Ready, Set, Plan! Planning to Goal Sets Using Generalized Bayesian Inference

Abstract: Many robotic tasks can have multiple and diverse solutions and, as such, are naturally expressed as goal sets. Examples include navigating to a room, finding a feasible placement location for an object, or opening a drawer enough to reach inside. Using a goal set as a planning objective requires that a model for the objective be explicitly given by the user. However, some goals are intractable to model, leading to uncertainty over the goal (e.g. stable grasping of an object). In this work, we propose a technique for planning directly to a set of sampled goal configurations. We formulate a planning as inference problem with a novel goal likelihood evaluated against the goal samples. To handle the intractable goal likelihood, we employ generalized Bayesian inference to approximate the trajectory distribution. The result is a fully differentiable cost which generalizes across a diverse range of goal set objectives for which samples can be obtained. We show that by considering all goal samples throughout the planning process, our method reliably finds plans on manipulation and navigation problems where heuristic approaches fail.

Keywords: Planning as inference, Variational inference, Nonparametric learning

1 Introduction

In order to accomplish diverse tasks in unstructured environments, robots need the ability to generate motion plans to different user-specified goals. Such goals are often naturally expressed as *goal regions* in the workspace, rather than single point goals (e.g. navigate to a certain room, place an object on a shelf). Traditional trajectory planning techniques require the goal regions to be explicitly specified as planning objectives. This specification is challenging in cases where the goal region is *uncertain*. Examples of uncertain goal regions include those which are intractable to model explicitly, such as stable grasp poses across a diverse objects [1, 2] or user preferences [3, 4].

We seek to solve planning problems under goal region uncertainty given only a fixed set of goal demonstrations. The key insight of our approach is to model these demonstrations as samples coming from an underlying goal distribution. Our *goal set planner* plans directly to these goal samples, resulting in a generalizable, data-driven planning objective which can be employed across goal regions, eliminating the need to explicitly model the goal. Planning to goal distributions has been formulated as a flexible representation for a number of objectives [5], but requires a fully specified parametric goal distribution. Our approach instead treats the goal as an *implicit distribution*, from which we can obtain samples (e.g. successful grasp poses [6]) but which cannot be evaluated explicitly. We formulate this problem as inference-based planning [7, 8, 9], and seek a distribution of trajectories which terminate in high-density regions of the goal distribution (see Figure 1). Due to the complex, multi-modal nature of the planning problem, we consider particle-based trajectory distributions and perform inference using Stein Variational Gradient Descent [10, 11]. In contrast to previous data-driven approaches to intractable planning objectives [1, 2, 3, 4, 6, 12, 13], we require no upfront learning before planning begins.

Goal Samples   **Simulated Rollouts**   **Trajectory Execution**

Figure 1: Our *goal set planner* considers a discrete set of goal samples as the planning objective (top left). By considering multiple possible goals within the trajectory optimization process (bottom left), we can handle challenging environments with clutter and dynamic obstacles.

Primary contributions. We formulate the problem of goal set planning as *generalized Bayesian inference* (GBI), an emerging statistical technique which enables inference for intractable likelihoods. GBI refers to a class of problems in which the likelihood function is specified through a generic loss, typically a divergence between two distributions evaluated against observations [14, 15, 16]. We demonstrate that divergences can be computed over implicit goal distributions, which allows us to leverage a loss function that quantifies the divergence between the terminal state samples and the goal samples to infer the posterior trajectory distribution [17]. We then propose a Stein variational inference methodology to solve the problem as a sequence of gradient updates of a set of trajectories.

Secondary contributions. We present a number of potential loss functions which can be applied within this framework. We additionally compare to several previously proposed heuristic approaches [5, 18]. We demonstrate the applicability of our approach in planar navigation and in a high-dimensional grasping task. By considering the full set of goal samples throughout the planning process, our planner reliably finds reachable grasps in scenarios where heuristic approaches fail to reach their selected pose from the goal set. Additional demos and data are available at https://sites.google.com/view/goal-sets.

2 Related Work

Trajectory planning to a goal can be stated as an optimization problem with the objective of finding a minimum cost path to a point in the robot workspace and can be efficiently solved using differentiable costs and dynamics [19, 20, 21]. Casting planning as probabilistic inference has become a popular technique for achieving robust control under uncertainty [22, 11, 23, 9, 24]. One popular approach is to employ a nonparametric, particle-based representation of the trajectory distribution, iteratively updating the posterior through importance sampling [23, 9]. Other methods use variational inference to efficiently infer the trajectory distribution [22, 11, 25].

Most related to our work is that of Conkey and Hermans, who formulate trajectory planning to goals defined as a probability distribution [5]. Our proposed approach also considers the goal to be a distribution, but we represent the goal as an implicit (sampled) distribution [17] eliminating the need for the distribution to be explicitly specified by the user. The problem of planning to a set of goals has been considered by the manipulation community for robotic grasping. A popular approach for grasp planning to a goal set has been to formulate trajectory optimization and grasp selection as independent steps [26, 27, 18]. These methods first select a single goal from the set prior to planning based on a scoring metric, then treat the selected sample as a point goal. Similar approaches have been used to heuristically guide sampling-[28] and graph-based [29] motion planners to goal samples from sets in other contexts. Another approach is to jointly perform trajectory optimization and grasp selection. Dragan et al. [30] add a manually defined goal set constraint to the problem of trajectory optimization. Wang et al. [31] perform grasp selection and refinement online by estimating the grasp distribution. These methods require domain-specific information about the grasping problem and do not generalize across planning problems. Data-driven approaches to represent intractable planning

objecteives either employ high-fidelity simulation to measure success (e.g of grasps) [1, 2, 6, 13], or demonstrations to represent user preferences [3, 4, 12]. In these techniques, an upfront computational cost is required before planning begins, and must be performed again whenever environmental conditions, such as observability or dynamic environments, necessitate replanning.

3 Goal Sample Set Planning as Inference

Given a robot with state $x_t \in \mathcal{X}$ at time t, we consider the problem of planning to a *goal set* within the state space, $\mathcal{G} \subset \mathcal{X}$. We consider the case of arbitrarily complex goal regions $\mathcal{G}$ that do not admit explicit analytic specification. Instead we can obtain a set of valid goal samples, $\tilde{G} = \{x_g^{(i)}\}_{i=1:N}$, $x_g^{(i)} \in \mathcal{G}$. Examples of this type of goal include data labelled by simulation or human demonstration.

We seek a policy $u_t = \pi(x_t)$ such that the terminal state after a time horizon T lies within the goal region, $x_T \in \mathcal{G}$, given only goal samples $\tilde{G}$. The robot trajectory is defined as a sequence of states and actions, $\tau = \{(x_t, u_t)\}_{t=0:T}$. We seek the minimum cost trajectory, τ^*, which solves:

$$\tau^* = \arg\min_{\tau} \ C\left(x_T; \tilde{G}\right) + \sum_{t=0}^{T-1} c_t(x_t, u_t; z), \tag{1}$$

where z denotes the environmental observation. The running cost, $c_t(x_t, u_t; z)$, is chosen based on the domain (e.g. quadratic cost, obstacle cost, etc.). Our primary contribution is the formulation of the terminal cost when the goal region is represented only by a set of samples, $\tilde{G}$.

The planning objective in Eq. 1 can be restated in terms of probabilistic inference using the formalism of planning as inference [7, 8, 22, 9]. In planning as variational inference, we seek to find a distribution over trajectories $q(\tau) \in \mathcal{Q}$ that minimizes the divergence with the posterior distribution of trajectories, $p_O(\tau \mid z)$, defined from the cost in Eq. (1):

$$q^*(\tau) = \arg\min_{q \in \mathcal{Q}} D_{KL}\left(q(\tau) \,\|\, p_O(\tau \mid z)\right). \tag{2}$$

We can factorize the posterior over trajectories, τ, given an environment observation, z, and a set of goal samples, $\tilde{G} = \{x_g^{(i)}\}_{i=1:N}$, as:

$$p_O(\tau \mid z, \tilde{G}) \propto p_{\text{goal}}(\tilde{G} \mid x_T) \prod_{t=1}^{T-1} p(z \mid x_t) \prod_{t=1}^{T-1} p(x_{t+1} \mid x_t, u_t) \tag{3}$$

$$= p_{\text{goal}}(\tilde{G} \mid x_T) p_O(z \mid \tau') p(\tau') \tag{4}$$

where τ' denotes the portion of the trajectory up to time $T-1$, $\tau' = \{(x_t, u_t)\}_{t=0:T-1}$, x_T denotes the terminal state of the trajectory, and $p_{\text{goal}}(\tilde{G} \mid x_T)$ is the goal likelihood given the implicit goal distribution $\tilde{G}$. We use the fact that $p_O(\tau' \mid z) \propto p(z \mid \tau') p(\tau')$ for optimal trajectories. We model the likelihood $p_O(z \mid \tau')$ in terms of the trajectory cost yielding:

$$p_O(z \mid \tau') \approx \frac{1}{Z} \exp\left(-\alpha \sum_{t=0}^{T-1} c_t(x_t, u_t; z)\right), \tag{5}$$

where Z is a normalization term. The likelihood $p_{\text{goal}}(\tilde{G} \mid x_T)$ in Eq. (4) is intractable due to the implicit goal distribution. The following section addresses the key technical question of this work: how can we perform inference on the intractable posterior arising from an implicit goal distribution?

4 Planning to Implicit Goal Distributions as Generalized Bayesian Inference

We propose applying generalized Bayesian inference to directly approximate the solution of Eq. (4). While one could fit a parametric model of $\hat{p}_{\text{goal}}(\mathcal{G} \mid x_T)$ to the sampled data $\tilde{G}$, we seek a generic objective for planning directly to the implicit distribution that avoids task-specific pre-processing.

Figure 2: Our SVGD-based planner takes as input a set of goal samples, $\tilde{G}$, an environmental observation, z, and an initial state, x_0 and iteratively updates a set of trajectory particles, τ. At each iteration, the particles are rolled out using a simulator. Each rollout is then evaluated in terms of its running cost, c. A goal loss , $\mathcal{L}_{\text{goal}}$, computes the discrepancy between the terminal points of the trajectory distribution and the goal set. SVGD updates the trajectory particles using the gradients of the combined running and goal losses.

We now give a brief overview of generalized Bayesian inference and Stein variational inference before explaining how we combine them to solve the goal set planning problem. Figure 2 visualizes our approach to planning as generalized Bayesian inference using SVGD.

Generalized Bayesian Inference. Generalized Bayesian inference (GBI) accounts for intractable inference problems arising from evaluating a loss function instead of a likelihood that factorizes over each observation [16]. Given a posterior $p(x \mid \mathbb{Y})$, where x are arbitrary random variables and $\mathbb{Y} = \{y^{(i)}\}_{i=1:N}$ is a set of N observed data points, GBI approximates the posterior as follows:

$$p_{\mathcal{L}}(x \mid \mathbb{Y}) \propto p(x) \exp\left(-\beta \mathcal{L}(x, \mathbb{Y})\right) \tag{6}$$

where $\mathcal{L}(x, \mathbb{Y})$ defines a loss between the random variables and observed data. Typically, the loss measures divergence (e.g. KL) between the data and query variables. The posterior $p_{\mathcal{L}}(x \mid \mathbb{Y})$ recovers the conventional Bayesian posterior when $\beta = 1$ and $\mathcal{L}(x, \mathbb{Y}) = -\sum_{i=1}^{N} \log p(y^{(i)}|x)$. We can compute the approximate posterior by solving the following variational optimization problem:

$$q_{\mathcal{L}}(x \mid \mathbb{Y}) = \arg\min_{q \in \mathcal{Q}} \beta \, \mathbb{E}_{x \sim q}\left[\mathcal{L}(x, \mathbb{Y})\right] + D_{KL}\Big(q(x) \,\|\, p(x)\Big) \tag{7}$$

Stein Variational Inference. We build on work that approximately solves Eq. (2) using Stein variational inference [11, 32]. Stein variational inference represents the candidate distribution $q(\tau)$ nonparametrically by a set of particles, each representing a trajectory, $\{\tau^{(i)}\}_{i=1:M}$. Stein variational gradient descent (SVGD) [10] employs gradient-based optimization over the particle set to minimize the kernelized Stein discrepancy [33] between the true and the approximate posterior. SVGD can solve high-dimensional planning problems involving complex, multi-modal posteriors [11, 32]. SVGD is an iterative algorithm which applies the following update to each particle i at iteration k:

$$\tau_k^{(i)} \leftarrow \tau_{k-1}^{(i)} + \gamma \phi(\tau_{k-1}^{(i)}) \tag{8}$$

$$\phi(\tau) = \frac{1}{M} \sum_{j=1}^{M} k(\tau^{(j)}, \tau) \nabla_{\tau^{(j)}} \log p_O(\tau^{(j)} \mid z) + \nabla_{\tau^{(j)}} k(\tau^{(j)}, \tau) \tag{9}$$

where $k(\tau^{(j)}, \tau)$ is a kernel function between trajectories. We can interpret the first term inside the summation of Eq. (9) as an attractive force that moves particles according to the gradient of the log-posterior, while the second term acts as a repulsive term keeping particles from collapsing to a single point estimate. Thus SVGD leverages parallel gradient-based optimization to generate a diverse set of samples more efficiently than MCMC samplers [9].

Terminal Losses for Goal Sets. We leverage results from the GBI literature which approximate the optimization problem in Eq. (7) using a divergence metric between two implicit distributions [17]. We define the implicit terminal distribution as $\tilde{X}_T = \{x_T^{(i)}\}_{i=1:M}$ and solve:

$$q_{\mathcal{L}}(x_T \mid \tilde{G}) := \arg\min_{q \in \mathcal{Q}} \mathcal{L}_{\text{goal}}(\tilde{X}_T, \tilde{G}) \tag{10}$$

Figure 3: (Left) Pass rate for different distance thresholds between the final state and the nearest goal sample. Dashed lines represent baseline methods. (Right) Path length for each terminal cost for the planar navigation task. The bar heights indicate average values and the points indicate individual samples. Only successful trajectories are included, where success is defined as getting to within 0.15 meter from any sample within the goal set. The numbers on the bars indicate the number of successful runs out of 50 in each category.

The loss function $\mathcal{L}_{\text{goal}}(\tilde{X}_T, \tilde{G})$ measures the discrepancy between the terminal state distribution and the distribution of goal states, represented implicitly by samples $\tilde{X}_T$ and $\tilde{G}$ respectively.

We note that the loss defined in Eq. (10) provides a measure over the set of terminal states, rather than an individual likelihood per trajectory particle required by Eq. (4). A key insight of our approach is to solve the resulting planning as inference task with SVGD, which employs the gradient of the particle likelihood, without the need to explicitly evaluate the goal loss per particle. We define the gradient in Eq. (9) as follows, combining the results from Eq. (4) and Eq. (10):

$$\nabla_{\tau^{(j)}} \log p_O(\tau^{(j)} \mid z) = \nabla_{\tau^{(j)}} \log p_O(z \mid \tau^{(j)}) + \nabla_{\tau^{(j)}} \log p(\tau^{(j)}) - \beta \nabla_{\tau^{(j)}} \mathcal{L}_{\text{goal}}(\tilde{X}_T, \tilde{G}) \quad (11)$$

The terminal loss must be differentiable with respect to the trajectory samples and efficient to compute so it can be used in a control loop. We consider the following *two sample* tests which meet this criteria: The kernel MMD [34], a KL divergence approximation based on classification [35, 36], a smooth k-Nearest Neighbor test [37], and the energy statistic [38]. Details of the derivations of these works can be found in the supplemental material.

5 Experimental Results

We evaluate our method in a simulated 2D planar navigation problem and manipulation experiments both in simulation and on a physical robot. Video results of the experiments are available in the supplementary material.

Baselines. Each baseline involves inference using SVGD [10, 11] with the same running costs, but with different heuristic terminal costs based on previously published work, as defined below:

– *Closest Point:* The nearest goal sample to the start configuration is selected as the goal. The terminal cost is the squared Euclidean distance to this point. A *dynamic* version of this cost is obtained by recomputing the closest point at the beginning of each planning timestep [18].

– *True Goal Distribution:* Given the true goal distribution, the objective is formulated as maximizing the density of the goal distribution at the terminal point [5]. This method is only applicable to the 2D planar navigation task when the distribution is known.

– *Mixture Model:* We define a goal likelihood with a Gaussian mixture model obtained by placing each component centered on each goal sample, with a fixed covariance–akin to a kernel density estimator. The goal loss is then the cross entropy over the mixture model which is equivalent to maximizing the likelihood [5].

Planar Navigation. We implement a planar navigation task in a 2D environment and a known goal distribution. We define goal distributions over 2D position for each scene using a parametric distribution (Gaussian, mixture of Gaussians, or uniform). We give the planner a random set of goal samples drawn from the distribution. We execute 10 runs per method per scene across 5 scenes.

Figure 4: Qualitative results for various simulation environments for the goal set planner with MMD terminal loss. Each sequence represents intermediate trajectory distributions at various timesteps followed by the final trajectory. The red points represent the goal samples and the orange contours represent the true goal distribution. The velocity (m/s) is shown in the bar to the right of the trajectories.

Figure 5: Selected results for the Cross Entropy (left) and Closest Point (right) baselines. The final trajectory is shown in blue, while intermediate trajectory distributions are purple. The true distribution is shown in orange contours. The red 'x' denotes the closest point. Both baselines fall into local minima failing to reach the goal.

To evaluate the goal set planner, we measure the Euclidean distance between the terminal state in the trajectory and the nearest goal sample for each run. The pass rate for various distance thresholds is visualized in Figure 3. The closest point baselines have a high failure rate on the challenging environments, particularly with collisions near the goal (see Figure 5). In the environment on the second row in Figure 4, the closest point chosen is in collision with the environment. While it would be trivial to check for collisions while finding the closest point in the planar environment, we avoid this additional domain-dependent pre-processing step for this experiment to better replicate environments in which collisions and reachability are expensive or intractable to compute. The cross entropy baseline performs close to the set-based methods with respect to pass rate, but is more prone to falling into local minima. An example of this case is shown in Figure 5.

We also show the total Euclidean path length for each method in Figure 3. We exclude paths for which the terminal state is not within 15 cm of any goal sample. The methods achieve similar path lengths, with the closest point baselines consistently resulting in slightly shorter paths. This is unsurprising given the distance-based goal selection bias, but comes at the cost of being less robust to different environments. Figure 4 shows example trajectory distributions for the goal set planner using the MMD cost. Both the Smooth k-NN and the Kernel MMD terminal losses display moment-matching behaviour, which we posit allows them to be more robust to challenging environments. We posit that the kernel embedding in the MMD loss enables more flexible representation of the goal region compared to the other losses. We select MMD for all manipulation experiments.

Distribution matching: We hypothesize that our goal set planning achieves higher success rate in challenging environments due to its ability to better approximate the underlying goal distribution. To verify this, we measure the Chamfer distance between the terminal states in the trajectory distribution and the goal samples. The results are shown in Fig. 6. The cross entropy baseline, using the true goal distribution, achieves lower Chamfer distance than the other baselines, but has higher distance than the goal set planners due to its mode-seeking behavior. Kernel MMD and smooth k-NN terminal set losses perform better on this metric than the energy statistic and the KL approximation.

Robotic Grasping. The grasping task aims to demonstrate the ability of the goal set planner to reason about high-dimensional goal distributions which are intractable to model. We assume that the environment and target object are fully observed. Experiments are performed using the Franka Panda

Figure 6: Chamfer Distance at each iteration for different terminal costs. (left) Comparison between the baseline methods and our goal set planner with MMD terminal loss. (right) Comparison between different terminal costs. The best performing baseline (Cross Entropy) is shown for reference.

Figure 7: Pass rate data for the grasping experiments: the Euclidean distance (left) and the L2-norm of the rotation matrix (right) between the final end-effector orientation and the goal. For the Mixture Model and Goal Set planners, the distance is measured between the nearest goal point.

manipulator in the Isaac Gym simulator [39]. We use the STORM library for GPU-accelerated, fully differentiable dynamics and costs evaluation [40]. The goal set is represented by $N = 100$ samples of valid grasp configurations drawn from the Acronym dataset [6]. The goal set planner is used to plan to a pre-grasp pose to offset grasp locations in an MPC framework over a fixed horizon. After convergence, we use inverse kinematics (IK) to move to the nearest grasp sample, and then lift the object. We evaluate over 5 runs across 10 scenes containing 8 different objects and 5 environments.

Grasp success results are shown in Table 1. We observe that a primary source of failure is due to failure in the IK grasp planner stage. To better understand the performance of the set planner, we measure the planning success rate of our planner to the pre-grasp pose, computed as the distance from the terminal end-effector pose to the nearest goal sample. We present the pass rate for each method in Fig 7. The goal set planner achieves a higher pass rate at thresholds over approximately 4 cm. The closest point method may select goal points which are in collision in some of the example scenes, increasing the distance error, which also accounts for the discrepancy in grasp success. The closest point method fails all 5 runs on this scene, while the goal set planner succeeds for 4 out of 5 runs. The rotation error is lower for closest point because it matches the orientation of the goal consistently, including for unreachable poses. We posit that the mixture model poorly approximates the true goal distribution explaining its failure.

Table 1: Grasp success rate

	Success (%)
Closest Pt.	50
Mixture Model	40
Goal Set Planner (MMD)	54

Robot Demonstration. We demonstrate that our simulated manipulation experiments are transferable to the real robot, under the assumptions of known object and obstacle locations and geometries,

Figure 8: Execution runs of the goal set planner on the real robot platform in an environment with no collisions (left three) and with collisions (right three). Trajectories are planned offline and executed on the robot.

Figure 9: An example sequence of the sequential placement task. As the robot places subsequent mugs, it automatically adjusts its trajectory distribution to avoid obstacles (2, 3). The final configuration is shown in (4).

through a robotic grasping demonstration. Figure 8 shows two successful grasping runs of the planner, one in an environment without obstacles and one with obstacles.

Placement. We posit that our planner is more effective than point selection heuristics at dealing with changing environments due to its ability to consider multiple possible goal regions within the trajectory distribution. To examine this claim, we design a qualitative demonstration to sequentially place mugs on a surface, such that each subsequent placement operation requires the consideration of a new obstacle in the scene. This is inspired by approaches which express valid placement as discrete points on a surface [41]. We generate the point goal set once at the beginning of the task, and do not update it at subsequent iterations. We perform a qualitative demonstration for this task using the MMD terminal loss.

Figure 9 shows the trajectory distribution of the set planner at various stages of the task. The trajectories converge to modes which avoid the mugs which have been placed on the table. The robot finds a free space to put the next mug on the table dynamically without any modification to the cost or goal model. Qualitatively, we observe failures when the planner oscillates between modes. In these cases, the planner times out and the placement might fail. This could be mitigated by detecting the trajectory modes and selecting one, rather than performing trajectory selection at each timestep.

6 Discussion & Conclusion

In this work, we present a formulation for a goal set planner, which considers the planning problem in which an uncertain goal region is represented as a set of samples. The planner is generalizable across goals which can be represented by implicit distributions, without the need for domain-specific, user-defined heuristics. We find that independent of the specific loss used, the GBI formulation outperforms previously proposed approaches for planning to goal sets and distributions. Our method outperforms all baselines on a 2D navigation problem and yields improved plan-to-grasp success on a robotic manipulation platform.

In order to remain true to the general planning problem, our method does not consider domain-specific techniques commonly used for the experiments considered. Our grasping results would likely be improved by integration of pre-planning steps for checking reachability or collisions [42], or pre-grasping refinement of the final pose [31]. We hypothesize that combining these approaches with goal set planning for the application of grasping would improve robustness.

Limitations: Maintaining modes in the trajectory distribution can lend added robustness in challenging environments, but it also raises challenges in selecting a single trajectory to execute. We attribute the larger errors in the final pre-grasp pose of our method to the challenge of detecting convergence given the multi-modality of the goal set planner. Furthermore, the terminal point is not guaranteed to be close to one of the points in the set. This could be mitigated by switching to a point-based cost once close to the goal set. To deploy our planner to novel environments, particularly for real-world applications, we need to investigate how to deal explicitly with perceptual uncertainty.

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