mid f_{nn}, \mathbf{w}, X) p(f_{nn} \mid \mathbf{y}^*_{dclm}, \mathbf{w}, X) \tag{10}$$

If $\mathbf{w}$ and X are given and the loss function $\phi_r(\mathbf{y}_t, f_{nn}) = -\frac{1}{2}\sum_l \mid \mathbf{y}_t - f_{nn} \mid^2$, the conditional probability distribution function

$$p(\mathbf{y}_t \mid f_{nn}, \mathbf{w}, X) = \frac{\exp(\phi_r(\mathbf{y}_t, f_{nn}))}{\Xi_1} \tag{11}$$

Meanwhile,

$$p(f_{nn} \mid \mathbf{y}^*_{dclm}, \mathbf{w}, X) = \frac{\exp(-\phi_d(\mathbf{y}^*_{dclm}, f_{nn}))}{\Xi_2} \tag{12}$$

where Ξ_1 and Ξ_2 are partition functions. Then by maximizing a likelihood function of $p(\mathbf{w} \mid X, \mathbf{y}_t)$ the optimal w can be obtained. In particular, assuming that $\mathbf{w}$ follows Gaussian distribution, we get:

$$C_{\mathbf{w}}(X, \mathbf{y}_t) = arg\max_{\mathbf{w}}[-\frac{\alpha}{2} \parallel \mathbf{w} \parallel^2 + \phi_r(\mathbf{y}_t, f_{nn}) - \log(\Xi_1) - \phi_d(\mathbf{y}^*_{dclm}, f_{nn}) - \log(\Xi_2)] \tag{13}$$

where α is a meta-parameter determined by the variance of the selected Gaussian distributions. Turn it into a minimization problem:

$$C_{\mathbf{w}}(X, y_t) = arg\min_{\mathbf{w}}[\frac{\alpha}{2} \parallel \mathbf{w} \parallel^2 - \phi_r(\mathbf{y}_t, f_{nn}) + \log(\Xi_1) + \phi_d(\mathbf{y}^*_{dclm}, f_{nn}) + \log(\Xi_2)] \tag{14}$$

The iterative optimization algorithm is shown as follows:

6 Experimental verification

We designed two experiments to verify the effectiveness of the proposed method. The first experiment verified whether the self-explanatory method could improve the interpretability performance

of the CNN without sacrificing its generalization performance. The second experiment verified whether the proposed method can tend towards stability and convergence in the game process.

In the experiments, CNN3(includes 3 convolution layers,3 MaxPooling layers, 3 fully connect layers(FCLs) and 1 output layer),CNN5(includes 5 convolution layers,5 MaxPooling layers, 3 FCLs and 1 output layer), and CNN8(includes 8 convolution layers,8 MaxPooling layers, 3 FCLs and 1 output layer) were used. Traditional training methods on the three types CNN were named as CNN3-Trad, CNN5-Trad and CNN8-Trad respectively. By contrast, our proposed methods on these CNNs were named as CNN3-DCLM, CNN5-DCLM and CNN8-DCLM respectively. All experiments used Mnist(Lecun et al., 1998), FashionMnist(Zalando, 2017), and Cifar-10(Krizhevsky, 2009) benchmark data sets. All algorithms were implemented in Python using the Pytorch library(Paszke et al., 2019). All experiments ran on a server with Intel Xeon 4110(2.1GHz) Silver Processor, 20GB RAM and Nvidia Telsa T4.

Experiment 1: Performance verification of the proposed method on CNN. We replaced the discrimination part of three traditionally trained CNNs with soft decision tree(SDT)(Frosst & Hinton, 2017) and designated these methods as CNN3-Trad-SDT, CNN5-Trad-SDT and CNN8-Trad-SDT respectively. The accuracy of CNN, accuracy of STD or DCLM, and interpretation distance corresponding to all methods were shown in Table 1. Some values are ”—”, which indicates that these results do not exist.

It is observed in Table 1 that the accuracy of all CNN-DCLMs are higher than the two interpretable models, such as SDT and DCLM, on all benchmark data sets and are around 1.4 percentage points lower than those of CNN-Trads. But it is worth noticing that on the most of data sets the interpretation distances of all CNN-DCLMs are lower than interpretation distances of all CNN-Trad-SDTs. These might prove that that the self-explanatory method can improve the interpretability performance of the discrimination part of a CNN without largely reducing its generalization performance. The accuracies of the DCLMs are higher than those of the SDTs except CNN3-DCLM on FashionMnist data set and CNN3-DCLM on Mnist. These results might prove that the proposed method can find more excellent interpretable model than CNN-Trad-SDT.

Experiment 2: Convergence test of the proposed method We designed the experiments to demonstrate convergence of the proposed method. CNN3-Trad, CNN5-Trad and CNN8-Trad were used for comparing with CNN3-DCLM, CNN5-DCLM, and CNN8-DCLM respectively. Every training works out 25 epochs. Experiment results were measured at every epoch and shown in four figures, Fig.2, Fig.3, Fig.5 and Fig.4. Every figure includes nine subplots. The three subplots on the left column were shown for the experiment results on Mnist data set. These subplots on the middle column were for FashionMnist data set and these subplots on the right column were for Cifar-10 data set.

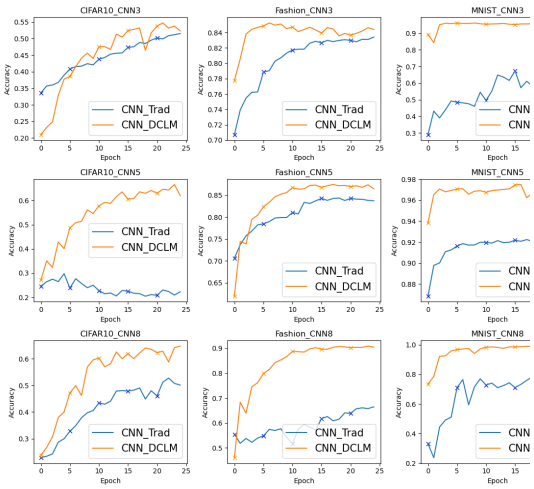

Figure 2: Accuracies of $DCLM$ and CNN trained by the proposed method

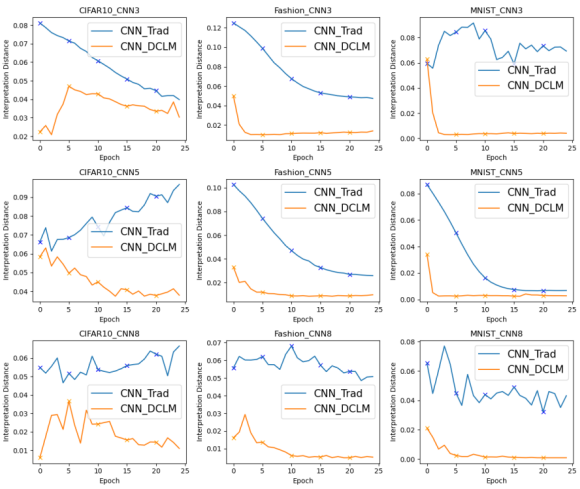

Figure 3: Interpretation distance of CNNs from the tradition training method and the proposed method respectively

Table 1: Classification accuracies and interpretation distances.

DATA SET: MNIST			
METHOD	ACCURACY	ACCURACY OF SDT OR DCLM	INTERPRETATION DISTANCE
CNN3-TRAD	0.971±0.06	—	—
CNN3-TRAD-SDT	—	0.970±0.017	0.005±0.0039
CNN3-DCLM	0.993±0.006	0.958±0.017	0.004±0.0010
CNN5-TRAD	0.993±0.006	—	—
CNN5-TRAD-SDT	—	0.968±0.017	0.005±0.0046
CNN5-DCLM	0.993±0.007	0.973±0.014	0.003±0.0010
CNN8-TRAD	0.984±0.007	—	—
CNN8-TRAD-SDT	—	0.978±0.012	0.003±0.0019
CNN8-DCLM	0.992±0.009	0.989±0.018	0.001±0.0010

DATA SET: FASHIONMNIST			
METHOD	ACCURACY	ACCURACY OF SDT OR DCLM	INTERPRETATION DISTANCE
CNN3-TRAD	0.921±0.021	—	—
CNN3-TRAD-SDT	—	0.842±0.041	0.083±0.0355
CNN3-DCLM	0.920±0.024	0.824±0.032	0.014±0.0030
CNN5-TRAD	0.923±0.024	—	—
CNN5-TRAD-SDT	—	0.792±0.041	0.064±0.0119
CNN5-DCLM	0.921±0.022	0.873±0.036	0.009±0.0020
CNN8-TRAD	0.931±0.022	—	—
CNN8-TRAD-SDT	—	0.785±0.043	0.122±0.0397
CNN8-DCLM	0.914±0.020	0.905±0.032	0.005±0.0020

DATA SET: CIFAR-10			
METHOD	ACCURACY	ACCURACY OF SDT OR DCLM	INTERPRETATION DISTANCE
CNN3-TRAD	0.681±0.040	—	—
CNN3-TRAD-SDT	—	0.538±0.057	0.044±0.0060
CNN3-DCLM	0.643±0.038	0.601±0.040	0.022±0.0030
CNN5-TRAD	0.744±0.041	—	—
CNN5-TRAD-SDT	—	0.609±0.038	0.036±0.0040
CNN5-DCLM	0.729±0.037	0.684±0.039	0.027±0.0040
CNN8-TRAD	0.754±0.035	—	—
CNN8-TRAD-SDT	—	0.674±0.035	0.040±0.0040
CNN8-DCLM	0.682±0.038	0.661±0.036	0.014±0.0030

In Fig.2, the accuracies of the DCLMs and the CNN-DCLMs of every epoch in the game process were shown. From these figures, it is obvious that accuracies of the DCLMs and these CNN-DCLMs steadily increase in the early stage. In the next stage, their accuracies tend to be stable. This reflects that the game method do not affect the improvement of the generalization performances of these DCLMs and these CNNs. We also find that the accuracy gap for the FashionMnist data set is much more than other two data sets. The main reason is that for the FashionMnist data set the logical relationships between the activation states of the input neurons and those of the output neurons of the discrimination part of the CNNs are more complex than the other data sets. Even so, the proposed method also improves the accuracies of the DCLMs and these CNN-DCLMs steadily.

Fig.3 shows the interpretation distances of the CNNs trained by the traditional method and the CNNs trained by the proposed method. As seen from these subplots, the interpretation distances of CNNs trained by the traditional method are greater than those of the other CNNs at the most of the epochs, especially by the end of the training. The results indicate that the game method can effectively improve the interpretability performance of CNNs. From these subplots in Fig.2and Fig.3, we also can see that after the 15th epoch, interpretation distances of CNNs from the proposed method tend to converge. The phenomenon indicates that the discrimination part of the CNNs can be explained by its DCLMs at every epoch after the fifteenth epoch.

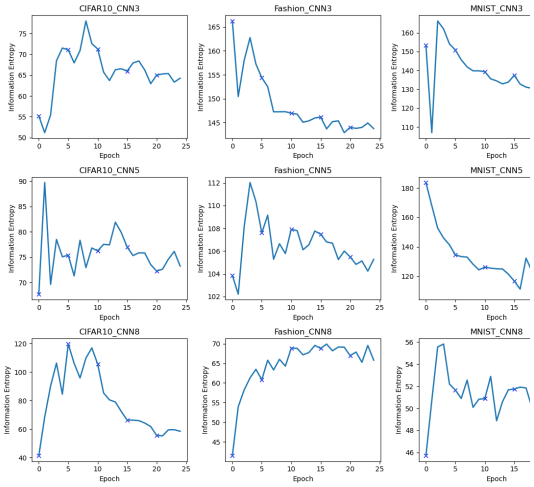

Figure 4: Information Entrop of $DCLM$

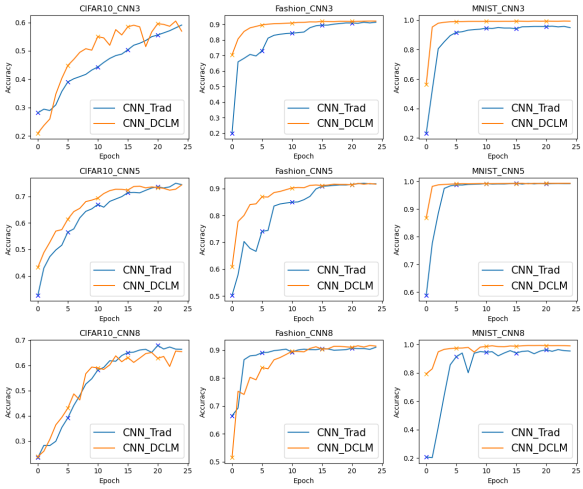

Figure 5: Accuracies on the tradition training method and the proposed method

From Fig.4, it is evident that all DCLMs from CNN3-DCLMs, CNN5-DCLMs, and CNN8-DCLMs have been found to have stable information entropies at the end of the game,which calculate the diversity of disjunction normal forms in the DCLMs obtained by the game method. On Mnist data set, the entropies finally converge between 120 and 145. On FashionMnist data set, the entropies finally converge between 125 and 136. On Cifar-10 data set, the entropies finally converge between 100 and 120. The results indicate that the game algorithm can ensure that the DCLMs converges to a stable state. The DCLMs from the proposed method have sparser disjunction normal forms than these not participating in the game process, which indicates they are more robust.

In Fig.5,the accuracies of the CNNs trained by the traditional method and the CNNs trained by the proposed method at every epoch were shown. From these subplots, it can be seen that its accuracies steadily increase in the early stage and accuracies of the CNNs trained by the proposed method are lower than the other CNNs. But in the following stage, their accuracies tend to be stable and consistent. The main reason is that in the early stage, a tradeoff problem between the generalization performance and interpretability performance of the discrimination part of a CNN inevitably reduces its generalization performance in order to increase its interpretability performance. But the proposed game method can effectively reduce the gap between two performances. This reflects that the proposed method is effective for the tradeoff problem.

7 Conclusion

The performance of the proposed method was demonstrated by experiments on benchmark data sets. The proposed method showed prominent advantages over traditional learning algorithm on CNN for improving the generalization performance and the interpretability performance of the discrimination part of the CNN.

In practical engineering, the proposed method may provide a new learning paradigm. The method can not only predict an relatively accurate result for new input data but also provide a reasonable causal interpretation for the prediction of the discrimination part of a CNN. We suppose that it can solve the black box problem in the discrimination part. We believe that the proposed method provide a way to understand the discrimination part.

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Algorithm 2 Construction Logic Net(Its time complexity is $O(scn)$ where s is number of iterations of MAX-SAT,c is number of sample categories and n is number of samples.)

Input: feature maps X, weight vectors F,y_{nn},batch size m.
Initialize L_a ,$y_{dclm} = y_{nn}$,$c = 0$.
repeat
  for $i = 1$ **to** sizeof(X) **do**
    for $j = first(i)$ **to** $final(i)$ **do**
      $IP[i][j]$ = Cosine similarity($X[i]$,$F[j]$).
      $lamda[i][j] =| X[i] || F[j] |$
    end for
  end for
  for $i = 1$ **to** sizeof(y_m) **do**
    $NewL_a[i] = maxsat(IP[][j], lamda[][j], y_{dclm}[i])$.
    $L_a[i] = NewL_a[i] \cap L_a[i]$.
  end for
  $c++$.
until $c == m$ is $true$

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A Appendix: A Maximum A Posterior(MAP) algorithm for extracting a DCLM

A Maximum A Posterior(MAP) algorithm on the Maximum Satisfiability Problem (MAX-SAT) for extracting an DCLM is as follows.

B Appendix: Sufficient and necessary condition for consistent convergence of the generalization performance and the interpretability performance

If the minimization of loss function of CNN can guarantee the minimum of its interpretability performance, learning algorithm of the CNN can improve its interpretability performance. If not, the tradeoff problem between the generalization performance and the interpretability performance will exit. For proving the existence of the problem, we focus on a neuron of CNN. From the foot

we may judge of Hercules. If an input channel $f(x)$ of the neuron is seen as a kernel function $K(x, w)$(w is weight vector including a bias of the neuron), it will span a kernel Hilbert space $\mathcal{H}_K = \{f(x) \in \mathcal{L}_\nu^2(X) \mid f(x) = K(x, w) = \sum_{k=1}^{\infty} a_k \phi_k(x)\}$ for the neuron. $\mathcal{H}_K$ can be regarded as a linear function set on $\mathcal{L}_\nu^2(X)$. It is a solution space of the neuron. The necessary and sufficient conditions for consistent convergence between the generalization performance and the interpretability performance are discussed below in $\mathcal{L}_\nu^2(X)$ based on the following lemmas.

Lemma 1. Continuous linear functional set on a separable Hilbert space X is nowhere dense in a square integrable function space $\mathcal{L}_\nu^2(X)$.

Lemma 2. Continuous nonlinear functional set of the separable Hilbert space X is everywhere dense in $\mathcal{L}_\nu^2(X)$.

When the optimal input channel $f^*(x)$ approximate a linear functional in $\mathcal{H}_K$ while the optimal interpretable model $P^*(x)$ don't approximate any linear functional or do not exist in $\mathcal{H}_K$, traditional training process cannot guarantee $f(x)$ approximates $P^*(x)$ according Lemma 1. From Lemma 2, the approximate will cannot converge until $P^*(x)$ approximate $f^*(x)$. Here, if we define the approximation as the similarity between the shapes of function curve of $f^*(x)$ and $P^*(x)$, the sufficient and necessary condition for the consistent convergence of the two performances is $\phi_d(P^*, f^*) = 0$. For the discrimination part of the CNN, the sufficient and necessary condition is still true. However because of data bias and noisy data in training data set, the condition is difficult to to be ensured in the majority of engineering applications. The tradeoff problem always exists between the two performances of the discrimination part.

According to the above conclusion, in order to completely solve the tradeoff problem, the $\phi_d(P^*, f^*)$(Here $f^*(x)$ is the optimal discrimination part) should be reduced. However P^* and $f^*(x)$ are unknown. Therefore, in training process, extracting a interpretable model $P(x)$ from a discrimination part $f(x)$ and then iteratively reducing $\phi_d(P, f)$ may be a convenient method.